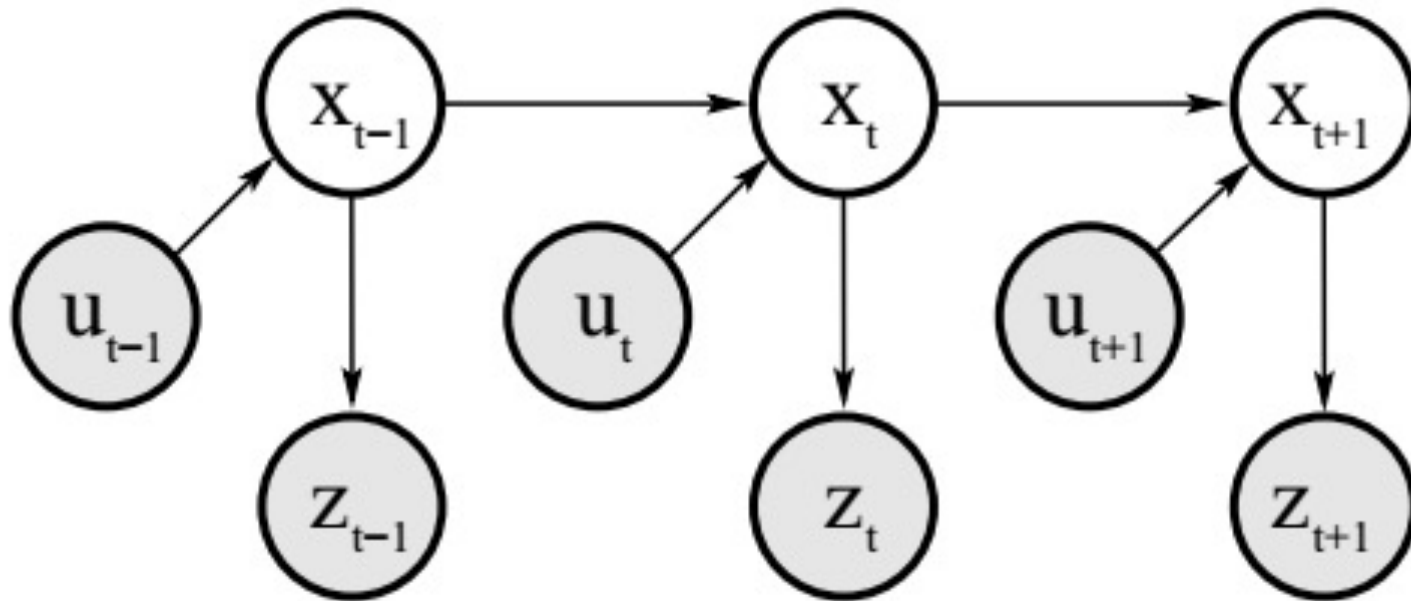


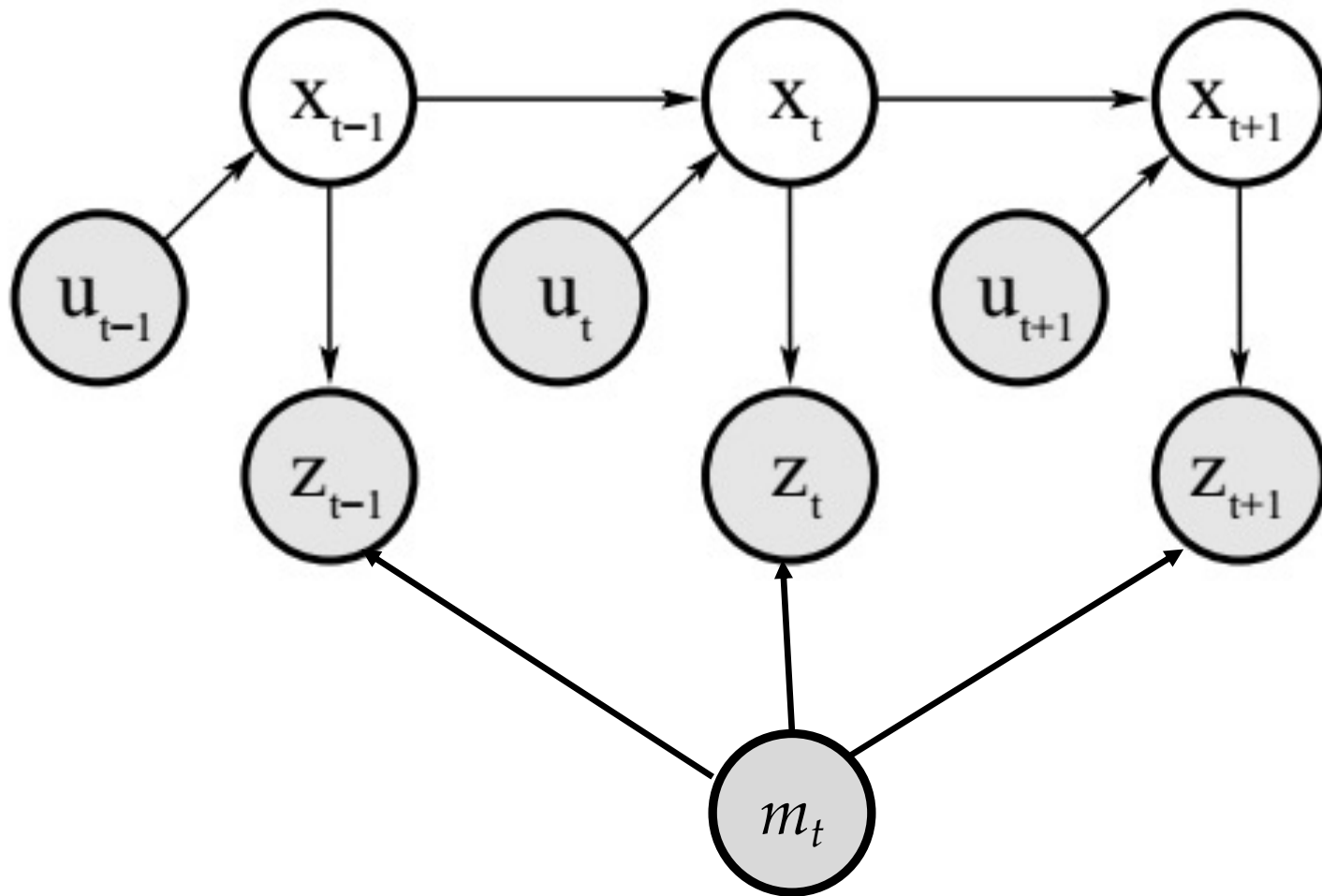
Localization and Mapping

CSCI 545 Introduction to Robotics
Instructor: Stefanos Nikolaidis

Localization



Localization



Dynamics

$$\begin{pmatrix} \hat{v}_t \\ \hat{\omega}_t \end{pmatrix} = \begin{pmatrix} v_t \\ \omega_t \end{pmatrix} + \begin{pmatrix} \epsilon_{a_1|v_t|+a_2|\omega_t|} \\ \epsilon_{a_3|v_t|+a_4|\omega_t|} \end{pmatrix}$$

$$x_t = x_{t-1} - \frac{\hat{v}_t}{\hat{\omega}_t} \sin(\theta_{t-1}) + \frac{\hat{v}_t}{\hat{\omega}_t} \sin(\theta_{t-1} + \hat{\omega}_t * \Delta_t)$$

$$y_t = y_{t-1} + \frac{\hat{v}_t}{\hat{\omega}_t} \cos(\theta_{t-1}) - \frac{\hat{v}_t}{\hat{\omega}_t} \cos(\theta_{t-1} + \hat{\omega}_t * \Delta_t)$$

$$\theta_t = \theta_{t-1} + \hat{\omega}_t * \Delta_t$$

Dynamics

$$x_t = x_{t-1} - \frac{v_t}{\omega_t} \sin(\theta_{t-1}) + \frac{v_t}{\omega_t} \sin(\theta_{t-1} + \omega_t * \Delta_t) + \epsilon_x$$


$$y_t = y_{t-1} + \frac{v_t}{\omega_t} \cos(\theta_{t-1}) - \frac{v_t}{\omega_t} \cos(\theta_{t-1} + \omega_t * \Delta_t) + \epsilon_y$$

$$\theta_t = \theta_{t-1} + \omega_t * \Delta_t + \epsilon_\theta$$

EKF

```
1:   Algorithm Extended_Kalman_filter( $\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$ ):  
2:      $\bar{\mu}_t = g(u_t, \mu_{t-1})$   
3:      $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$   
4:      $K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1}$   
5:      $\mu_t = \bar{\mu}_t + K_t(z_t - h(\bar{\mu}_t))$   
6:      $\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$   
7:     return  $\mu_t, \Sigma_t$ 
```

EKF

```
1:   Algorithm Extended_Kalman_filter( $\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$ ):  
2:     $\bar{\mu}_t = g(u_t, \mu_{t-1})$   
3:    $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$   
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5:    $\mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t))$   
6:    $\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$   
7:   return  $\mu_t, \Sigma_t$ 
```

Dynamics

$$x_t = x_{t-1} - \frac{v_t}{\omega_t} \sin(\theta_{t-1}) + \frac{v_t}{\omega_t} \sin(\theta_{t-1} + \omega_t * \Delta_t) + \epsilon_x$$

$$y_t = y_{t-1} + \frac{v_t}{\omega_t} \cos(\theta_{t-1}) - \frac{v_t}{\omega_t} \cos(\theta_{t-1} + \omega_t * \Delta_t) + \epsilon_y$$

$$\theta_t = \theta_{t-1} + \omega_t * \Delta_t + \epsilon_\theta$$

EKF

$$\bar{\mu}_t = \mu_{t-1} + \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \mu_{t-1,\theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1,\theta} + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \mu_{t-1,\theta} - \frac{v_t}{\omega_t} \cos(\mu_{t-1,\theta} + \omega_t \Delta t) \\ \omega_t \Delta t \end{pmatrix}$$

EKF

1: **Algorithm Extended_Kalman_filter**($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$):
2: $\bar{\mu}_t = g(u_t, \mu_{t-1})$
3: → $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$
4: $K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1}$
5: $\mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t))$
6: $\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$
7: return μ_t, Σ_t

Dynamics

$$x_t = x_{t-1} - \frac{v_t}{\omega_t} \sin(\theta_{t-1}) + \frac{v_t}{\omega_t} \sin(\theta_{t-1} + \omega_t * \Delta_t) + \epsilon_x$$

$$y_t = y_{t-1} + \frac{v_t}{\omega_t} \cos(\theta_{t-1}) - \frac{v_t}{\omega_t} \cos(\theta_{t-1} + \omega_t * \Delta_t) + \epsilon_y$$

$$\theta_t = \theta_{t-1} + \omega_t * \Delta_t + \epsilon_\theta$$

EKF

$$G_t = \begin{pmatrix} 1 & 0 & \frac{v_t}{\omega_t} \cos \mu_{t-1,\theta} - \frac{v_t}{\omega_t} \cos(\mu_{t-1,\theta} + \omega_t \Delta t) \\ 0 & 1 & \frac{v_t}{\omega_t} \sin \mu_{t-1,\theta} - \frac{v_t}{\omega_t} \sin(\mu_{t-1,\theta} + \omega_t \Delta t) \\ 0 & 0 & 1 \end{pmatrix}$$

EKF

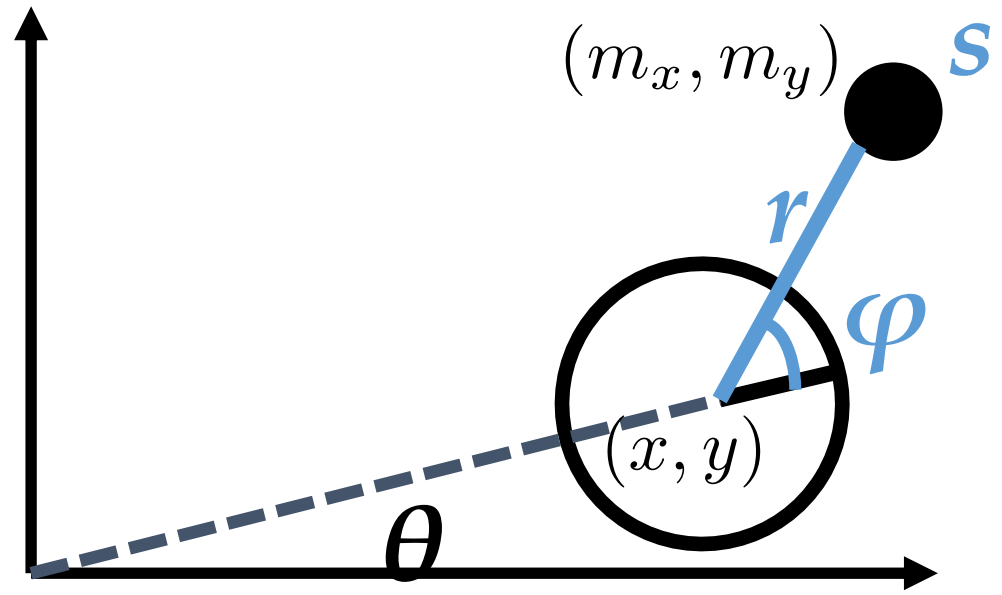
1: **Algorithm Extended_Kalman_filter**($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$):
2: $\bar{\mu}_t = g(u_t, \mu_{t-1})$
3: $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$
4: → $K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1}$
5: $\mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t))$
6: $\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$
7: return μ_t, Σ_t

Measurements

$$\underbrace{\begin{pmatrix} r_t^i \\ \phi_t^i \\ s_t^i \end{pmatrix}}_{z_t^i} = \underbrace{\begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \text{atan2}(m_{j,y} - y, m_{j,x} - x) - \theta \\ m_{j,s} \end{pmatrix}}_{h(x_t, j, m)} + \mathcal{N}(0, Q_t),$$

Feature-Based Sensor Models

$$\begin{pmatrix} r_t^i \\ \phi_t^i \\ s_t^i \end{pmatrix} = \begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \text{atan2}(m_{j,y} - y, m_{j,x} - x) - \theta \\ s_j \end{pmatrix} + \begin{pmatrix} \varepsilon_{\sigma_r^2} \\ \varepsilon_{\sigma_\phi^2} \\ \varepsilon_{\sigma_s^2} \end{pmatrix}$$



Measurements

$$\underbrace{\begin{pmatrix} r_t^i \\ \phi_t^i \\ s_t^i \end{pmatrix}}_{z_t^i} = \underbrace{\begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \text{atan2}(m_{j,y} - y, m_{j,x} - x) - \theta \\ m_{j,s} \end{pmatrix}}_{h(x_t, j, m)} + \mathcal{N}(0, Q_t),$$

EKF

$$Q_t = \begin{pmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & \sigma_s^2 \end{pmatrix}$$

$$H_t^i = \frac{\partial h(\bar{\mu}_t, j, m)}{\partial x_t} = \begin{pmatrix} \frac{\partial r_t^i}{\partial \bar{\mu}_{t,x}} & \frac{\partial r_t^i}{\partial \bar{\mu}_{t,y}} & \frac{\partial r_t^i}{\partial \bar{\mu}_{t,\theta}} \\ \frac{\partial \phi_t^i}{\partial \bar{\mu}_{t,x}} & \frac{\partial \phi_t^i}{\partial \bar{\mu}_{t,y}} & \frac{\partial \phi_t^i}{\partial \bar{\mu}_{t,\theta}} \\ \frac{\partial s_t^i}{\partial \bar{\mu}_{t,x}} & \frac{\partial s_t^i}{\partial \bar{\mu}_{t,y}} & \frac{\partial s_t^i}{\partial \bar{\mu}_{t,\theta}} \end{pmatrix} = \begin{pmatrix} -\frac{m_{j,x} - \bar{\mu}_{t,x}}{\sqrt{q}} & -\frac{m_{j,y} - \bar{\mu}_{t,y}}{\sqrt{q}} & 0 \\ \frac{m_{j,y} - \bar{\mu}_{t,y}}{q} & -\frac{m_{j,x} - \bar{\mu}_{t,x}}{q} & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$q_t = (m_{j,x} - \bar{\mu}_{t,x})^2 + (m_{j,y} - \bar{\mu}_{t,y})^2$$

EKF

```
1:   Algorithm Extended_Kalman_filter( $\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$ ):  
2:      $\bar{\mu}_t = g(u_t, \mu_{t-1})$   
3:      $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$   
4:      $K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1}$   
5:      $\mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t))$   
6:      $\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$   
7:     return  $\mu_t, \Sigma_t$ 
```

EKF

1: **Algorithm EKF_localization_known_correspondences**($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, c_t, m$):

2: $\theta = \mu_{t-1, \theta}$

$$3: G_t = \begin{pmatrix} 1 & 0 & -\frac{v_t}{\omega_t} \cos \theta + \frac{v_t}{\omega_t} \cos(\theta + \omega_t \Delta t) \\ 0 & 1 & -\frac{v_t}{\omega_t} \sin \theta + \frac{v_t}{\omega_t} \sin(\theta + \omega_t \Delta t) \\ 0 & 0 & 1 \end{pmatrix}$$

$$4: V_t = \begin{pmatrix} \frac{-\sin \theta + \sin(\theta + \omega_t \Delta t)}{\omega_t} & \frac{v_t(\sin \theta - \sin(\theta + \omega_t \Delta t))}{\omega_t^2} + \frac{v_t \cos(\theta + \omega_t \Delta t) \Delta t}{\omega_t} \\ \frac{\cos \theta - \cos(\theta + \omega_t \Delta t)}{\omega_t} & -\frac{v_t(\cos \theta - \cos(\theta + \omega_t \Delta t))}{\omega_t^2} + \frac{v_t \sin(\theta + \omega_t \Delta t) \Delta t}{\omega_t} \\ 0 & \Delta t \end{pmatrix}$$

$$5: M_t = \begin{pmatrix} \alpha_1 v_t^2 + \alpha_2 \omega_t^2 & 0 \\ 0 & \alpha_3 v_t^2 + \alpha_4 \omega_t^2 \end{pmatrix}$$

$$6: \bar{\mu}_t = \mu_{t-1} + \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \theta + \frac{v_t}{\omega_t} \sin(\theta + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \theta - \frac{v_t}{\omega_t} \cos(\theta + \omega_t \Delta t) \\ \omega_t \Delta t \end{pmatrix}$$

$$7: \bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + \underbrace{V_t M_t V_t^T}_R$$

$$8: Q_t = \begin{pmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & \sigma_s^2 \end{pmatrix} R$$

9: *for all observed features* $z_t^i = (r_t^i \ \phi_t^i \ s_t^i)^T$ *do*

10: $j = c_t^i$

$$11: q = (m_{j,x} - \bar{\mu}_{t,x})^2 + (m_{j,y} - \bar{\mu}_{t,y})^2$$

$$12: \hat{z}_t^i = \begin{pmatrix} \sqrt{q} \\ \text{atan2}(m_{j,y} - \bar{\mu}_{t,y}, m_{j,x} - \bar{\mu}_{t,x}) - \bar{\mu}_{t,\theta} \\ m_{j,s} \end{pmatrix}$$

$$13: H_t^i = \begin{pmatrix} \frac{-m_{j,x} - \bar{\mu}_{t,x}}{\sqrt{q}} & \frac{-m_{j,y} - \bar{\mu}_{t,y}}{\sqrt{q}} & 0 \\ \frac{m_{j,y} - \bar{\mu}_{t,y}}{q} & \frac{-m_{j,x} - \bar{\mu}_{t,x}}{q} & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$14: S_t^i = H_t^i \bar{\Sigma}_t [H_t^i]^T + Q_t$$

$$15: K_t^i = \bar{\Sigma}_t [H_t^i]^T [S_t^i]^{-1}$$

$$16: \bar{\mu}_t = \bar{\mu}_t + K_t^i (z_t^i - \hat{z}_t^i)$$

$$17: \bar{\Sigma}_t = (I - K_t^i H_t^i) \bar{\Sigma}_t$$

18: *endfor*

$$19: \mu_t = \bar{\mu}_t$$

$$20: \Sigma_t = \bar{\Sigma}_t$$

$$21: p_{z_t} = \prod_i \det(2\pi S_t^i)^{-\frac{1}{2}} \exp\left\{-\frac{1}{2} (z_t^i - \hat{z}_t^i)^T [S_t^i]^{-1} (z_t^i - \hat{z}_t^i)\right\}$$

22: *return* μ_t, Σ_t, p_{z_t}

Example

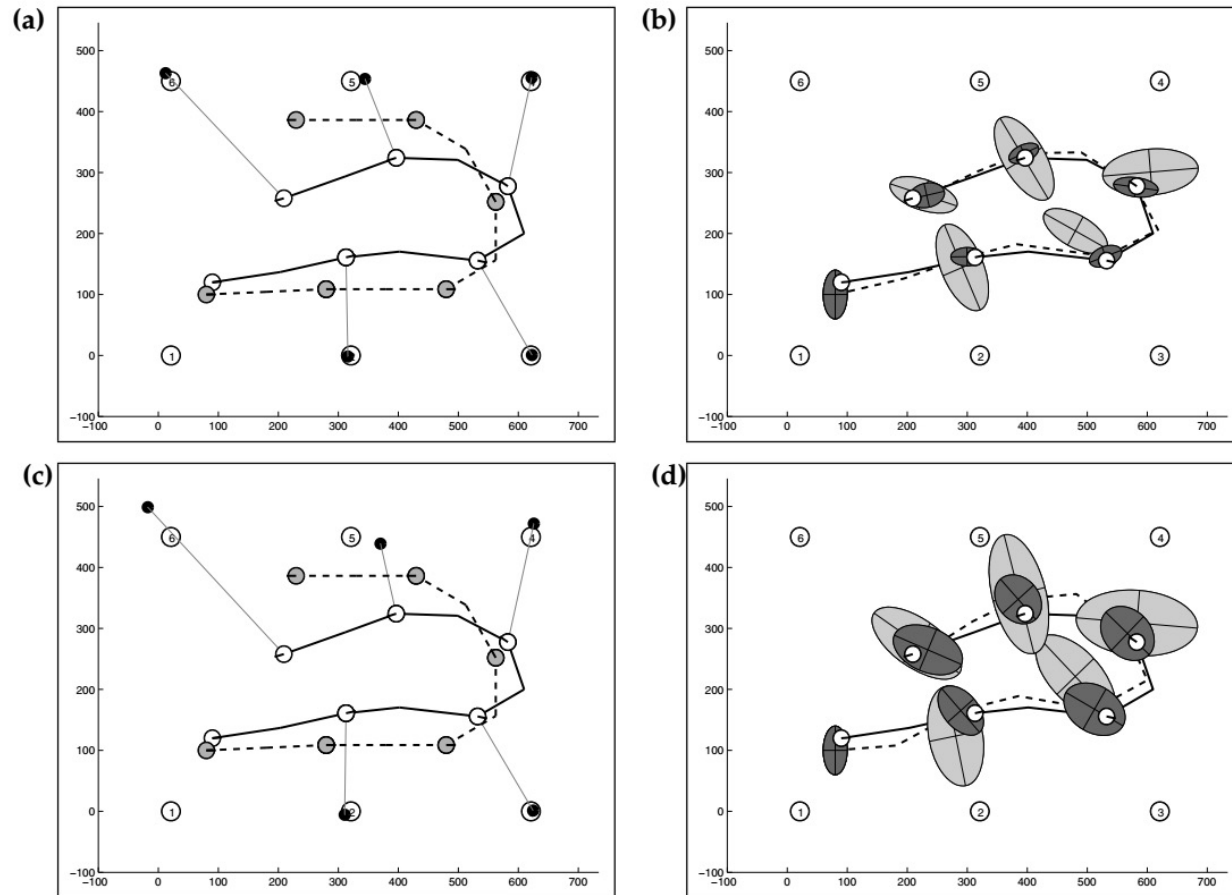


Figure 7.11 EKF-based localization with an accurate (upper row) and a less accurate (lower row) landmark detection sensor. The dashed lines in the left panel indicate the robot trajectories as estimated from the motion controls. The solid lines represent the true robot motion resulting from these controls. Landmark detections at five locations are indicated by the thin lines. The dashed lines in the right panels show the corrected robot trajectories, along with uncertainty before (light gray, $\bar{\Sigma}_t$) and after (dark gray, Σ_t) incorporating a landmark detection.

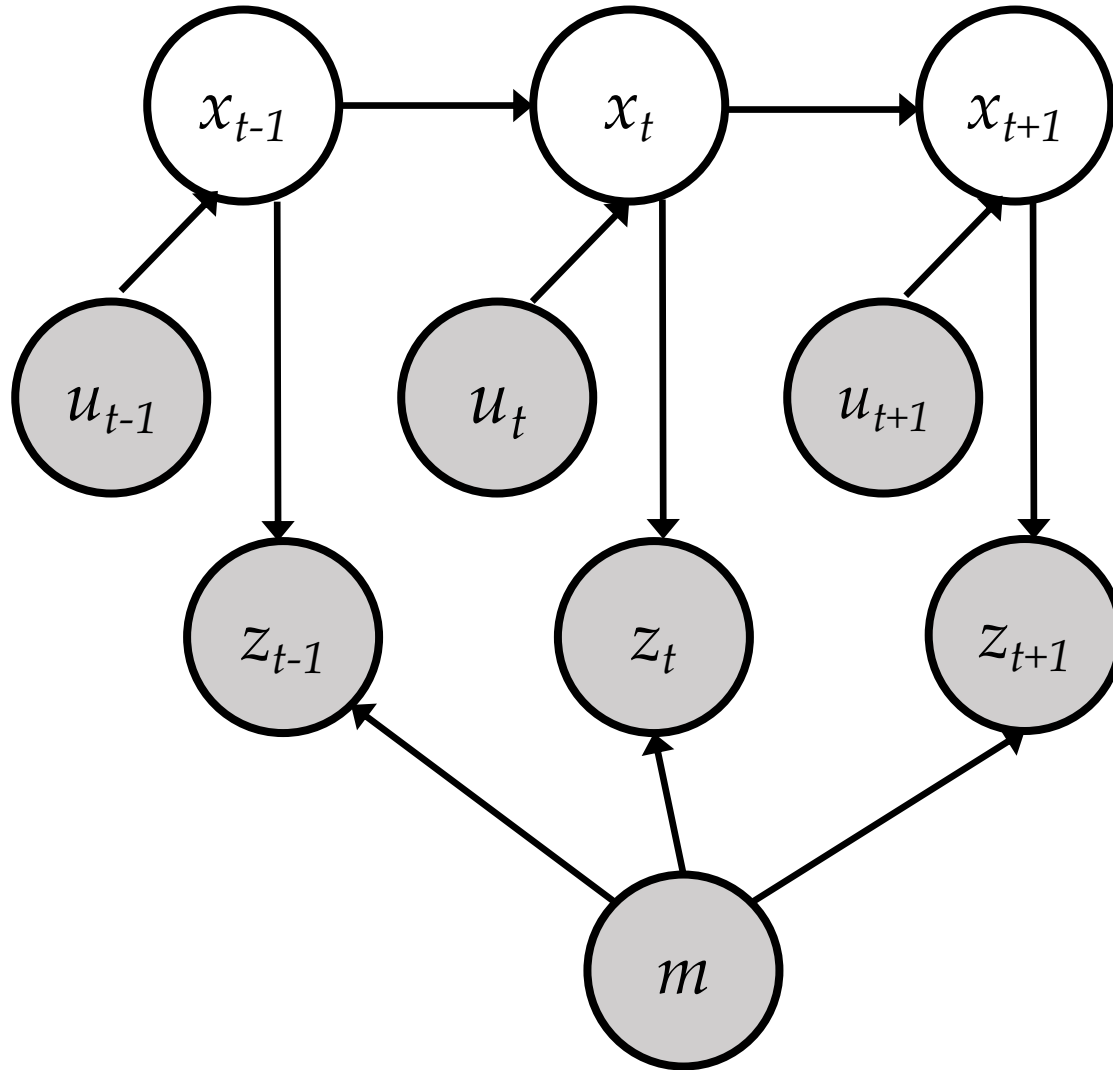
Localization with Unknown Correspondences

- How to do EKF-localization when the correspondences are not known?

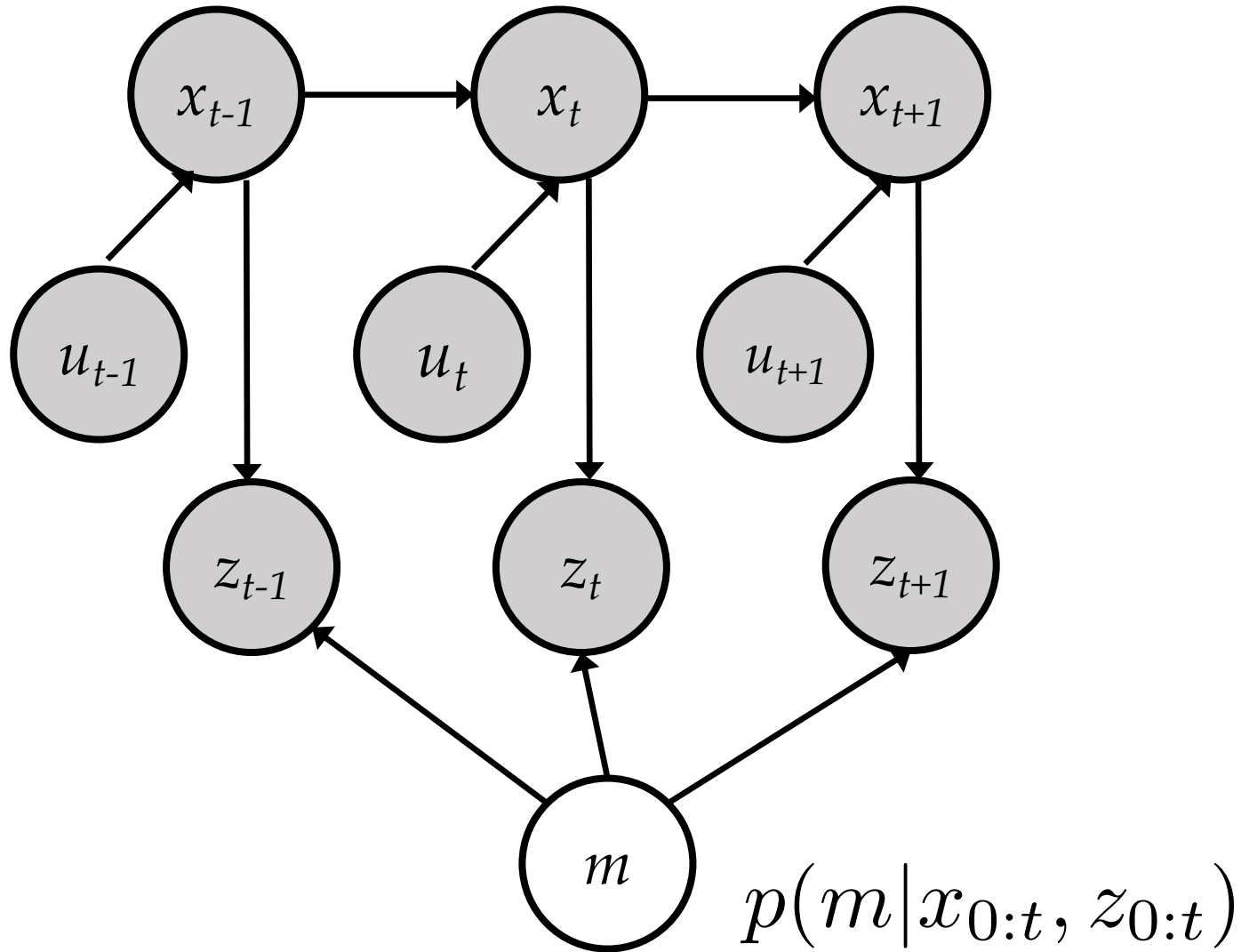
Localization with Unknown Correspondences

- How to do EKF-localization when the correspondences are not known?
- Maximum Likelihood Estimation

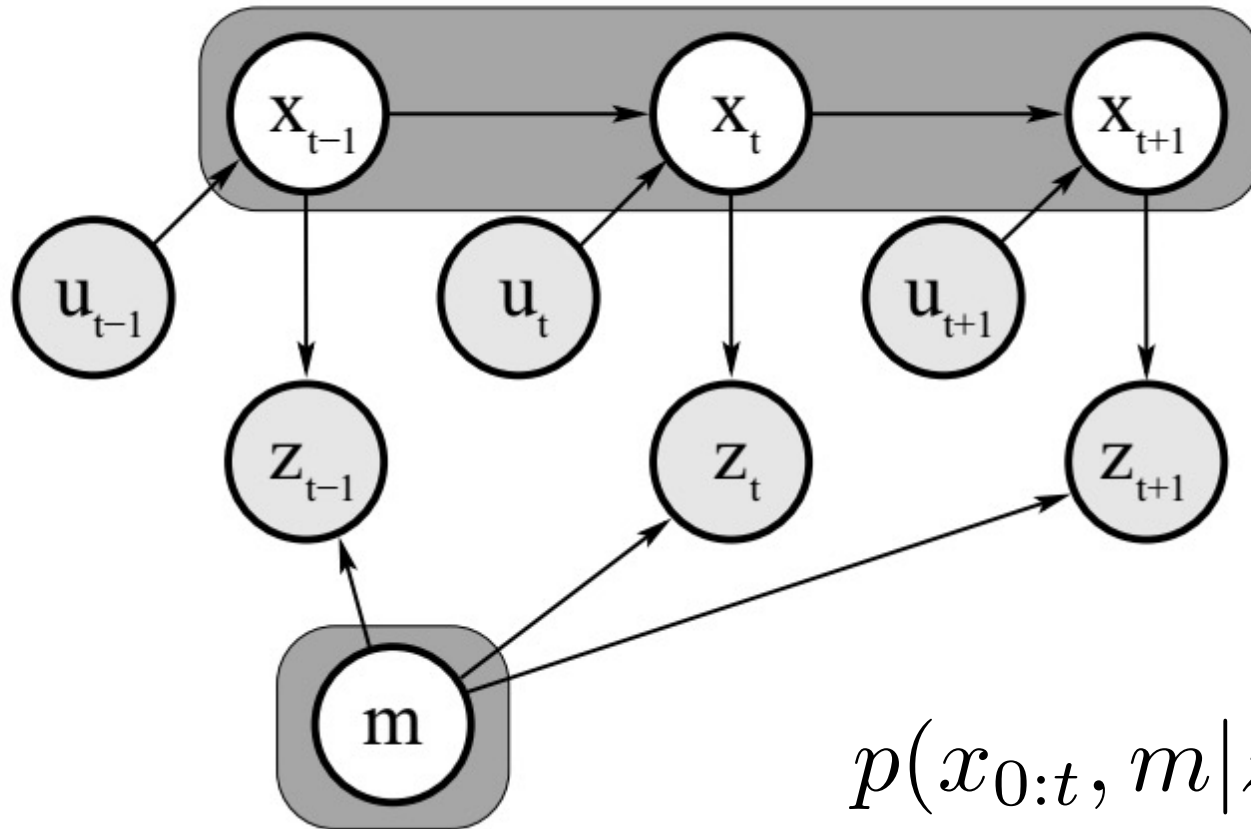
Localization



Mapping



Simultaneous Localization and Mapping



Simultaneous Localization and Mapping

Simultaneous Localization and Mapping

Autonomous Aerial Navigation in Confined Indoor Environments

Shaojie Shen, Nathan Michael, Vijay Kumar



EKF - SLAM

$$\begin{aligned} y_t &= \begin{pmatrix} x_t \\ m \end{pmatrix} \\ &= (x \ y \ \theta \ m_{1,x} \ m_{1,y} \ s_1 \ m_{2,x} \ m_{2,y} \ s_2 \ \dots \ m_{N,x} \ m_{N,y} \ s_N)^T \end{aligned}$$

Assumptions:

- Static map
- Known set of landmarks
- Known correspondences

EKF

```
1:   Algorithm Extended_Kalman_filter( $\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$ ):  
2:      $\bar{\mu}_t = g(u_t, \mu_{t-1})$   
3:      $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$   
4:      $K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1}$   
5:      $\mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t))$   
6:      $\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$   
7:     return  $\mu_t, \Sigma_t$ 
```

Initial Mean and Covariance

$$\mu_0 = (0 \ 0 \ 0 \ \dots \ 0)^T$$

$$\Sigma_0 = \begin{pmatrix} 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \infty & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & \infty \end{pmatrix}$$

Dynamics Model

$$y_t = y_{t-1} + \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \theta + \frac{v_t}{\omega_t} \sin(\theta + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \theta - \frac{v_t}{\omega_t} \cos(\theta + \omega_t \Delta t) \\ \omega_t \Delta t + \gamma_t \Delta t \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

Dynamics Model

$$y_t = y_{t-1} + \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \theta + \frac{v_t}{\omega_t} \sin(\theta + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \theta - \frac{v_t}{\omega_t} \cos(\theta + \omega_t \Delta t) \\ \omega_t \Delta t + \gamma_t \Delta t \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$y_t = y_{t-1} + F_x^T \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \theta + \frac{v_t}{\omega_t} \sin(\theta + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \theta - \frac{v_t}{\omega_t} \cos(\theta + \omega_t \Delta t) \\ \omega_t \Delta t + \gamma_t \Delta t \end{pmatrix}$$

$$F_x = \begin{pmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & \underbrace{0 & \dots & 0}_{3N \text{ columns}} \end{pmatrix}$$

Dynamics Model

$$y_t = y_{t-1} + \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \theta + \frac{v_t}{\omega_t} \sin(\theta + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \theta - \frac{v_t}{\omega_t} \cos(\theta + \omega_t \Delta t) \\ \omega_t \Delta t + \gamma_t \Delta t \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$y_t = y_{t-1} + \underbrace{F_x^T \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \theta + \frac{v_t}{\omega_t} \sin(\theta + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \theta - \frac{v_t}{\omega_t} \cos(\theta + \omega_t \Delta t) \\ \omega \Delta t \end{pmatrix}}_{g(u_t, y_{t-1})} + \mathcal{N}(0, F_x^T R_t F_x)$$

$$F_x = \begin{pmatrix} 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & \underbrace{0 & \dots & 0}_{3N \text{ columns}} \end{pmatrix}$$

Dynamics Model

$$y_t = y_{t-1} + \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \theta + \frac{v_t}{\omega_t} \sin(\theta + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \theta - \frac{v_t}{\omega_t} \cos(\theta + \omega_t \Delta t) \\ \omega_t \Delta t + \gamma_t \Delta t \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$g(u_t, y_{t-1}) \approx g(u_t, \mu_{t-1}) + G_t (y_{t-1} - \mu_{t-1})$$

EKF-SLAM

```

1:  Algorithm EKF_SLAM_known_correspondences( $\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, c_t$ ):
2:       $F_x = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & \underbrace{0 \cdots 0}_{3N} \end{pmatrix}$ 
3:       $\bar{\mu}_t = \mu_{t-1} + F_x^T \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \mu_{t-1, \theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1, \theta} + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \mu_{t-1, \theta} - \frac{v_t}{\omega_t} \cos(\mu_{t-1, \theta} + \omega_t \Delta t) \\ \omega_t \Delta t \end{pmatrix}$ 
4:       $G_t = I + F_x^T \begin{pmatrix} 0 & 0 & -\frac{v_t}{\omega_t} \cos \mu_{t-1, \theta} + \frac{v_t}{\omega_t} \cos(\mu_{t-1, \theta} + \omega_t \Delta t) \\ 0 & 0 & -\frac{v_t}{\omega_t} \sin \mu_{t-1, \theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1, \theta} + \omega_t \Delta t) \\ 0 & 0 & 0 \end{pmatrix} F_x$ 
5:       $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + F_x^T R_t F_x$ 
6:       $Q_t = \begin{pmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & \sigma_s^2 \end{pmatrix}$ 
7:      for all observed features  $z_t^i = (r_t^i \ \phi_t^i \ s_t^i)^T$  do
8:           $j = c_t^i$ 
9:          if landmark  $j$  never seen before
10:              $\begin{pmatrix} \bar{\mu}_{j,x} \\ \bar{\mu}_{j,y} \\ \bar{\mu}_{j,s} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \\ s_t^i \end{pmatrix} + \begin{pmatrix} r_t^i \cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i \sin(\phi_t^i + \bar{\mu}_{t,\theta}) \\ 0 \end{pmatrix}$ 
11:         endif
12:          $\delta = \begin{pmatrix} \delta_x \\ \delta_y \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{j,x} - \bar{\mu}_{t,x} \\ \bar{\mu}_{j,y} - \bar{\mu}_{t,y} \end{pmatrix}$ 
13:          $q = \delta^T \delta$ 
14:          $\hat{z}_t^i = \begin{pmatrix} \sqrt{q} \\ \text{atan2}(\delta_y, \delta_x) - \bar{\mu}_{t,\theta} \\ \bar{\mu}_{j,s} \end{pmatrix}$ 
15:          $F_{x,j} = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 1 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & \underbrace{0 \cdots 0}_{3j-3} & 0 & 0 & 1 & \underbrace{0 \cdots 0}_{3N-3j} \end{pmatrix}$ 
16:          $H_t^i = \frac{1}{q} \begin{pmatrix} -\sqrt{q} \delta_x & -\sqrt{q} \delta_y & 0 & +\sqrt{q} \delta_x & \sqrt{q} \delta_y & 0 \\ \delta_y & -\delta_x & -q & -\delta_y & +\delta_x & 0 \\ 0 & 0 & 0 & 0 & 0 & q \end{pmatrix} F_{x,j}$ 
17:          $K_t^i = \bar{\Sigma}_t H_t^{iT} (H_t^i \bar{\Sigma}_t H_t^{iT} + Q_t)^{-1}$ 
18:          $\bar{\mu}_t = \bar{\mu}_t + K_t^i (z_t^i - \hat{z}_t^i)$ 
19:          $\bar{\Sigma}_t = (I - K_t^i H_t^i) \bar{\Sigma}_t$ 
20:     endfor
21:      $\mu_t = \bar{\mu}_t$ 
22:      $\Sigma_t = \bar{\Sigma}_t$ 
23:     return  $\mu_t, \Sigma_t$ 

```

Table 10.1 The EKF algorithm for the SLAM problem (with known correspondences).

Measurement Model

$$z_t^i = \underbrace{\begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \text{atan2}(m_{j,y} - y, m_{j,x} - x) - \theta \\ m_{j,s} \end{pmatrix}}_{h(y_t, j)} + \mathcal{N}\left(0, \underbrace{\begin{pmatrix} \sigma_r & 0 & 0 \\ 0 & \sigma_\phi & 0 \\ 0 & 0 & \sigma_s \end{pmatrix}}_{Q_t}\right)$$

$$h(y_t, j) \approx h(\bar{\mu}_t, j) + H_t^i (y_t - \bar{\mu}_t)$$

Measurement Model

$$z_t^i = \underbrace{\begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \text{atan2}(m_{j,y} - y, m_{j,x} - x) - \theta \\ m_{j,s} \end{pmatrix}}_{h(y_t, j)} + \mathcal{N}\left(0, \underbrace{\begin{pmatrix} \sigma_r & 0 & 0 \\ 0 & \sigma_\phi & 0 \\ 0 & 0 & \sigma_s \end{pmatrix}}_{Q_t}\right)$$

$$h(y_t, j) \approx h(\bar{\mu}_t, j) + H_t^i (y_t - \bar{\mu}_t)$$

What if landmark is seen for the first time?

Measurement Model

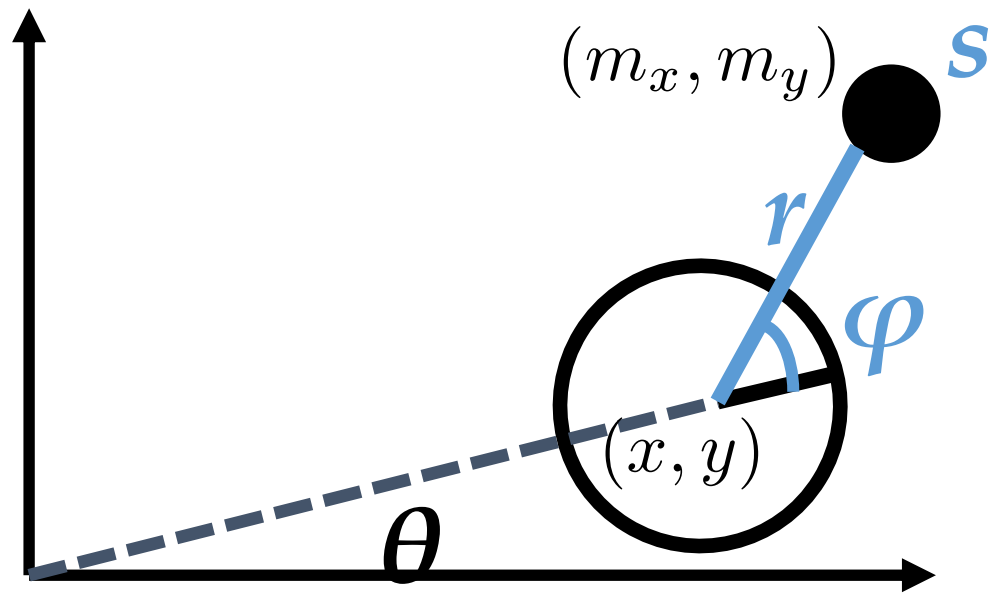
$$z_t^i = \underbrace{\begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \text{atan2}(m_{j,y} - y, m_{j,x} - x) - \theta \\ m_{j,s} \end{pmatrix}}_{h(y_t, j)} + \mathcal{N}\left(0, \underbrace{\begin{pmatrix} \sigma_r & 0 & 0 \\ 0 & \sigma_\phi & 0 \\ 0 & 0 & \sigma_s \end{pmatrix}}_{Q_t}\right)$$

$$h(y_t, j) \approx h(\bar{\mu}_t, j) + H_t^i (y_t - \bar{\mu}_t)$$

What if landmark is seen for the first time?

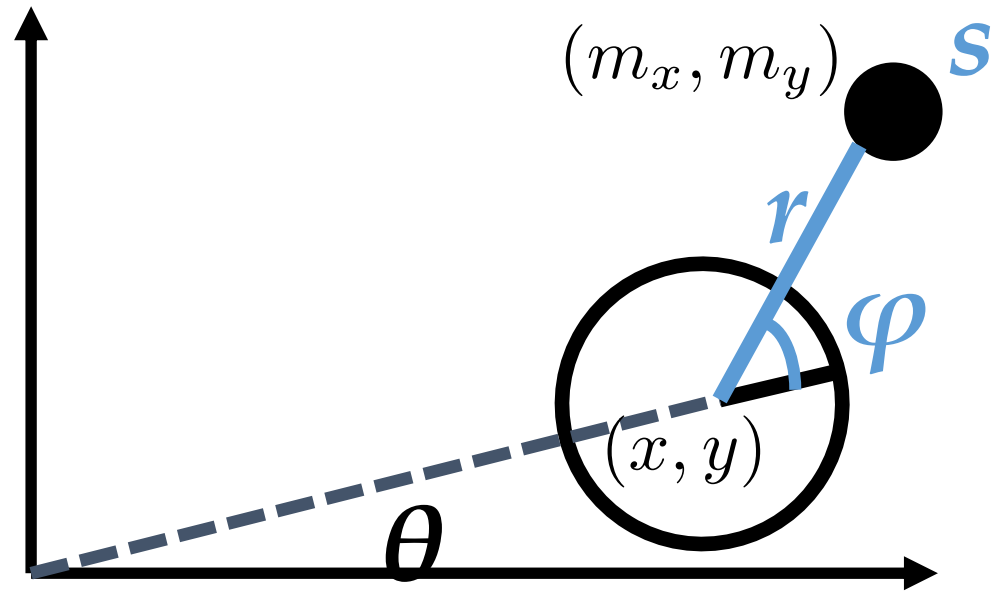
Estimate it based on the robot's position!

Measurement Model



Measurement Model

$$\begin{pmatrix} \bar{\mu}_{j,x} \\ \bar{\mu}_{j,y} \\ \bar{\mu}_{j,s} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \\ s_t^i \end{pmatrix} + \begin{pmatrix} r_t^i \cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i \sin(\phi_t^i + \bar{\mu}_{t,\theta}) \\ 0 \end{pmatrix}$$



Measurement Model

$$z_t^i = \underbrace{\begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \text{atan2}(m_{j,y} - y, m_{j,x} - x) - \theta \\ m_{j,s} \end{pmatrix}}_{h(y_t, j)} + \mathcal{N}\left(0, \underbrace{\begin{pmatrix} \sigma_r & 0 & 0 \\ 0 & \sigma_\phi & 0 \\ 0 & 0 & \sigma_s \end{pmatrix}}_{Q_t}\right)$$

$$h(y_t, j) \approx h(\bar{\mu}_t, j) + H_t^i (y_t - \bar{\mu}_t)$$

What if landmark is seen for the first time?

Estimate it based on the robot's position!

Otherwise use previous estimate

EKF-SLAM

```

1:  Algorithm EKF_SLAM_known_correspondences( $\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, c_t$ ):
2:       $F_x = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & \underbrace{0 \cdots 0}_{3N} \end{pmatrix}$ 
3:       $\bar{\mu}_t = \mu_{t-1} + F_x^T \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \mu_{t-1, \theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1, \theta} + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \mu_{t-1, \theta} - \frac{v_t}{\omega_t} \cos(\mu_{t-1, \theta} + \omega_t \Delta t) \\ \omega_t \Delta t \end{pmatrix}$ 
4:       $G_t = I + F_x^T \begin{pmatrix} 0 & 0 & -\frac{v_t}{\omega_t} \cos \mu_{t-1, \theta} + \frac{v_t}{\omega_t} \cos(\mu_{t-1, \theta} + \omega_t \Delta t) \\ 0 & 0 & -\frac{v_t}{\omega_t} \sin \mu_{t-1, \theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1, \theta} + \omega_t \Delta t) \\ 0 & 0 & 0 \end{pmatrix} F_x$ 
5:       $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + F_x^T R_t F_x$ 
6:       $Q_t = \begin{pmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & \sigma_s^2 \end{pmatrix}$ 
7:      for all observed features  $z_t^i = (r_t^i \ \phi_t^i \ s_t^i)^T$  do
8:           $j = c_t^i$ 
9:          if landmark  $j$  never seen before
10:              $\begin{pmatrix} \bar{\mu}_{j,x} \\ \bar{\mu}_{j,y} \\ \bar{\mu}_{j,s} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \\ s_t^i \end{pmatrix} + \begin{pmatrix} r_t^i \cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i \sin(\phi_t^i + \bar{\mu}_{t,\theta}) \\ 0 \end{pmatrix}$ 
11:         endif
12:          $\delta = \begin{pmatrix} \delta_x \\ \delta_y \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{j,x} - \bar{\mu}_{t,x} \\ \bar{\mu}_{j,y} - \bar{\mu}_{t,y} \end{pmatrix}$ 
13:          $q = \delta^T \delta$ 
14:          $\hat{z}_t^i = \begin{pmatrix} \sqrt{q} \\ \text{atan2}(\delta_y, \delta_x) - \bar{\mu}_{t,\theta} \\ \bar{\mu}_{j,s} \end{pmatrix}$ 
15:          $F_{x,j} = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 1 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & \underbrace{0 \cdots 0}_{3j-3} & 0 & 0 & 1 & \underbrace{0 \cdots 0}_{3N-3j} \end{pmatrix}$ 
16:          $H_t^i = \frac{1}{q} \begin{pmatrix} -\sqrt{q} \delta_x & -\sqrt{q} \delta_y & 0 & +\sqrt{q} \delta_x & \sqrt{q} \delta_y & 0 \\ \delta_y & -\delta_x & -q & -\delta_y & +\delta_x & 0 \\ 0 & 0 & 0 & 0 & 0 & q \end{pmatrix} F_{x,j}$ 
17:          $K_t^i = \bar{\Sigma}_t H_t^{iT} (H_t^i \bar{\Sigma}_t H_t^{iT} + Q_t)^{-1}$ 
18:          $\bar{\mu}_t = \bar{\mu}_t + K_t^i (z_t^i - \hat{z}_t^i)$ 
19:          $\bar{\Sigma}_t = (I - K_t^i H_t^i) \bar{\Sigma}_t$ 
20:     endfor
21:      $\mu_t = \bar{\mu}_t$ 
22:      $\Sigma_t = \bar{\Sigma}_t$ 
23:     return  $\mu_t, \Sigma_t$ 

```

Table 10.1 The EKF algorithm for the SLAM problem (with known correspondences).

EKF-SLAM

```

1:  Algorithm EKF_SLAM_known_correspondences( $\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, c_t$ ):
2:       $F_x = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & \underbrace{0 \cdots 0}_{3N} \end{pmatrix}$ 
3:       $\bar{\mu}_t = \mu_{t-1} + F_x^T \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \mu_{t-1, \theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1, \theta} + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \mu_{t-1, \theta} - \frac{v_t}{\omega_t} \cos(\mu_{t-1, \theta} + \omega_t \Delta t) \\ \omega_t \Delta t \end{pmatrix}$ 
4:       $G_t = I + F_x^T \begin{pmatrix} 0 & 0 & -\frac{v_t}{\omega_t} \cos \mu_{t-1, \theta} + \frac{v_t}{\omega_t} \cos(\mu_{t-1, \theta} + \omega_t \Delta t) \\ 0 & 0 & -\frac{v_t}{\omega_t} \sin \mu_{t-1, \theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1, \theta} + \omega_t \Delta t) \\ 0 & 0 & 0 \end{pmatrix} F_x$ 
5:       $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + F_x^T R_t F_x$ 
6:       $Q_t = \begin{pmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & \sigma_s^2 \end{pmatrix}$ 
7:      for all observed features  $z_t^i = (r_t^i \ \phi_t^i \ s_t^i)^T$  do
8:           $j = c_t^i$ 
9:          if landmark  $j$  never seen before
10:              $\begin{pmatrix} \bar{\mu}_{j,x} \\ \bar{\mu}_{j,y} \\ \bar{\mu}_{j,s} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \\ s_t^i \end{pmatrix} + \begin{pmatrix} r_t^i \cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i \sin(\phi_t^i + \bar{\mu}_{t,\theta}) \\ 0 \end{pmatrix}$ 
11:          endif
12:           $\delta = \begin{pmatrix} \delta_x \\ \delta_y \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{j,x} - \bar{\mu}_{t,x} \\ \bar{\mu}_{j,y} - \bar{\mu}_{t,y} \end{pmatrix}$ 
13:           $q = \delta^T \delta$ 
14:           $\hat{z}_t^i = \begin{pmatrix} \sqrt{q} \\ \text{atan2}(\delta_y, \delta_x) - \bar{\mu}_{t,\theta} \\ \bar{\mu}_{j,s} \end{pmatrix}$ 
15:           $F_{x,j} = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 1 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & \underbrace{0 \cdots 0}_{3j-3} & 0 & 0 & 1 & \underbrace{0 \cdots 0}_{3N-3j} \end{pmatrix}$ 
16:           $H_t^i = \frac{1}{q} \begin{pmatrix} -\sqrt{q} \delta_x & -\sqrt{q} \delta_y & 0 & +\sqrt{q} \delta_x & \sqrt{q} \delta_y & 0 \\ \delta_y & -\delta_x & -q & -\delta_y & +\delta_x & 0 \\ 0 & 0 & 0 & 0 & 0 & q \end{pmatrix} F_{x,j}$ 
17:           $K_t^i = \bar{\Sigma}_t H_t^{iT} (H_t^i \bar{\Sigma}_t H_t^{iT} + Q_t)^{-1}$ 
18:           $\bar{\mu}_t = \bar{\mu}_t + K_t^i (z_t^i - \hat{z}_t^i)$ 
19:           $\bar{\Sigma}_t = (I - K_t^i H_t^i) \bar{\Sigma}_t$ 
20:      endfor
21:       $\mu_t = \bar{\mu}_t$ 
22:       $\Sigma_t = \bar{\Sigma}_t$ 
23:      return  $\mu_t, \Sigma_t$ 

```

Table 10.1 The EKF algorithm for the SLAM problem (with known correspondences).

Measurement Model

$$z_t^i = \underbrace{\begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \text{atan2}(m_{j,y} - y, m_{j,x} - x) - \theta \\ m_{j,s} \end{pmatrix}}_{h(y_t, j)} + \mathcal{N}\left(0, \underbrace{\begin{pmatrix} \sigma_r & 0 & 0 \\ 0 & \sigma_\phi & 0 \\ 0 & 0 & \sigma_s \end{pmatrix}}_{Q_t}\right)$$

$$h(y_t, j) \approx h(\bar{\mu}_t, j) + H_t^i (y_t - \bar{\mu}_t)$$

$$H_t^i = h_t^i F_{x,j}$$

Measurement Model

$$z_t^i = \underbrace{\begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \text{atan2}(m_{j,y} - y, m_{j,x} - x) - \theta \\ m_{j,s} \end{pmatrix}}_{h(y_t, j)} + \mathcal{N}\left(0, \underbrace{\begin{pmatrix} \sigma_r & 0 & 0 \\ 0 & \sigma_\phi & 0 \\ 0 & 0 & \sigma_s \end{pmatrix}}_{Q_t}\right)$$

$$h(y_t, j) \approx h(\bar{\mu}_t, j) + H_t^i (y_t - \bar{\mu}_t)$$

$$H_t^i = h_t^i F_{x,j}$$

$$h_t^i = \begin{pmatrix} \frac{m_{j,x} - \bar{\mu}_{t,x}}{\sqrt{q_t}} & \frac{y_t - \bar{\mu}_{t,y}}{\sqrt{q_t}} & 0 & \frac{\bar{\mu}_{t,x} - m_{j,x}}{\sqrt{q_t}} & \frac{\bar{\mu}_{t,y} - y_t}{\sqrt{q_t}} & 0 \\ \frac{\bar{\mu}_{t,y} - y_t}{q_t} & \frac{m_{j,x} - \bar{\mu}_{t,x}}{q_t} & -1 & \frac{y_t - \bar{\mu}_{t,y}}{q_t} & \frac{\bar{\mu}_{t,x} - m_{j,x}}{q_t} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Measurement Model

$$z_t^i = \underbrace{\begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \text{atan2}(m_{j,y} - y, m_{j,x} - x) - \theta \\ m_{j,s} \end{pmatrix}}_{h(y_t, j)} + \mathcal{N}\left(0, \underbrace{\begin{pmatrix} \sigma_r & 0 & 0 \\ 0 & \sigma_\phi & 0 \\ 0 & 0 & \sigma_s \end{pmatrix}}_{Q_t}\right)$$

$$h(y_t, j) \approx h(\bar{\mu}_t, j) + H_t^i (y_t - \bar{\mu}_t)$$

$$H_t^i = h_t^i F_{x,j}$$

$$h_t^i = \begin{pmatrix} \frac{m_{j,x} - \bar{\mu}_{t,x}}{\sqrt{q_t}} & \frac{y_t - \bar{\mu}_{t,y}}{\sqrt{q_t}} & 0 & \frac{\bar{\mu}_{t,x} - m_{j,x}}{\sqrt{q_t}} & \frac{\bar{\mu}_{t,y} - y_t}{\sqrt{q_t}} & 0 \\ \frac{\bar{\mu}_{t,y} - y_t}{q_t} & \frac{m_{j,x} - \bar{\mu}_{t,x}}{q_t} & -1 & \frac{y_t - \bar{\mu}_{t,y}}{q_t} & \frac{\bar{\mu}_{t,x} - m_{j,x}}{q_t} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad F_{x,j} = \begin{pmatrix} 1 & 0 & 0 & 0 \dots 0 & 0 & 0 & 0 & 0 \dots 0 \\ 0 & 1 & 0 & 0 \dots 0 & 0 & 0 & 0 & 0 \dots 0 \\ 0 & 0 & 1 & 0 \dots 0 & 0 & 0 & 0 & 0 \dots 0 \\ 0 & 0 & 0 & 0 \dots 0 & 1 & 0 & 0 & 0 \dots 0 \\ 0 & 0 & 0 & 0 \dots 0 & 0 & 1 & 0 & 0 \dots 0 \\ 0 & 0 & 0 & \underbrace{0 \dots 0}_{2j-2} & 0 & 0 & 1 & \underbrace{0 \dots 0}_{2N-2j} \end{pmatrix}$$

EKF-SLAM

```

1: Algorithm EKF_SLAM_known_correspondences( $\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, c_t$ ):
2:    $F_x = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & \underbrace{0 \cdots 0}_{3N} \end{pmatrix}$ 
3:    $\bar{\mu}_t = \mu_{t-1} + F_x^T \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \mu_{t-1, \theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1, \theta} + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \mu_{t-1, \theta} - \frac{v_t}{\omega_t} \cos(\mu_{t-1, \theta} + \omega_t \Delta t) \\ \omega_t \Delta t \end{pmatrix}$ 
4:    $G_t = I + F_x^T \begin{pmatrix} 0 & 0 & -\frac{v_t}{\omega_t} \cos \mu_{t-1, \theta} + \frac{v_t}{\omega_t} \cos(\mu_{t-1, \theta} + \omega_t \Delta t) \\ 0 & 0 & -\frac{v_t}{\omega_t} \sin \mu_{t-1, \theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1, \theta} + \omega_t \Delta t) \\ 0 & 0 & 0 \end{pmatrix} F_x$ 
5:    $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + F_x^T R_t F_x$ 
6:    $Q_t = \begin{pmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & \sigma_s^2 \end{pmatrix}$ 
7:   for all observed features  $z_t^i = (r_t^i \ \phi_t^i \ s_t^i)^T$  do
8:      $j = c_t^i$ 
9:     if landmark  $j$  never seen before
10:       $\begin{pmatrix} \bar{\mu}_{j,x} \\ \bar{\mu}_{j,y} \\ \bar{\mu}_{j,s} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \\ s_t^i \end{pmatrix} + \begin{pmatrix} r_t^i \cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i \sin(\phi_t^i + \bar{\mu}_{t,\theta}) \\ 0 \end{pmatrix}$ 
11:    endif
12:     $\delta = \begin{pmatrix} \delta_x \\ \delta_y \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{j,x} - \bar{\mu}_{t,x} \\ \bar{\mu}_{j,y} - \bar{\mu}_{t,y} \end{pmatrix}$ 
13:     $q = \delta^T \delta$ 
14:     $\hat{z}_t^i = \begin{pmatrix} \sqrt{q} \\ \text{atan2}(\delta_y, \delta_x) - \bar{\mu}_{t,\theta} \\ \bar{\mu}_{j,s} \end{pmatrix}$ 
15:     $F_{x,j} = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 1 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & \underbrace{0 \cdots 0}_{3j-3} & 0 & 0 & 1 & \underbrace{0 \cdots 0}_{3N-3j} \end{pmatrix}$ 
16:     $H_t^i = \frac{1}{q} \begin{pmatrix} -\sqrt{q} \delta_x & -\sqrt{q} \delta_y & 0 & +\sqrt{q} \delta_x & \sqrt{q} \delta_y & 0 \\ \delta_y & -\delta_x & -q & -\delta_y & +\delta_x & 0 \\ 0 & 0 & 0 & 0 & 0 & q \end{pmatrix} F_{x,j}$ 
17:     $K_t^i = \bar{\Sigma}_t H_t^{iT} (H_t^i \bar{\Sigma}_t H_t^{iT} + Q_t)^{-1}$ 
18:     $\bar{\mu}_t = \bar{\mu}_t + K_t^i (z_t^i - \hat{z}_t^i)$ 
19:     $\bar{\Sigma}_t = (I - K_t^i H_t^i) \bar{\Sigma}_t$ 
20:  endfor
21:   $\mu_t = \bar{\mu}_t$ 
22:   $\Sigma_t = \bar{\Sigma}_t$ 
23:  return  $\mu_t, \Sigma_t$ 

```

Table 10.1 The EKF algorithm for the SLAM problem (with known correspondences).

EKF-SLAM

```

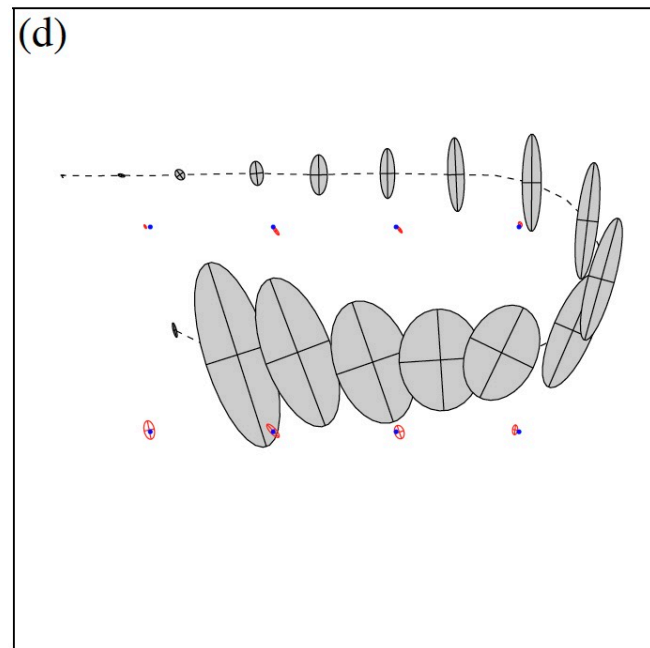
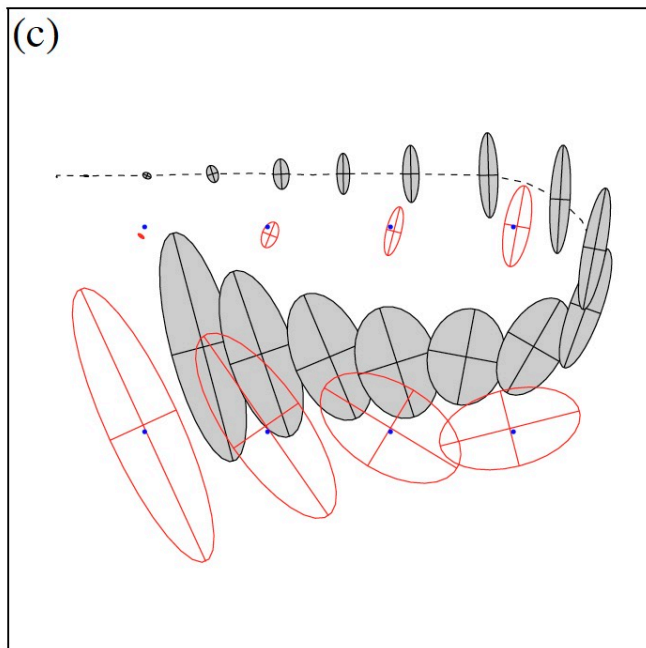
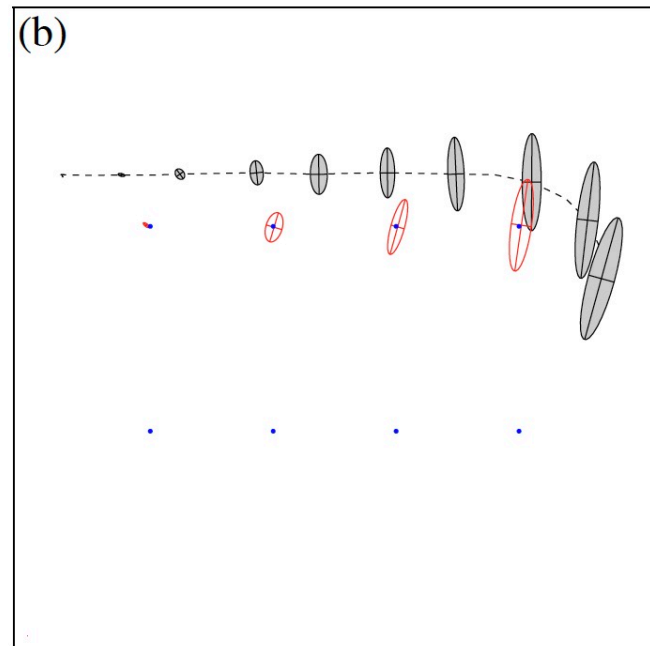
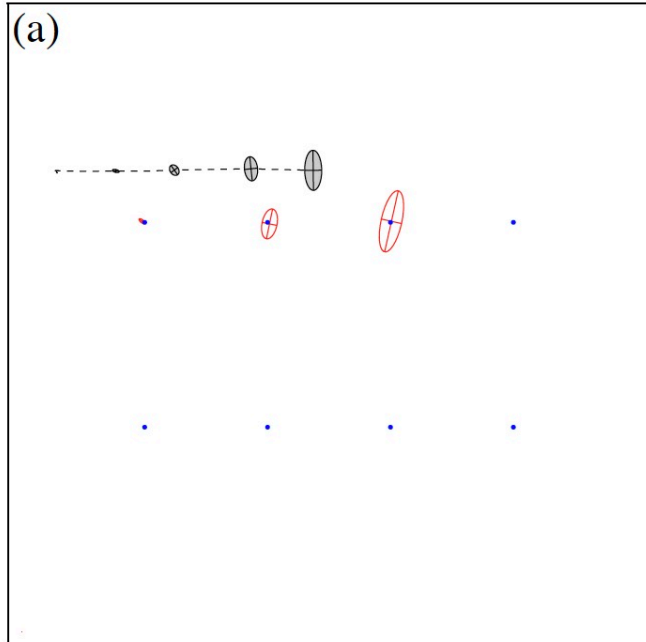
1: Algorithm EKF_SLAM_known_correspondences( $\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, c_t$ ):
2:    $F_x = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & \underbrace{0 \cdots 0}_{3N} \end{pmatrix}$ 
3:    $\bar{\mu}_t = \mu_{t-1} + F_x^T \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \mu_{t-1, \theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1, \theta} + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \mu_{t-1, \theta} - \frac{v_t}{\omega_t} \cos(\mu_{t-1, \theta} + \omega_t \Delta t) \\ \omega_t \Delta t \end{pmatrix}$ 
4:    $G_t = I + F_x^T \begin{pmatrix} 0 & 0 & -\frac{v_t}{\omega_t} \cos \mu_{t-1, \theta} + \frac{v_t}{\omega_t} \cos(\mu_{t-1, \theta} + \omega_t \Delta t) \\ 0 & 0 & -\frac{v_t}{\omega_t} \sin \mu_{t-1, \theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1, \theta} + \omega_t \Delta t) \\ 0 & 0 & 0 \end{pmatrix} F_x$ 
5:    $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + F_x^T R_t F_x$ 
6:    $Q_t = \begin{pmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & \sigma_s^2 \end{pmatrix}$ 
7:   for all observed features  $z_t^i = (r_t^i \ \phi_t^i \ s_t^i)^T$  do
8:      $j = c_t^i$ 
9:     if landmark  $j$  never seen before
10:        $\begin{pmatrix} \bar{\mu}_{j,x} \\ \bar{\mu}_{j,y} \\ \bar{\mu}_{j,s} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \\ s_t^i \end{pmatrix} + \begin{pmatrix} r_t^i \cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i \sin(\phi_t^i + \bar{\mu}_{t,\theta}) \\ 0 \end{pmatrix}$ 
11:     endif
12:      $\delta = \begin{pmatrix} \delta_x \\ \delta_y \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{j,x} - \bar{\mu}_{t,x} \\ \bar{\mu}_{j,y} - \bar{\mu}_{t,y} \end{pmatrix}$ 
13:      $q = \delta^T \delta$ 
14:      $\hat{z}_t^i = \begin{pmatrix} \sqrt{q} \\ \text{atan2}(\delta_y, \delta_x) - \bar{\mu}_{t,\theta} \\ \bar{\mu}_{j,s} \end{pmatrix}$ 
15:      $F_{x,j} = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 1 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & \underbrace{0 \cdots 0}_{3j-3} & 0 & 0 & 1 & \underbrace{0 \cdots 0}_{3N-3j} \end{pmatrix}$ 
16:      $H_t^i = \frac{1}{q} \begin{pmatrix} -\sqrt{q} \delta_x & -\sqrt{q} \delta_y & 0 & +\sqrt{q} \delta_x & \sqrt{q} \delta_y & 0 \\ \delta_y & -\delta_x & -q & -\delta_y & +\delta_x & 0 \\ 0 & 0 & 0 & 0 & 0 & q \end{pmatrix} F_{x,j}$ 
17:      $K_t^i = \bar{\Sigma}_t H_t^{iT} (H_t^i \bar{\Sigma}_t H_t^{iT} + Q_t)^{-1}$ 
18:      $\bar{\mu}_t = \bar{\mu}_t + K_t^i (z_t^i - \hat{z}_t^i)$ 
19:      $\bar{\Sigma}_t = (I - K_t^i H_t^i) \bar{\Sigma}_t$ 
20:   endfor
21:    $\mu_t = \bar{\mu}_t$ 
22:    $\Sigma_t = \bar{\Sigma}_t$ 
23:   return  $\mu_t, \Sigma_t$ 

```

Table 10.1 The EKF algorithm for the SLAM problem (with known correspondences).

EKF-SLAM Intuition

- Kalman gain is fully populated for all state variables.
- Observing the landmark improves the robot's pose estimate, which in turn improves the estimates of other landmarks.



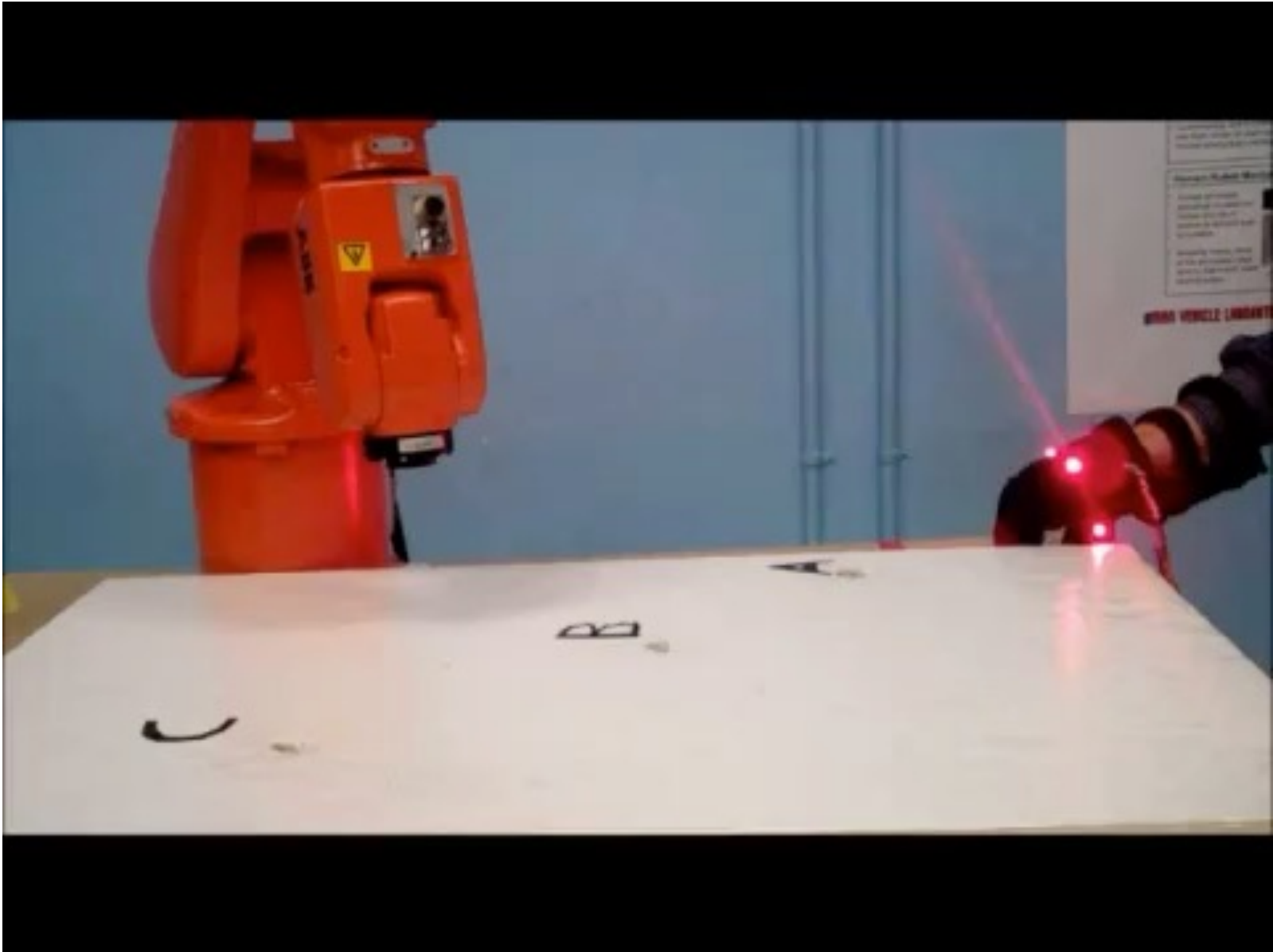
Explanation

- The position of the first landmark is well known
- Observing the first landmark again reduces uncertainty on the robot's pose
- This results in reduction of uncertainty in poses of other landmarks – from correlations represented by the non-diagonal elements of Σ_t

Explanation

- The position of the first landmark is well known
- Observing the first landmark again reduces uncertainty on the robot's pose
- This results in reduction of uncertainty in poses of other landmarks – from correlations represented by the non-diagonal elements of Σ_t
- Why does the uncertainty on previous robot poses not decrease?

Filtering Applications: Hand-Tracking for Human-Robot Interaction



Filtering Applications: Camera Calibration

The screenshot displays a Windows desktop environment used for camera calibration. The desktop features a sidebar with various application icons and a taskbar at the bottom. Several windows are open:

- Game Window:** Shows a top-down view of a green field with a red robot and a blue robot. Green numbers (37, -50, 28, 34) are overlaid on the field.
- Browser Window:** Displays the IPELA website, which is the IP Address Portable Environment for Learning and Experimentation. The address bar shows `http://192.168.1.101/10/Viewer.html`.
- Console Window:** Contains technical data for camera calibration, including estimated world position, screen coordinates, and evaluation function values for multiple cameras.
- Visualization Window:** Shows a green field with a red robot and a blue robot, similar to the game window, but with a different perspective.
- CamStudio Window:** A recording application window with the text "FenderBot CamStudio OPEN SOURCE".

The console window displays the following data:

```
Estimate world Position for (-2449,3422,2496)
Screen Coords of Active Camera [192.168.1.14] for (37, -50)
Screen Coords of Active Camera [192.168.1.12] for (45, -63)
Camera[192.168.1.10] is inactive
Camera[192.168.1.10] estimated coordinates are: (104,5)
Camera[192.168.1.11] is inactive
Camera[192.168.1.11] estimated coordinates are: (-194,-44)
Camera[192.168.1.10] evaluation function for: 0.080373
Camera[192.168.1.14] evaluation function for: 0.258465
Camera[192.168.1.11] evaluation function for: 0.000000
Camera[192.168.1.12] evaluation function for: 0.166834
```

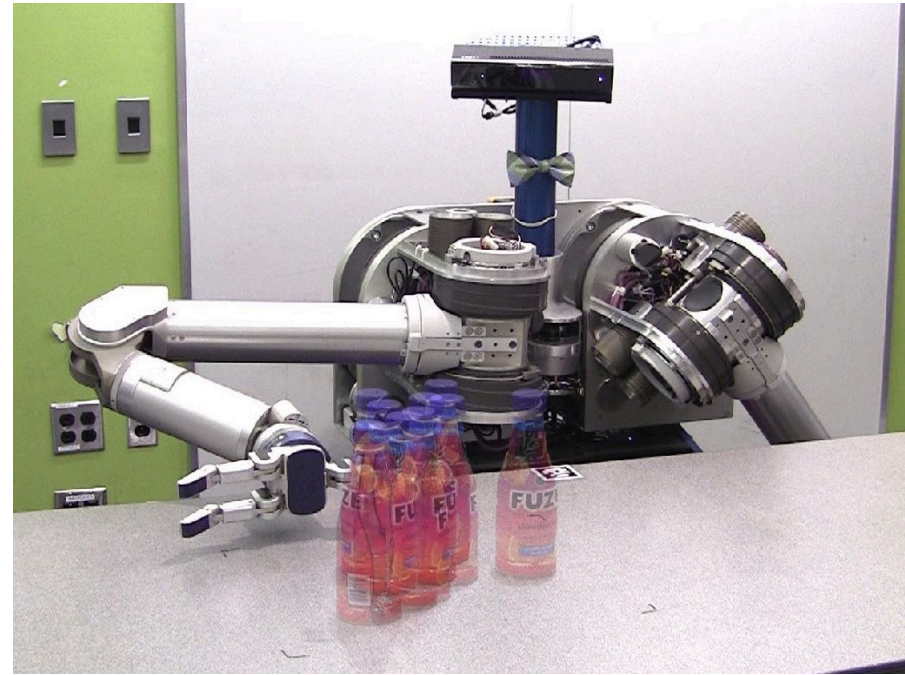
Filtering Applications: Fast Estimation



Filtering Applications: Navigation in Crowds



Filtering Applications: Object Location



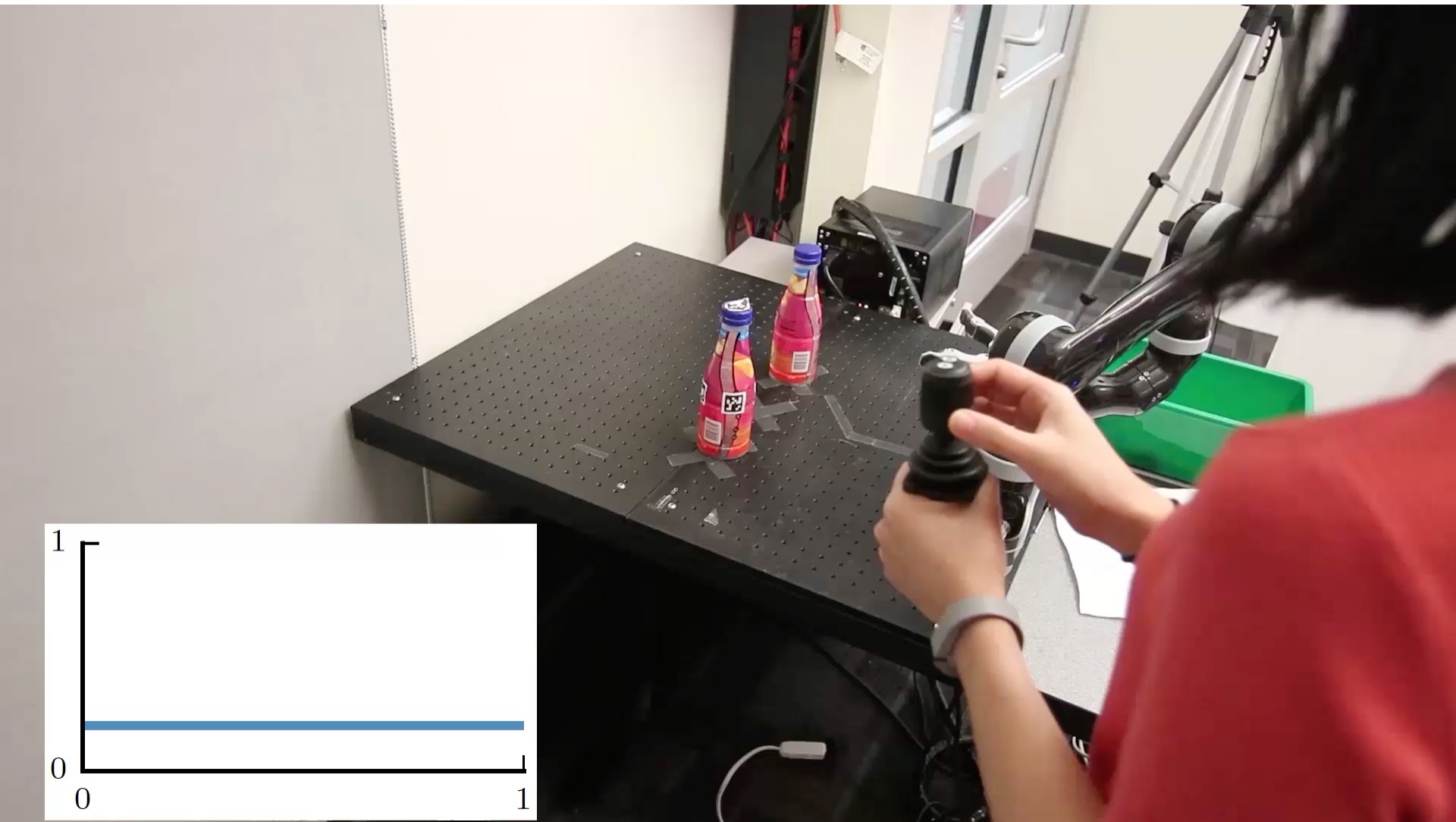
Filtering Applications: Object Properties



Filtering Applications: Human State Estimation



Filtering Applications: Human State Estimation



Extra

Mapping

