USC Viterbi School of Engineering

Particle Filters

CSCI 545 Introduction to Robotics
Instructor: Stefanos Nikolaidis

Test for a Mobile Robot Localization

Algorithm : MCL
Sensor : USAT(Ultra Sonic)

Chosun Univ. Instrument & Control
Autonomous Robot Control systoem Lab
Date : 2009. 03

Motivation

- What if the state distribution is not Gaussian?

- Performance of most conventional filters drops when the dynamics are non-linear

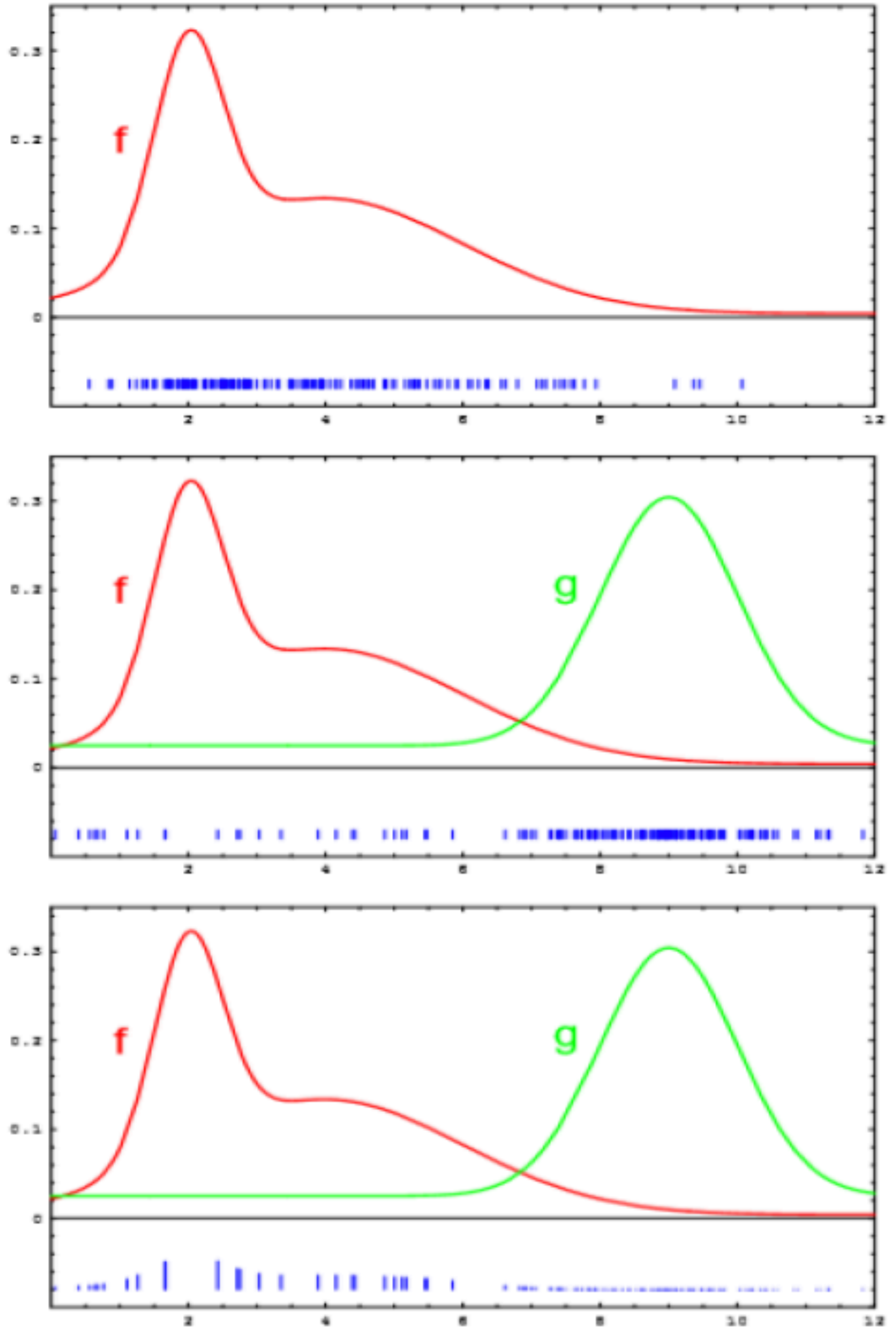
- Key Idea: Particle Filters represent the distribution with a number of samples

Importance Sampling

- Assume that we have a distribution $f(x)$ that is difficuklt to draw samples from

- We can specify a distribution $g(x)$, draw samples from g and multiply them with $\frac{f(x)}{g(x)}$

Example



Algorithm

Algorithm Particle_filter($\mathcal{X}_{t-1}, u_t, z_t$)**:**

$\bar{\mathcal{X}}_t = \mathcal{X}_t = \emptyset$

for $m = 1$ to M do

sample $x_t^{[m]} \sim p(x_t \mid u_t, x_{t-1}^{[m]})$

$w_t^{[m]} = p(z_t \mid x_t^{[m]})$

$\bar{\mathcal{X}}_t = \bar{\mathcal{X}}_t + \langle x_t^{[m]}, w_t^{[m]} \rangle$

endfor

for $m = 1$ to M do

draw i with probability $\propto w_t^{[i]}$

add $x_t^{[i]}$ to $\mathcal{X}_t$

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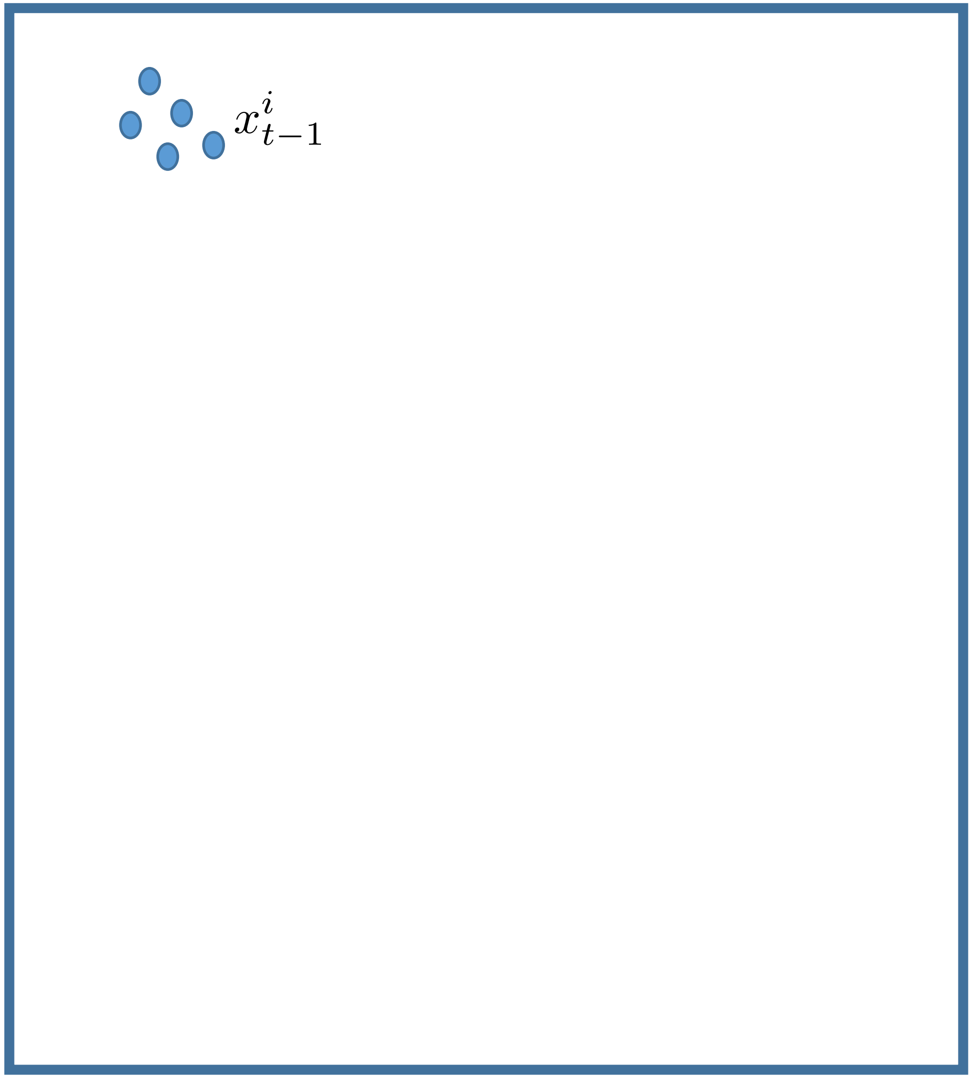
Initial Distribution



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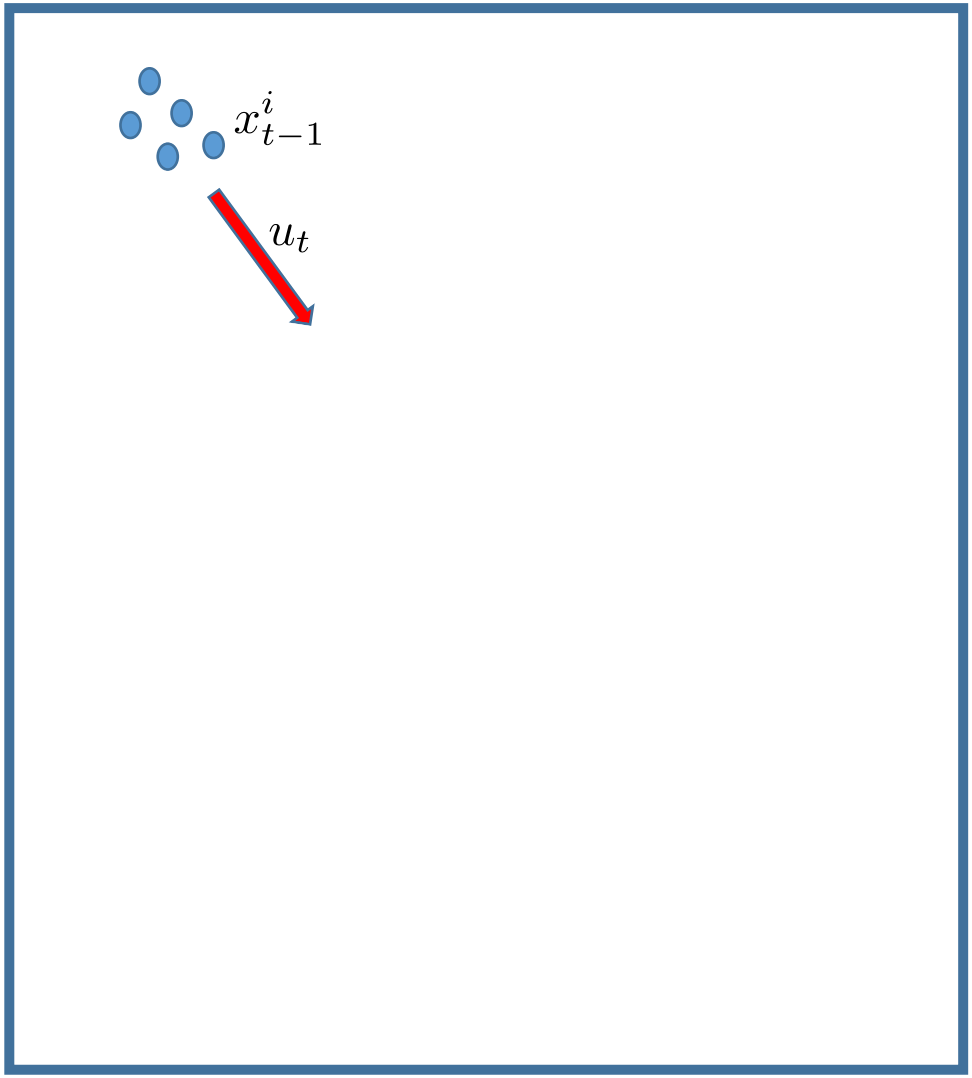
Motion Model



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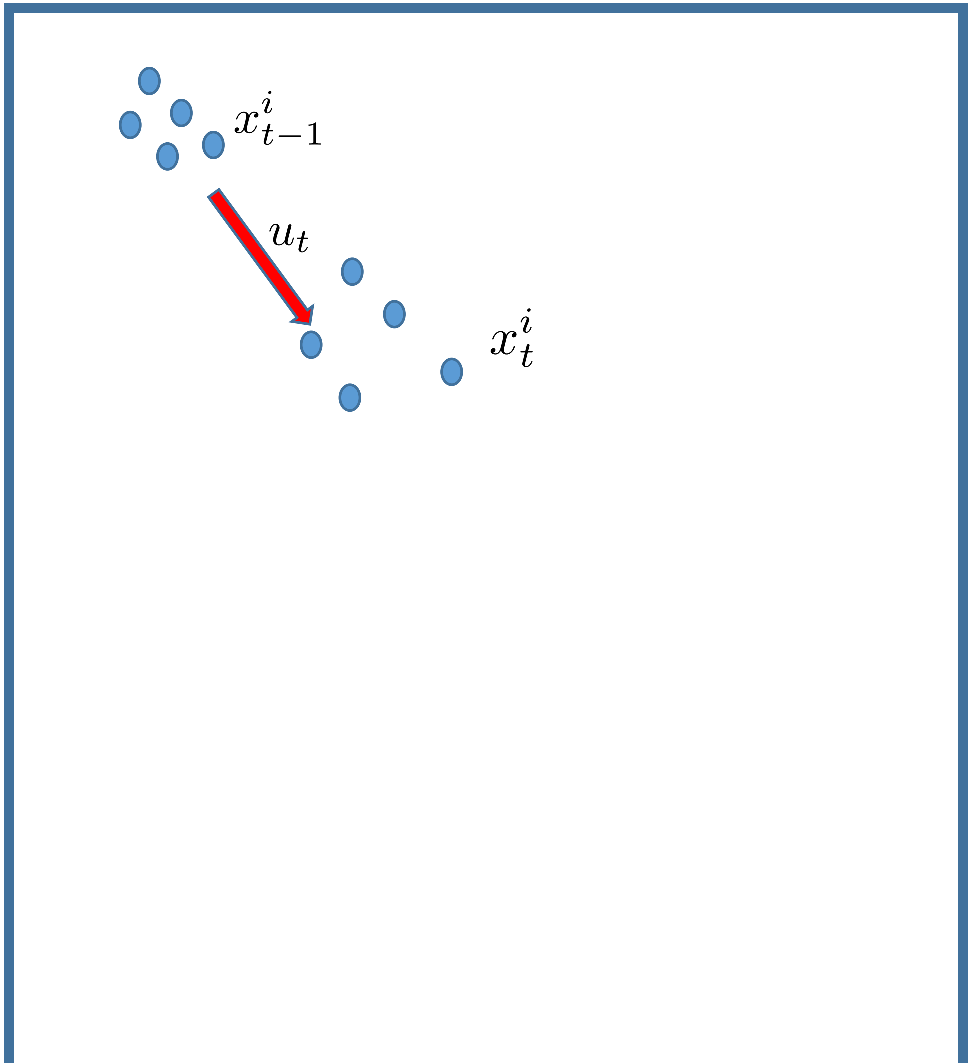

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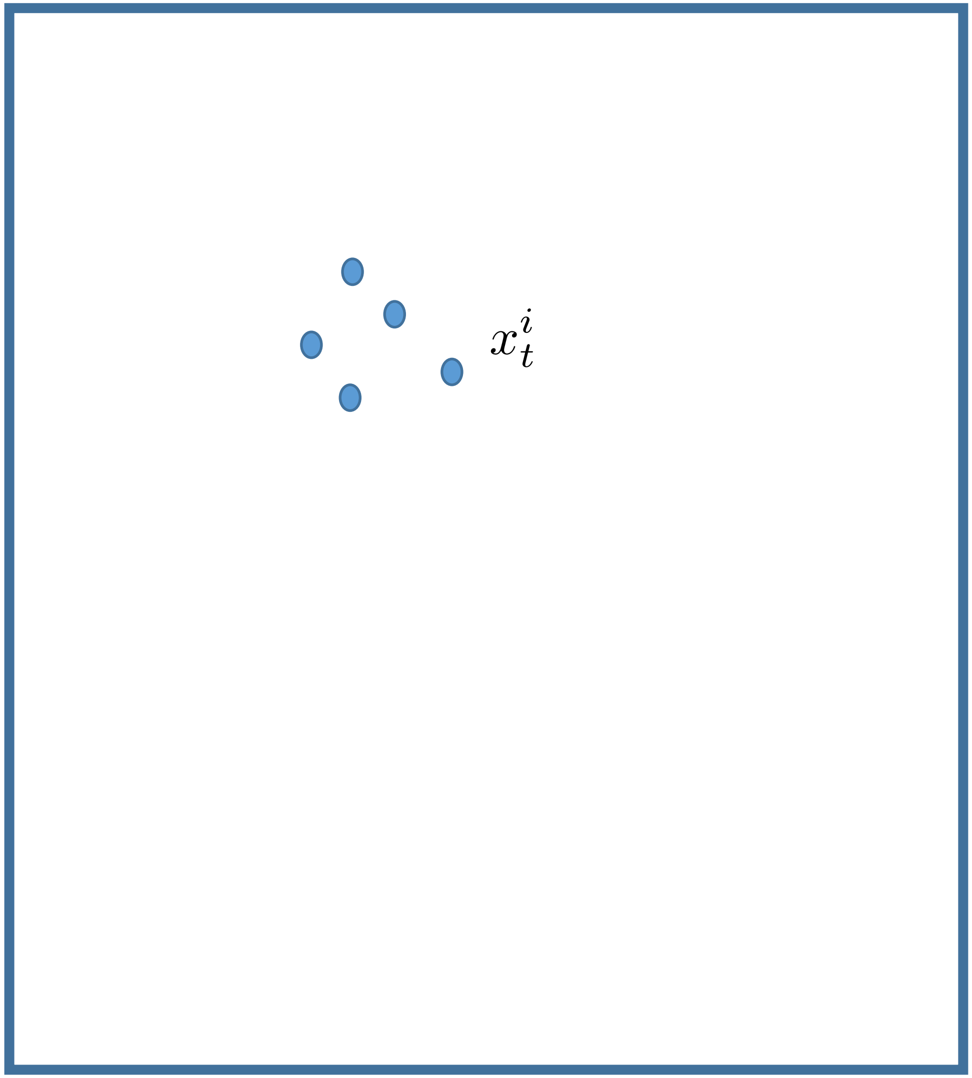

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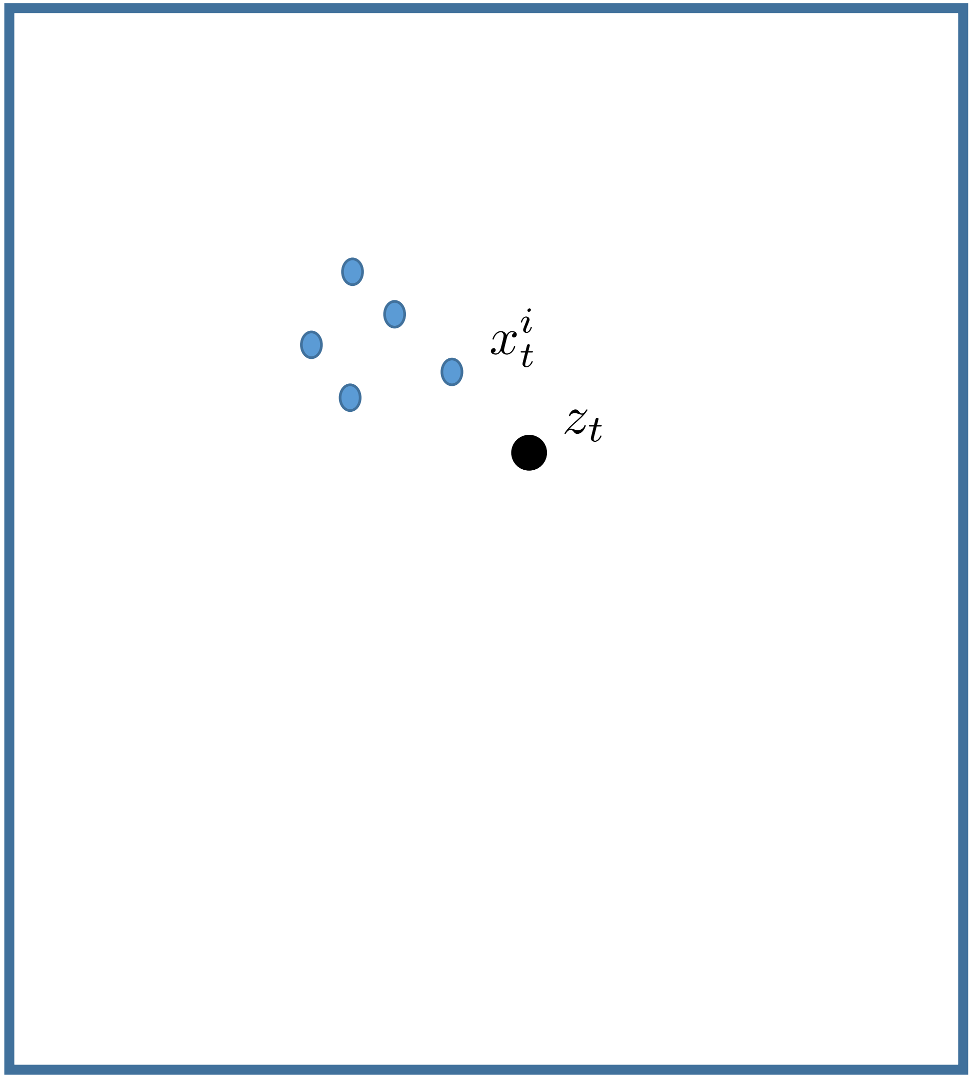
Observation



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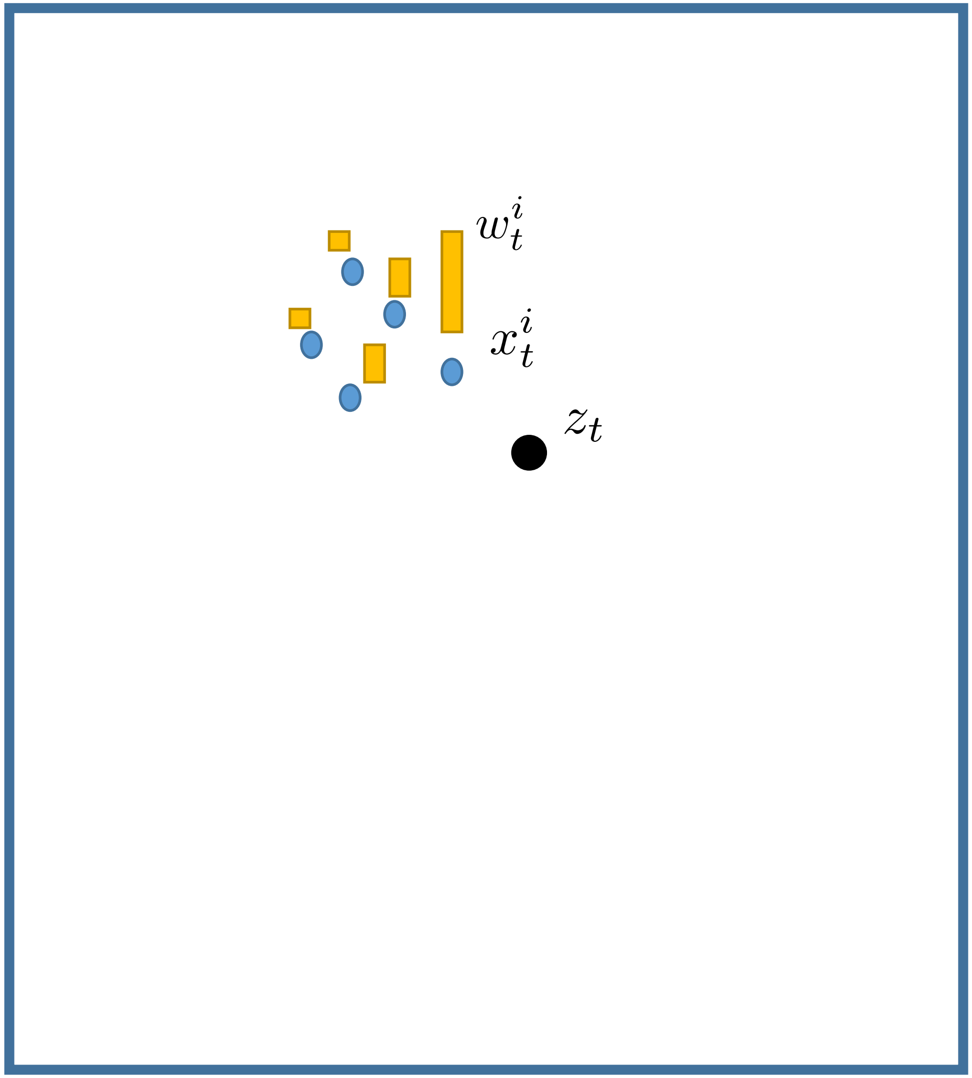
Weight Assignment



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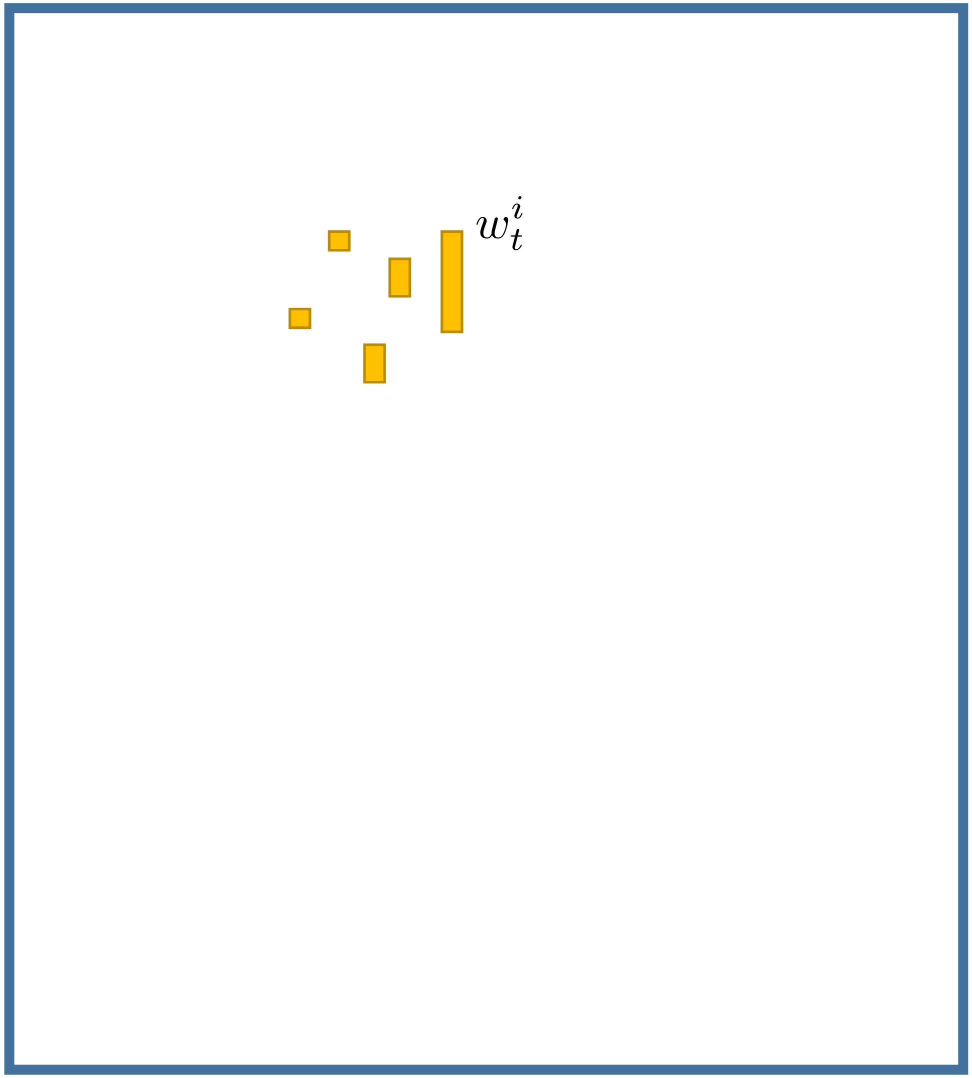
Resampling



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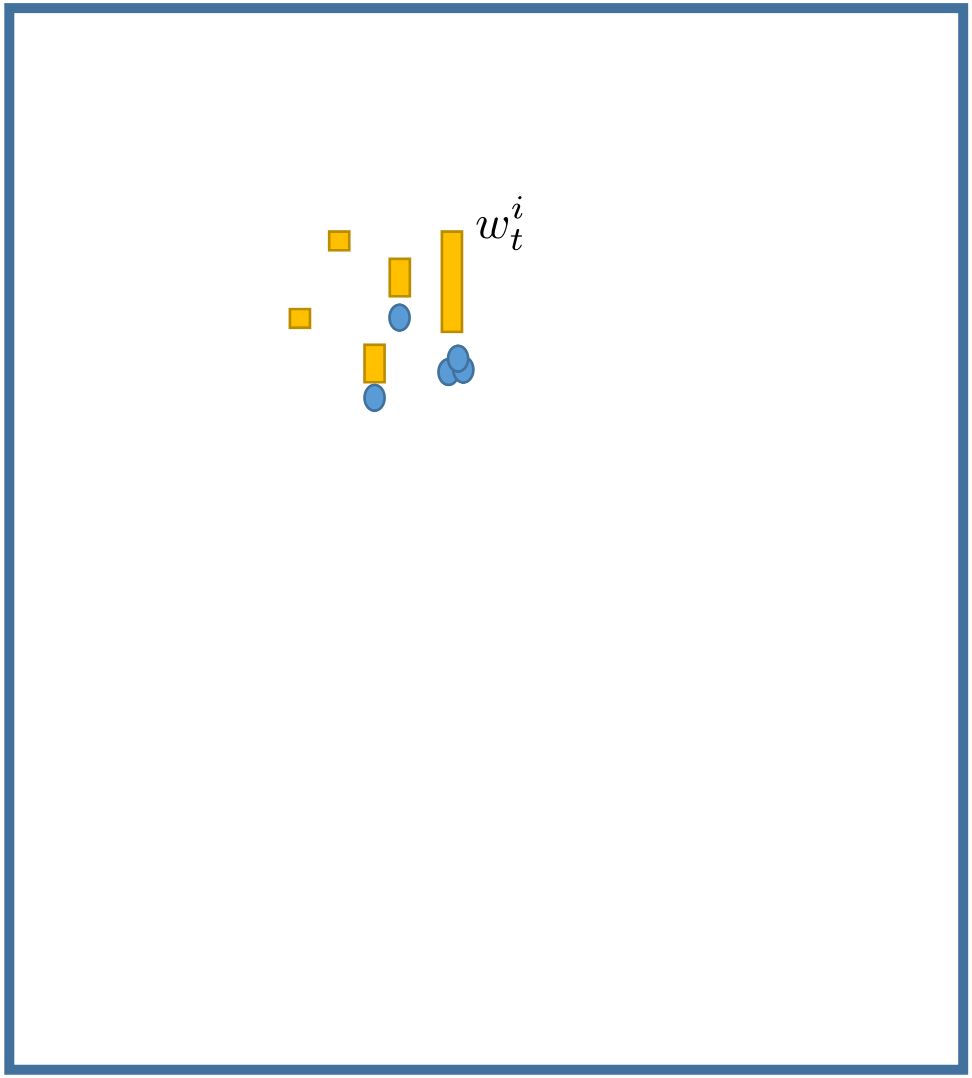

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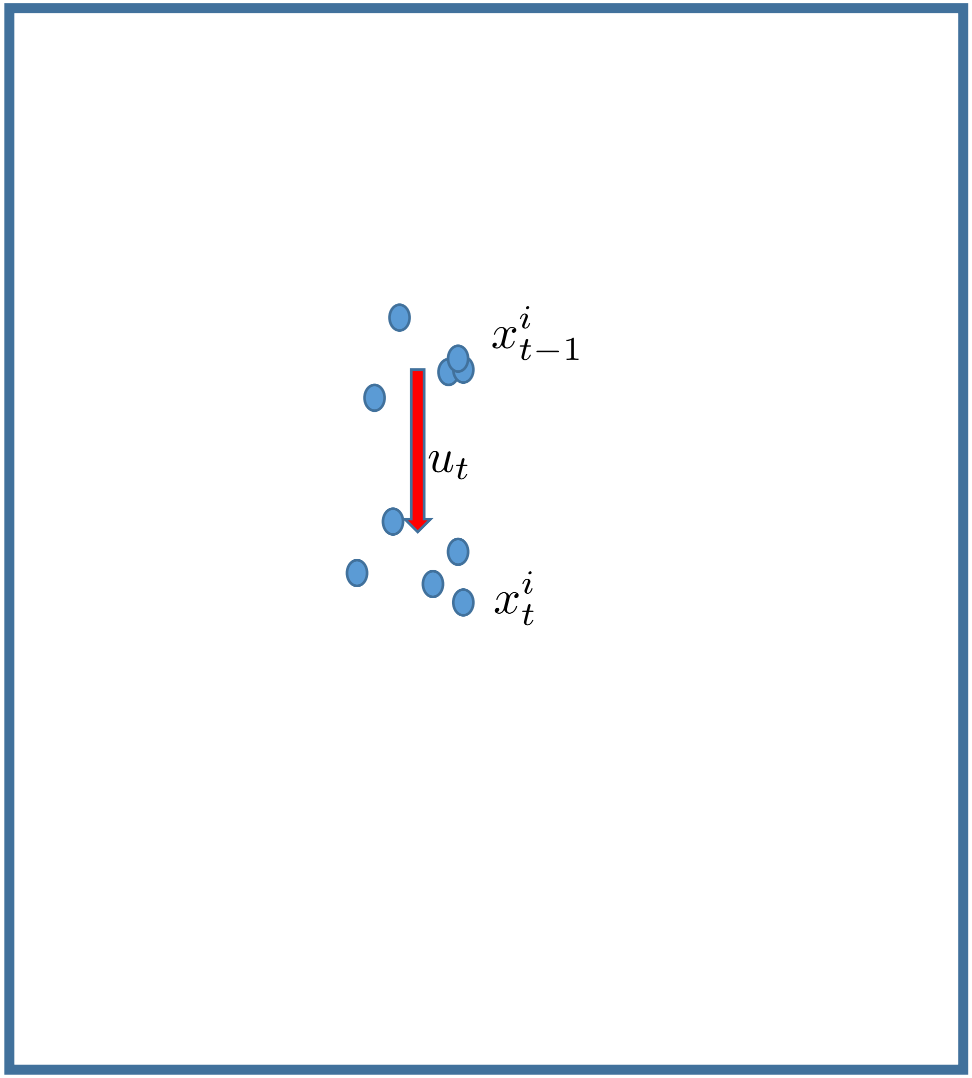

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Mathematical Derivation

$$b(x_{0:t}) = P(x_{0:t}|z_{1:t}, u_{1:t})$$

Mathematical Derivation

$$
\begin{aligned}
b(x_{0:t}) &= P(x_{0:t} | z_{1:t}, u_{1:t}) \\
&= P(x_{0:t} | z_{1:t-1}, z_t, u_{1:t})
\end{aligned}
$$

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b(x_{0:t}) &= P(x_{0:t}|z_{1:t}, u_{1:t}) \\
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&= \eta P(z_t|x_{0:t}, z_{1:t-1}, u_{1:t}) P(x_{0:t}|z_{1:t-1}, u_{1:t})
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Mathematical Derivation

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$$b(x_{0:t}) = \eta P(z_t|x_t)P(x_t|x_{t-1}, u_t)b(x_{0:t-1})$$

$$w_t^{[m]} = \frac{target\ distribution}{proposal\ distribution}$$

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$$= \frac{b(x_{0:t})}{P(x_t|x_{t-1}, u_t)b(x_{0:t-1})}$$

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&= \eta P(z_t|x_t)
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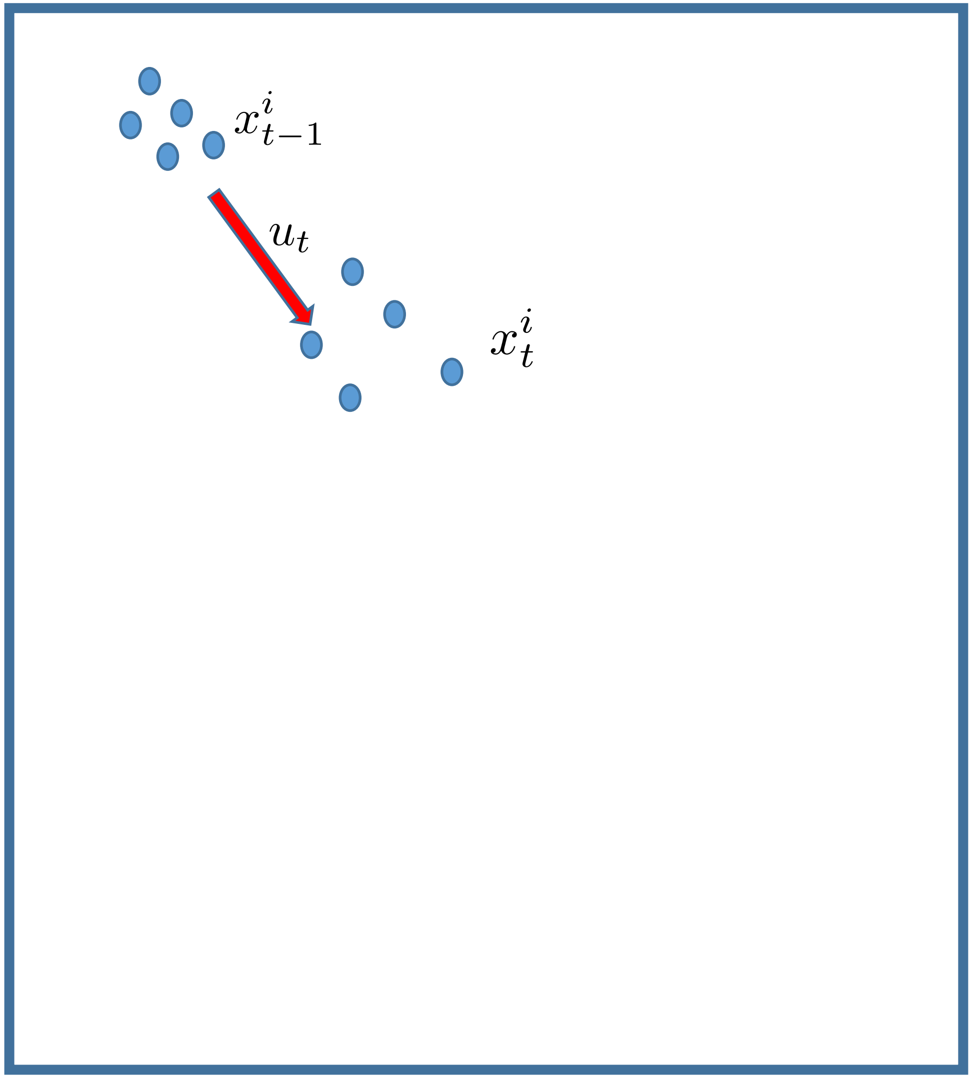
Why Resampling?



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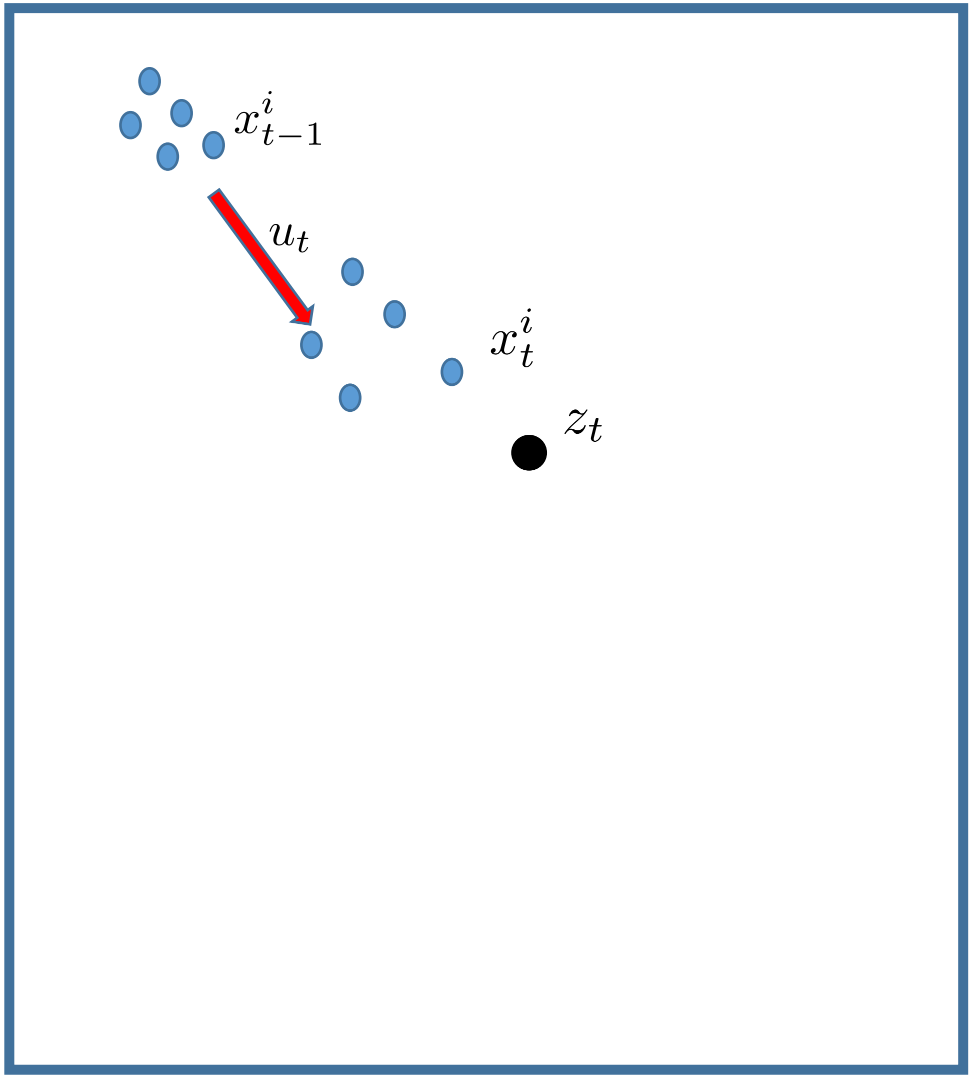

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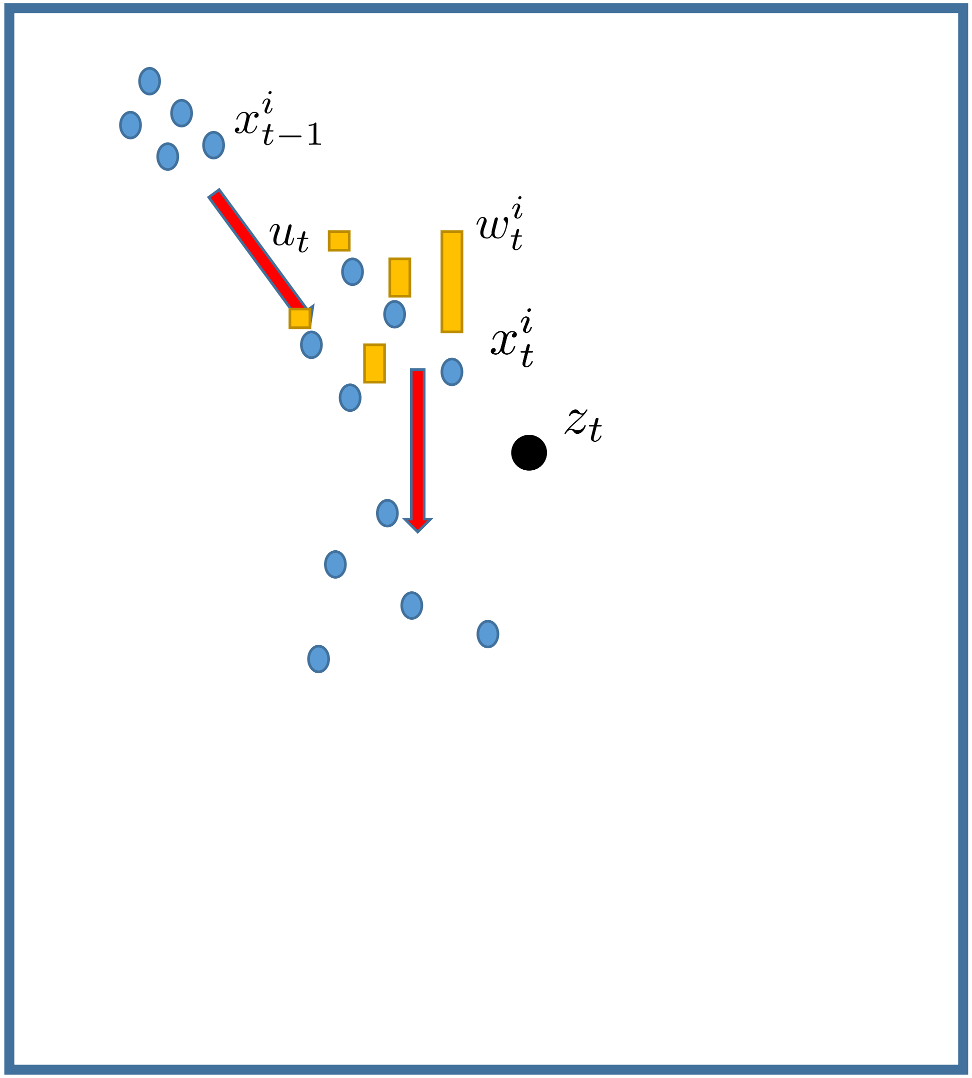
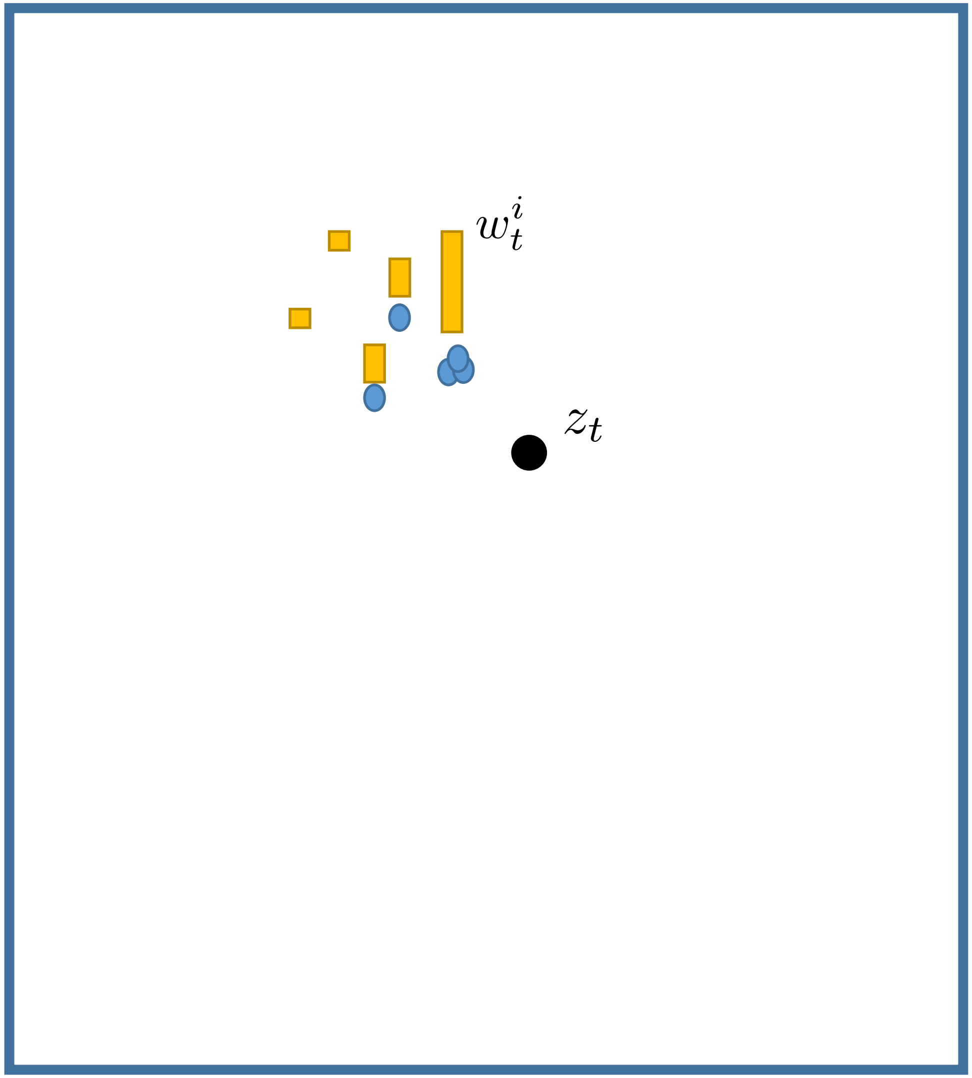


Issues With Particle Filter



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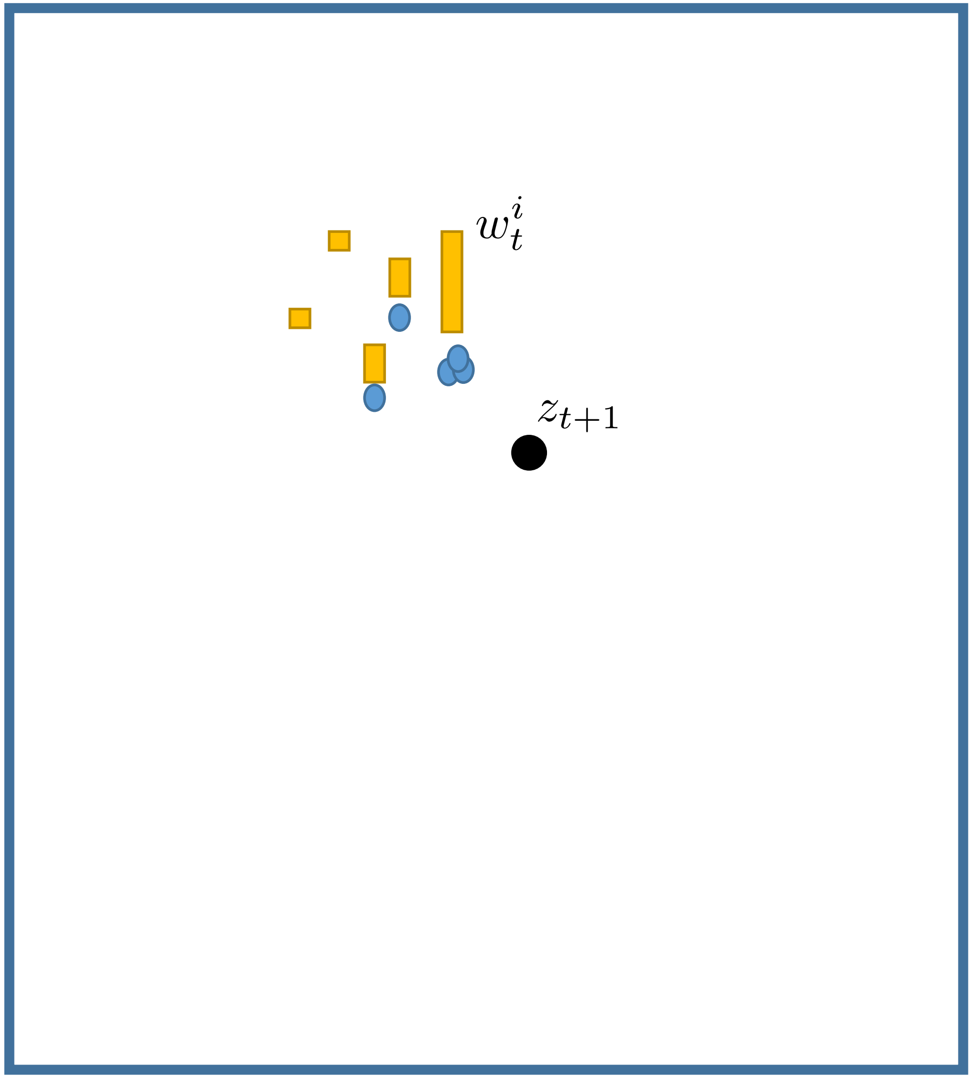

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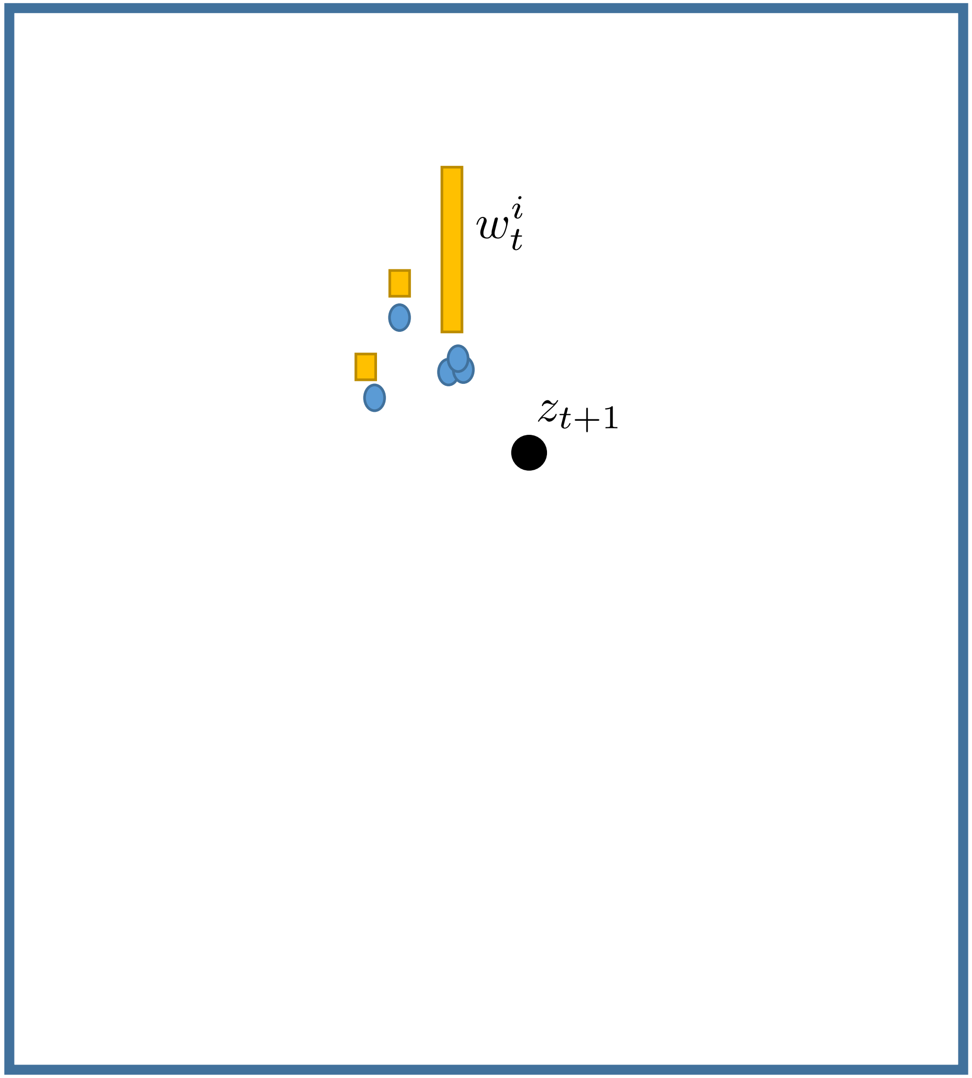

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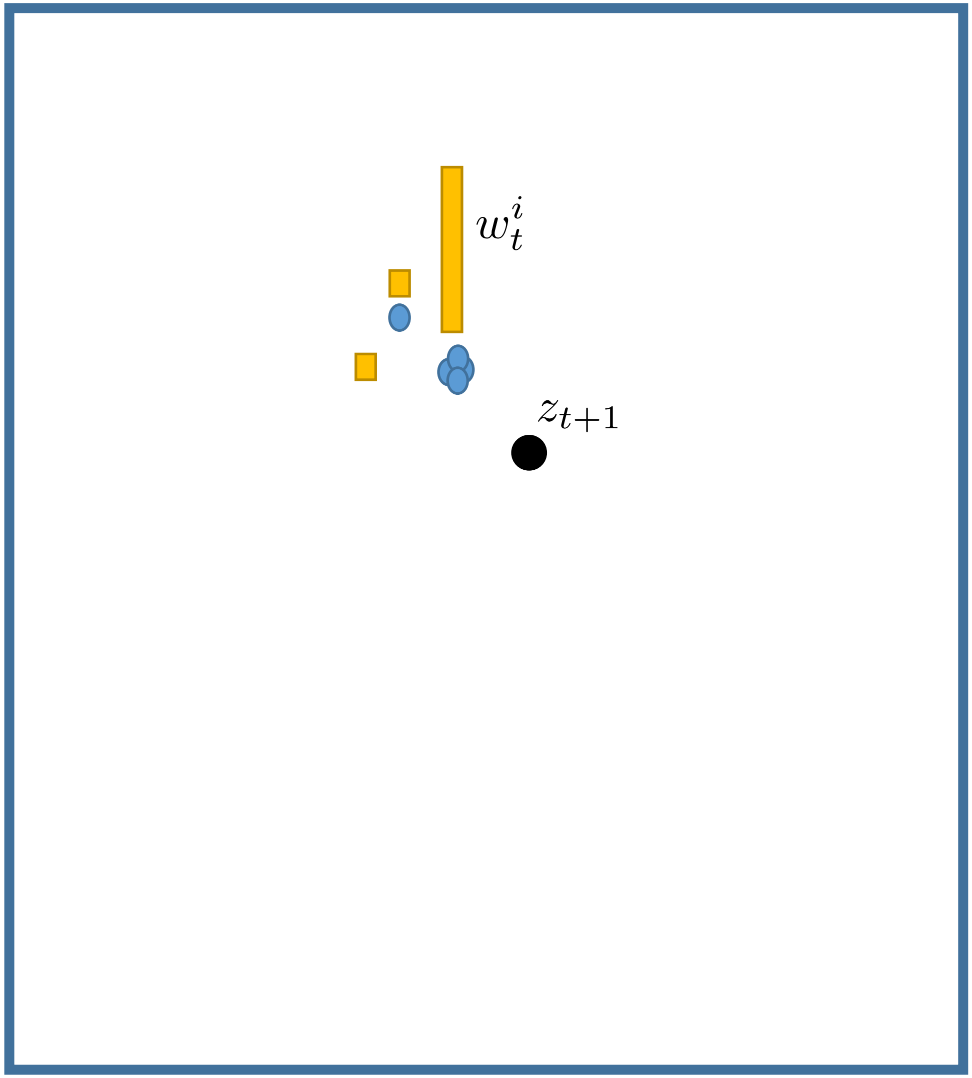

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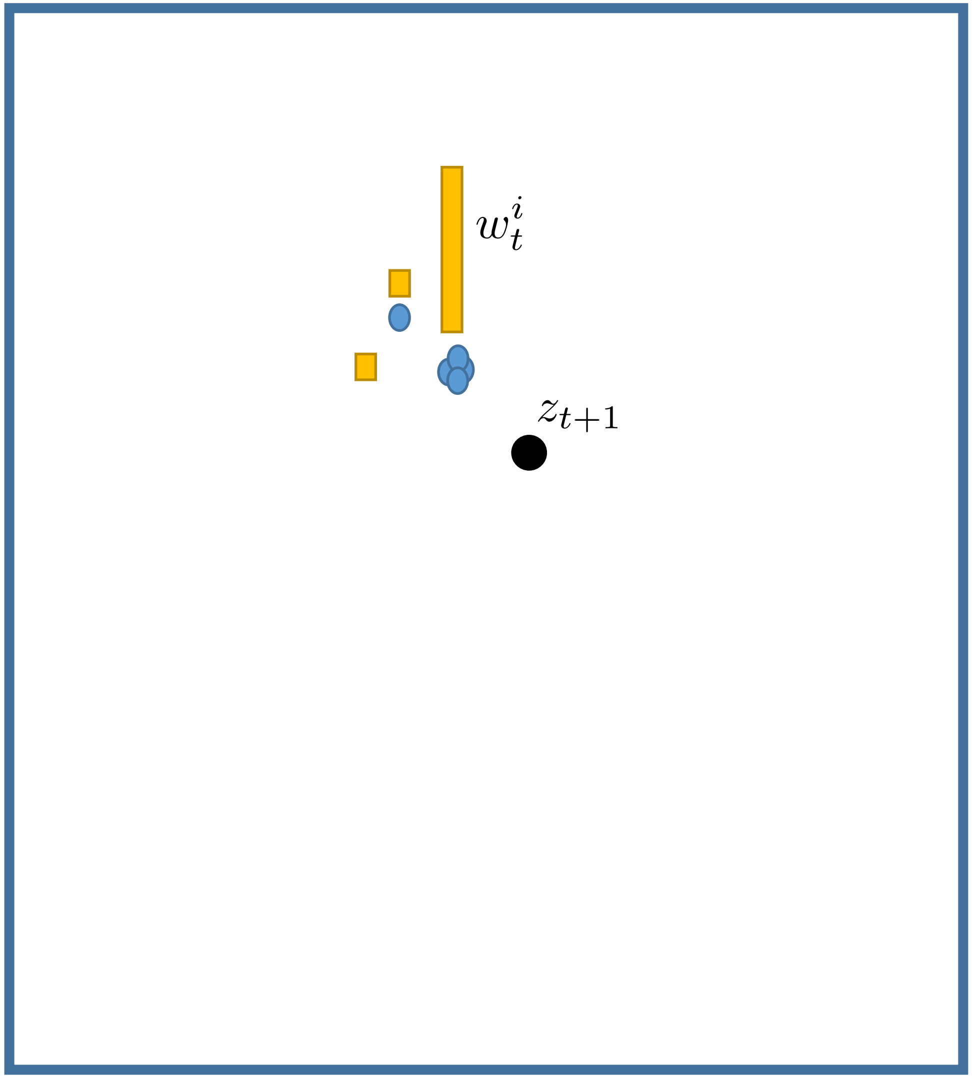

How can we address the sample impoverishment problem?