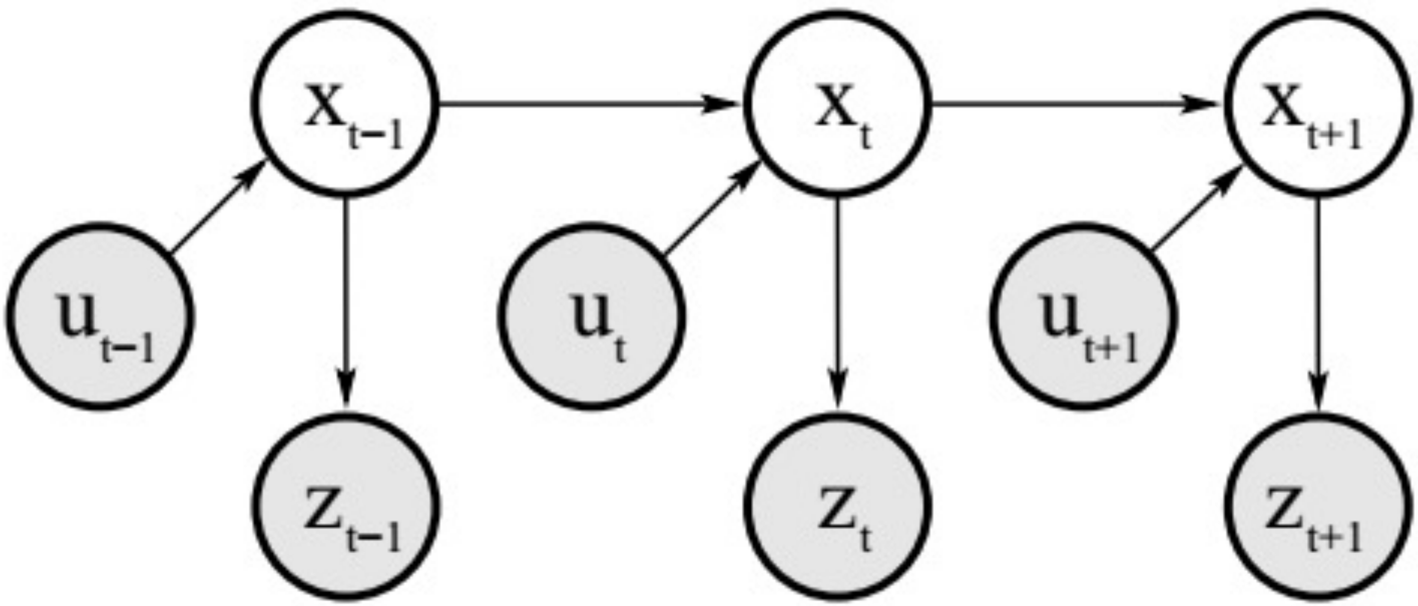
USC Viterbi School of Engineering

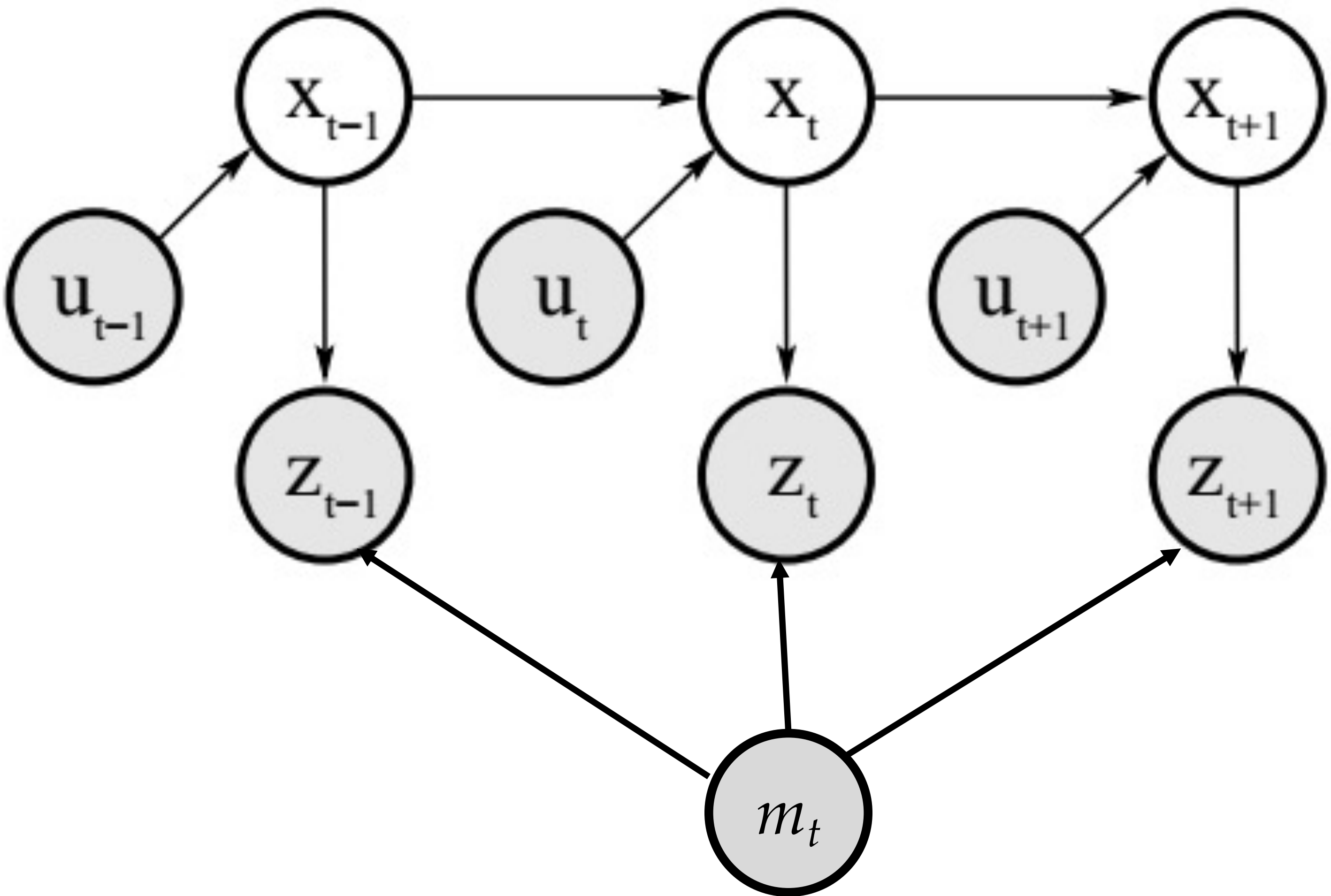
Localization and Mapping

CSCI 545 Introduction to Robotics
Instructor: Stefanos Nikolaidis

Localization



Localization



Dynamics

$$\begin{pmatrix} \hat{v}_t \\ \hat{\omega}_t \end{pmatrix} = \begin{pmatrix} v_t \\ \omega_t \end{pmatrix} + \begin{pmatrix} \epsilon_{a_1|v_t|+a_2|\omega_t|} \\ \epsilon_{a_3|v_t|+a_4|\omega_t|} \end{pmatrix}$$

$$x_t = x_{t-1} - \frac{\hat{v}_t}{\hat{\omega}_t} sin(\theta_{t-1}) + \frac{\hat{v}_t}{\hat{\omega}_t} sin(\theta_{t-1} + \hat{\omega}_t * \Delta_t)$$

$$y_t = y_{t-1} + \frac{\hat{v}_t}{\hat{\omega}_t} cos(\theta_{t-1}) - \frac{\hat{v}_t}{\hat{\omega}_t} cos(\theta_{t-1} + \hat{\omega}_t * \Delta_t)$$

$$\theta_t = \theta_{t-1} + \hat{\omega}_t * \Delta_t$$

Dynamics

$$x_t = x_{t-1} - \frac{v_t}{\omega_t} sin(\theta_{t-1}) + \frac{v_t}{\omega_t} sin(\theta_{t-1} + \omega_t * \Delta_t) + \epsilon_x$$

$$y_t = y_{t-1} + \frac{v_t}{\omega_t} cos(\theta_{t-1}) - \frac{v_t}{\omega_t} cos(\theta_{t-1} + \omega_t * \Delta_t) + \epsilon_y$$

$$\theta_t = \theta_{t-1} + \omega_t * \Delta_t + \epsilon_\theta$$

EKF

Algorithm Extended_Kalman_filter($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$)**:**

$\bar{\mu}_t = g(u_t, \mu_{t-1})$

$\bar{\Sigma}_t = G_t \, \Sigma_{t-1} \, G_t^T + R_t$

$K_t = \bar{\Sigma}_t \, H_t^T (H_t \, \bar{\Sigma}_t \, H_t^T + Q_t)^{-1}$

$\mu_t = \bar{\mu}_t + K_t(z_t - h(\bar{\mu}_t))$

$\Sigma_t = (I - K_t \, H_t) \, \bar{\Sigma}_t$

return μ_t, Σ_t

EKF

Algorithm Extended_Kalman_filter($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$)**:**

$\bar{\mu}_t = g(u_t, \mu_{t-1})$

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$\mu_t = \bar{\mu}_t + K_t(z_t - h(\bar{\mu}_t))$

$\Sigma_t = (I - K_t\ H_t)\ \bar{\Sigma}_t$

return μ_t, Σ_t

Dynamics

$$x_t = x_{t-1} - \frac{v_t}{\omega_t} sin(\theta_{t-1}) + \frac{v_t}{\omega_t} sin(\theta_{t-1} + \omega_t * \Delta_t) + \epsilon_x$$

$$y_t = y_{t-1} + \frac{v_t}{\omega_t} cos(\theta_{t-1}) - \frac{v_t}{\omega_t} cos(\theta_{t-1} + \omega_t * \Delta_t) + \epsilon_y$$

$$\theta_t = \theta_{t-1} + \omega_t * \Delta_t + \epsilon_\theta$$

EKF

$$\bar{\mu}_t = \mu_{t-1} + \begin{pmatrix} -\frac{v_t}{\omega_t}\sin\mu_{t-1,\theta} + \frac{v_t}{\omega_t}\ \sin(\mu_{t-1,\theta} + \omega_t \Delta t) \\ \frac{v_t}{\omega_t}\cos\mu_{t-1,\theta} - \frac{v_t}{\omega_t}\ \cos(\mu_{t-1,\theta} + \omega_t \Delta t) \\ \omega_t \Delta t \end{pmatrix}$$

EKF

Algorithm Extended_Kalman_filter($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$)**:**

$\bar{\mu}_t = g(u_t, \mu_{t-1})$

$\bar{\Sigma}_t = G_t \, \Sigma_{t-1} \, G_t^T + R_t$

$K_t = \bar{\Sigma}_t \, H_t^T (H_t \, \bar{\Sigma}_t \, H_t^T + Q_t)^{-1}$

$\mu_t = \bar{\mu}_t + K_t(z_t - h(\bar{\mu}_t))$

$\Sigma_t = (I - K_t \, H_t) \, \bar{\Sigma}_t$

return μ_t, Σ_t

Dynamics

$$x_t = x_{t-1} - \frac{v_t}{\omega_t} sin(\theta_{t-1}) + \frac{v_t}{\omega_t} sin(\theta_{t-1} + \omega_t * \Delta_t) + \epsilon_x$$

$$y_t = y_{t-1} + \frac{v_t}{\omega_t} cos(\theta_{t-1}) - \frac{v_t}{\omega_t} cos(\theta_{t-1} + \omega_t * \Delta_t) + \epsilon_y$$

$$\theta_t = \theta_{t-1} + \omega_t * \Delta_t + \epsilon_\theta$$

EKF

$$G_t = \begin{pmatrix} 1 & 0 & \frac{v_t}{\omega_t}\cos\mu_{t-1,\theta} - \frac{v_t}{\omega_t}\cos(\mu_{t-1,\theta} + \omega_t \Delta t) \\ 0 & 1 & \frac{v_t}{\omega_t}\sin\mu_{t-1,\theta} - \frac{v_t}{\omega_t}\sin(\mu_{t-1,\theta} + \omega_t \Delta t) \\ 0 & 0 & 1 \end{pmatrix}$$

EKF

Algorithm Extended_Kalman_filter($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$)**:**

$\bar{\mu}_t = g(u_t, \mu_{t-1})$

$\bar{\Sigma}_t = G_t \ \Sigma_{t-1} \ G_t^T + R_t$

$K_t = \bar{\Sigma}_t \ H_t^T (H_t \ \bar{\Sigma}_t \ H_t^T + Q_t)^{-1}$

$\mu_t = \bar{\mu}_t + K_t(z_t - h(\bar{\mu}_t))$

$\Sigma_t = (I - K_t \ H_t) \ \bar{\Sigma}_t$

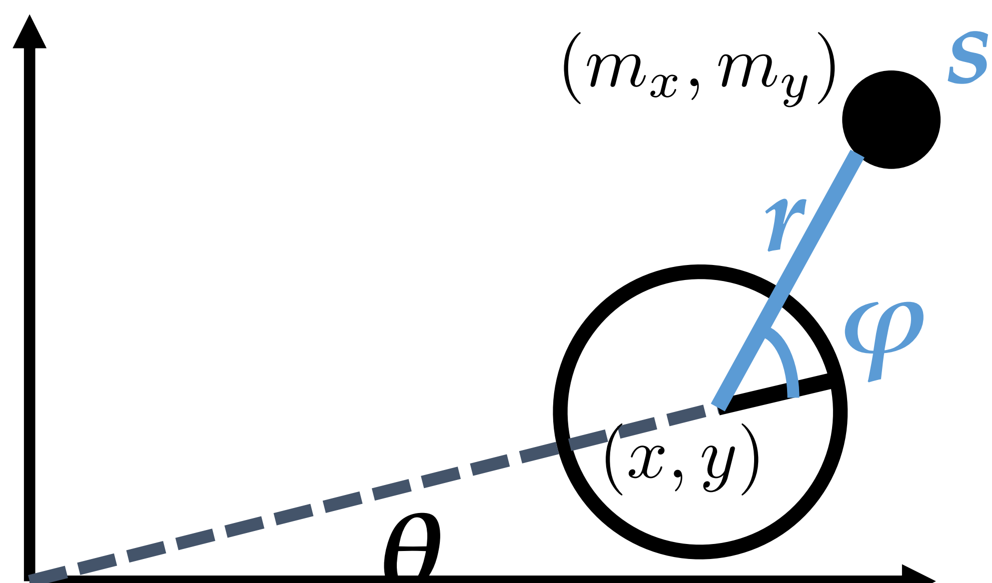
return μ_t, Σ_t

Measurements

$$\underbrace{\begin{pmatrix} r_t^i \\ \phi_t^i \\ s_t^i \end{pmatrix}}_{z_t^i} = \underbrace{\begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \operatorname{atan2}(m_{j,y} - y, m_{j,x} - x) - \theta \\ m_{j,s} \end{pmatrix}}_{h(x_t, j, m)} + \mathcal{N}(0, Q_t),$$

Feature-Based Sensor Models

$$\begin{pmatrix} r_t^i \\ \phi_t^i \\ s_t^i \end{pmatrix} = \begin{pmatrix} \sqrt{(m_{j,x}-x)^2+(m_{j,y}-y)^2} \\ \text{atan2}(m_{j,y}-y, m_{j,x}-x)-\theta \\ s_j \end{pmatrix} + \begin{pmatrix} \varepsilon_{\sigma_r^2} \\ \varepsilon_{\sigma_\phi^2} \\ \varepsilon_{\sigma_s^2} \end{pmatrix}$$



Measurements

$$\underbrace{\begin{pmatrix} r_t^i \\ \phi_t^i \\ s_t^i \end{pmatrix}}_{z_t^i} = \underbrace{\begin{pmatrix} \sqrt{(m_{j,x}-x)^2+(m_{j,y}-y)^2} \\ \operatorname{atan2}(m_{j,y}-y, m_{j,x}-x)-\theta \\ m_{j,s} \end{pmatrix}}_{h(x_t,j,m)} + \mathcal{N}(0, Q_t),$$

EKF

$$Q_t = \begin{pmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & \sigma_s^2 \end{pmatrix}$$

$$H_t^i = \frac{\partial h(\bar{\mu}_t, j, m)}{\partial x_t} = \begin{pmatrix} \frac{\partial r_t^i}{\partial \bar{\mu}_{t,x}} & \frac{\partial r_t^i}{\partial \bar{\mu}_{t,y}} & \frac{\partial r_t^i}{\partial \bar{\mu}_{t,\theta}} \\ \frac{\partial \phi_t^i}{\partial \bar{\mu}_{t,x}} & \frac{\partial \phi_t^i}{\partial \bar{\mu}_{t,y}} & \frac{\partial \phi_t^i}{\partial \bar{\mu}_{t,\theta}} \\ \frac{\partial s_t^i}{\partial \bar{\mu}_{t,x}} & \frac{\partial s_t^i}{\partial \bar{\mu}_{t,y}} & \frac{\partial s_t^i}{\partial \bar{\mu}_{t,\theta}} \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{m_{j,x}-\bar{\mu}_{t,x}}{\sqrt{q}} & -\frac{m_{j,y}-\bar{\mu}_{t,y}}{\sqrt{q}} & 0 \\ \frac{m_{j,y}-\bar{\mu}_{t,y}}{q} & -\frac{m_{j,x}-\bar{\mu}_{t,x}}{q} & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$q_t = (m_{j,x} - \bar{\mu}_{t,x})^2 + (m_{j,y} - \bar{\mu}_{t,y})^2$$

EKF

```
Algorithm Extended_Kalman_filter(μ_{t-1}, Σ_{t-1}, u_t, z_t):
    μ̄_t = g(u_t, μ_{t-1})
    Σ̄_t = G_t Σ_{t-1} G_t^T + R_t
    K_t = Σ̄_t H_t^T (H_t Σ̄_t H_t^T + Q_t)^{-1}
    μ_t = μ̄_t + K_t (z_t - h(μ̄_t))
    Σ_t = (I - K_t H_t) Σ̄_t
    return μ_t, Σ_t
```

Algorithm Extended_Kalman_filter($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$)**:**

$\bar{\mu}_t = g(u_t, \mu_{t-1})$

$\bar{\Sigma}_t = G_t\ \Sigma_{t-1}\ G_t^T + R_t$

$K_t = \bar{\Sigma}_t\ H_t^T (H_t\ \bar{\Sigma}_t\ H_t^T + Q_t)^{-1}$

$\mu_t = \bar{\mu}_t + K_t(z_t - h(\bar{\mu}_t))$

$\Sigma_t = (I - K_t\ H_t)\ \bar{\Sigma}_t$

return μ_t, Σ_t

EKF

Algorithm EKF_localization_known_correspondences($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, c_t, m$):

$$\theta = \mu_{t-1,\theta}$$

$$G_t = \begin{pmatrix} 1 & 0 & -\frac{v_t}{\omega_t}\cos\theta + \frac{v_t}{\omega_t}\cos(\theta + \omega_t \Delta t) \\ 0 & 1 & -\frac{v_t}{\omega_t}\sin\theta + \frac{v_t}{\omega_t}\sin(\theta + \omega_t \Delta t) \\ 0 & 0 & 1 \end{pmatrix}$$

$$V_t = \begin{pmatrix} \frac{-\sin\theta + \sin(\theta+\omega_t\Delta t)}{\omega_t} & \frac{v_t(\sin\theta - \sin(\theta+\omega_t\Delta t))}{\omega_t^2} + \frac{v_t\cos(\theta+\omega_t\Delta t)\Delta t}{\omega_t} \\ \frac{\cos\theta - \cos(\theta+\omega_t\Delta t)}{\omega_t} & -\frac{v_t(\cos\theta - \cos(\theta+\omega_t\Delta t))}{\omega_t^2} + \frac{v_t\sin(\theta+\omega_t\Delta t)\Delta t}{\omega_t} \\ 0 & \Delta t \end{pmatrix}$$

$$M_t = \begin{pmatrix} \alpha_1 v_t^2 + \alpha_2 \omega_t^2 & 0 \\ 0 & \alpha_3 v_t^2 + \alpha_4 \omega_t^2 \end{pmatrix}$$

$$\bar{\mu}_t = \mu_{t-1} + \begin{pmatrix} -\frac{v_t}{\omega_t}\sin\theta + \frac{v_t}{\omega_t}\sin(\theta+\omega_t\Delta t) \\ \frac{v_t}{\omega_t}\cos\theta - \frac{v_t}{\omega_t}\cos(\theta+\omega_t\Delta t) \\ \omega_t \Delta t \end{pmatrix}$$

$$\bar{\Sigma}_t = G_t\, \Sigma_{t-1}\, G_t{}^T + \underbrace{V_t\, M_t\, V_t{}^T}_{R}$$

$$Q_t = \begin{pmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & \sigma_s^2 \end{pmatrix}$$

for all observed features $z_t^i = (r_t^i \;\; \phi_t^i \;\; s_t^i)^T$ *do*

$$j = c_t^i$$

$$q = (m_{j,x} - \bar{\mu}_{t,x})^2 + (m_{j,y} - \bar{\mu}_{t,y})^2$$

$$\hat{z}_t^i = \begin{pmatrix} \sqrt{q} \\ \operatorname{atan2}(m_{j,y} - \bar{\mu}_{t,y}, m_{j,x} - \bar{\mu}_{t,x}) - \bar{\mu}_{t,\theta} \\ m_{j,s} \end{pmatrix}$$

$$H_t^i = \begin{pmatrix} -\frac{m_{j,x} - \bar{\mu}_{t,x}}{\sqrt{q}} & -\frac{m_{j,y} - \bar{\mu}_{t,y}}{\sqrt{q}} & 0 \\ \frac{m_{j,y} - \bar{\mu}_{t,y}}{q} & -\frac{m_{j,x} - \bar{\mu}_{t,x}}{q} & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$S_t^i = H_t^i\, \bar{\Sigma}_t\, [H_t^i]^T + Q_t$$

$$K_t^i = \bar{\Sigma}_t\, [H_t^i]^T [S_t^i]^{-1}$$

$$\bar{\mu}_t = \bar{\mu}_t + K_t^i (z_t^i - \hat{z}_t^i)$$

$$\bar{\Sigma}_t = (I - K_t^i\, H_t^i)\, \bar{\Sigma}_t$$

endfor

$$\mu_t = \bar{\mu}_t$$

$$\Sigma_t = \bar{\Sigma}_t$$

$$p_{z_t} = \prod_i \det\left(2\pi S_t^i\right)^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}\,(z_t^i - \hat{z}_t^i)^T [S_t^i]^{-1} (z_t^i - \hat{z}_t^i)\right\}$$

return μ_t, Σ_t, p_{z_t}

Example

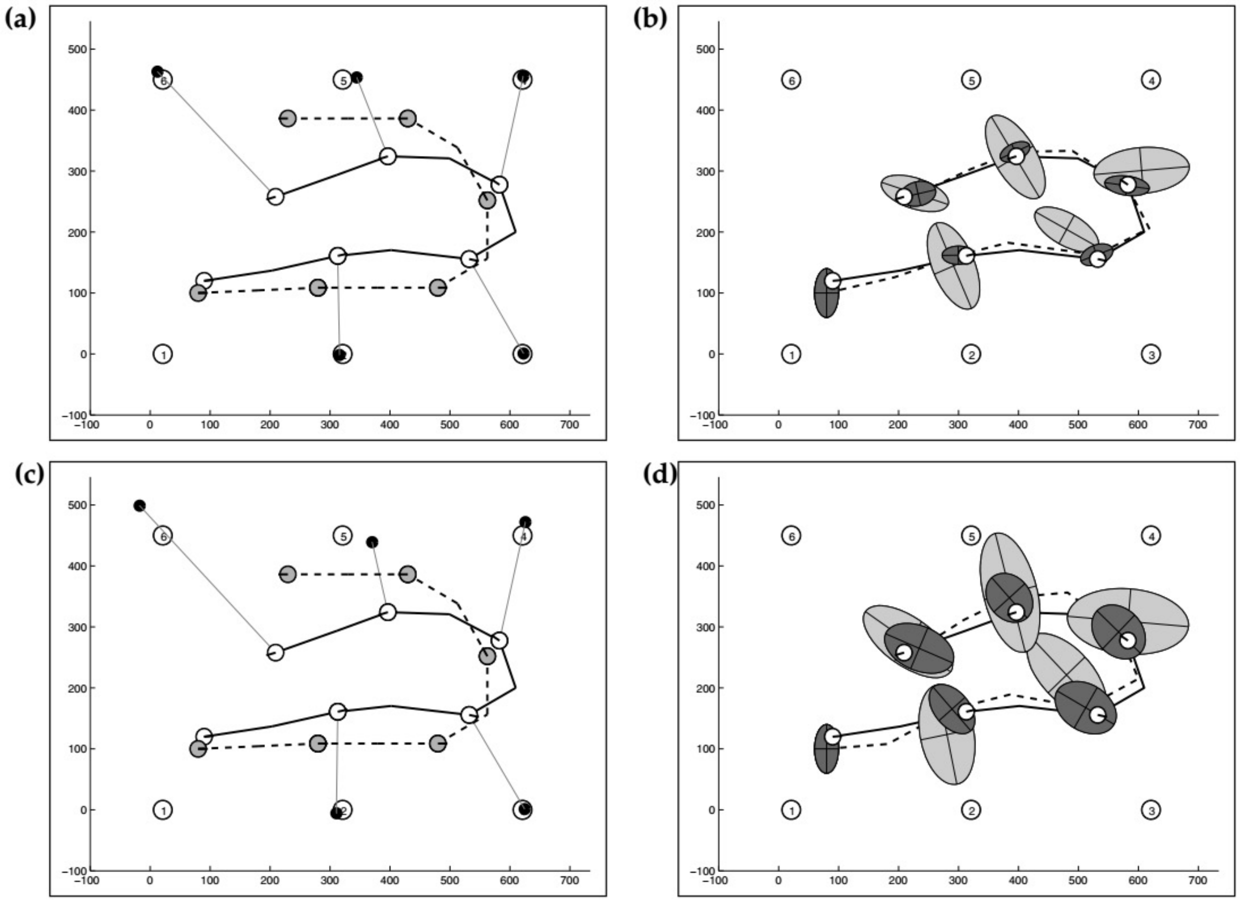


Figure 7.11 EKF-based localization with an accurate (upper row) and a less accurate (lower row) landmark detection sensor. The dashed lines in the left panel indicate the robot trajectories as estimated from the motion controls. The solid lines represent the true robot motion resulting from these controls. Landmark detections at five locations are indicated by the thin lines. The dashed lines in the right panels show the corrected robot trajectories, along with uncertainty before (light gray, $\bar{\Sigma}_t$) and after (dark gray, Σ_t) incorporating a landmark detection.

Localization with Unknown Correspondences

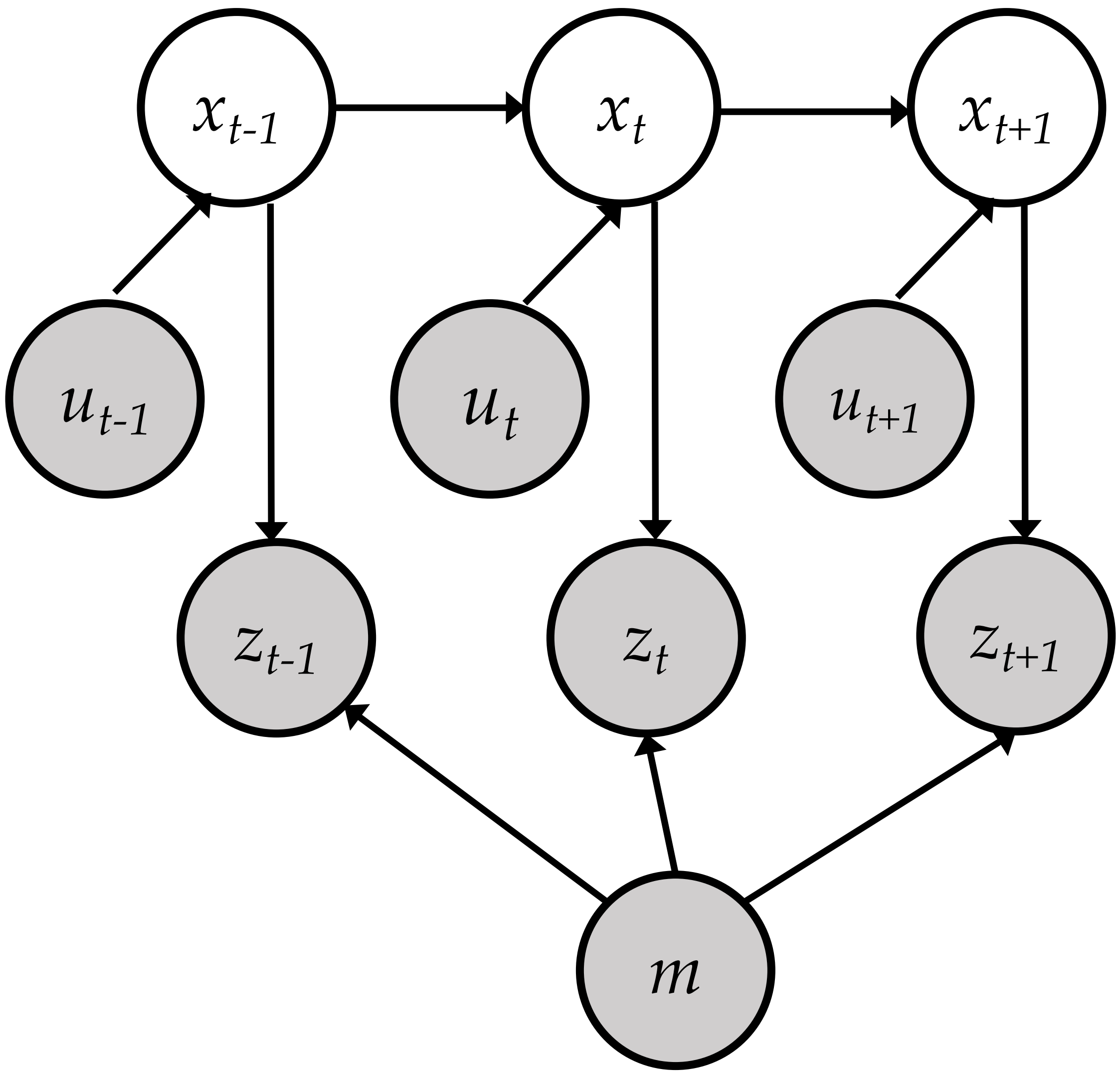
- How to do EKF-localization when the correspondences are not known?

Localization with Unknown Correspondences

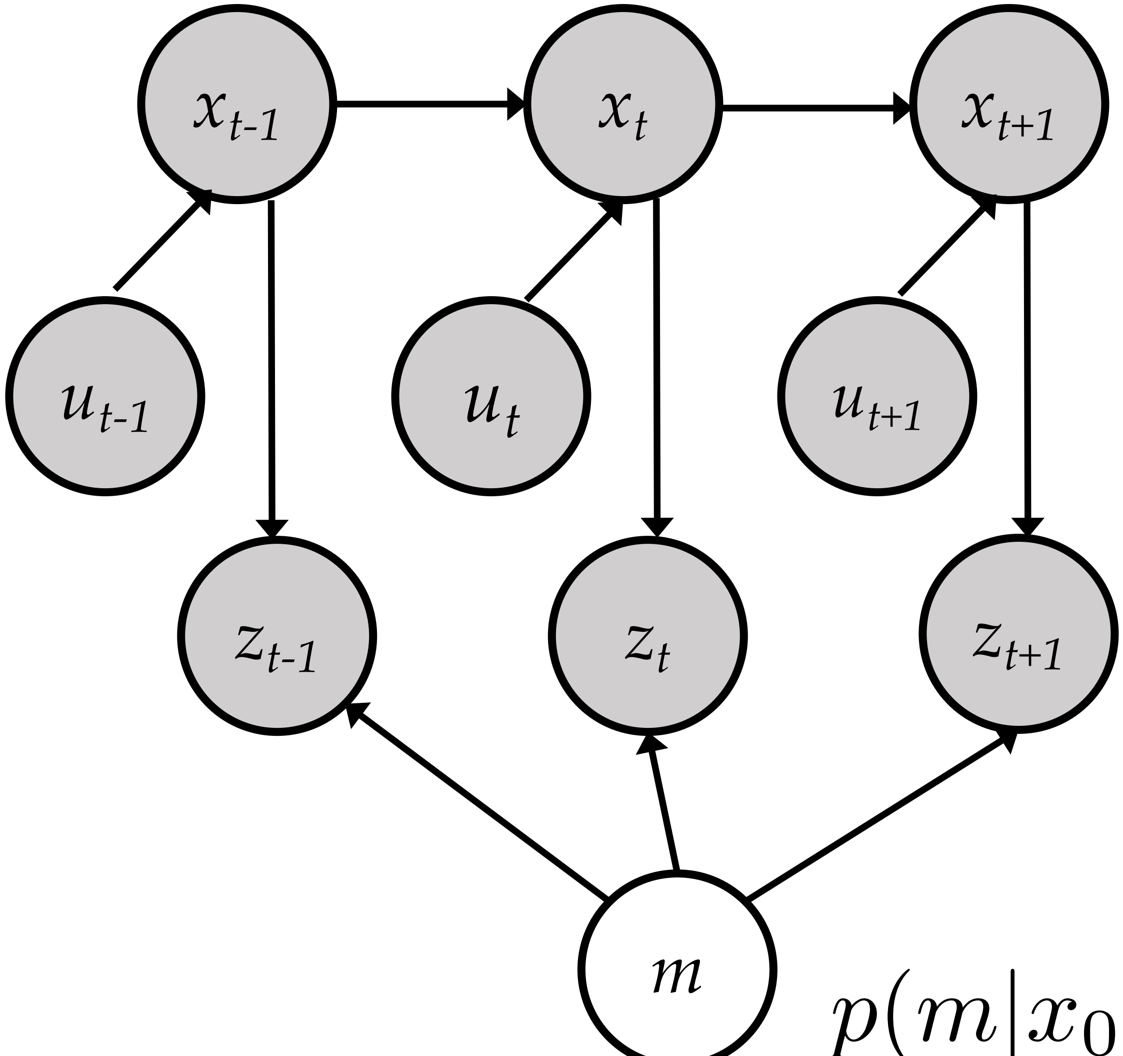
- How to do EKF-localization when the correspondences are not known?

- Maximum Likelihood Estimation

Localization

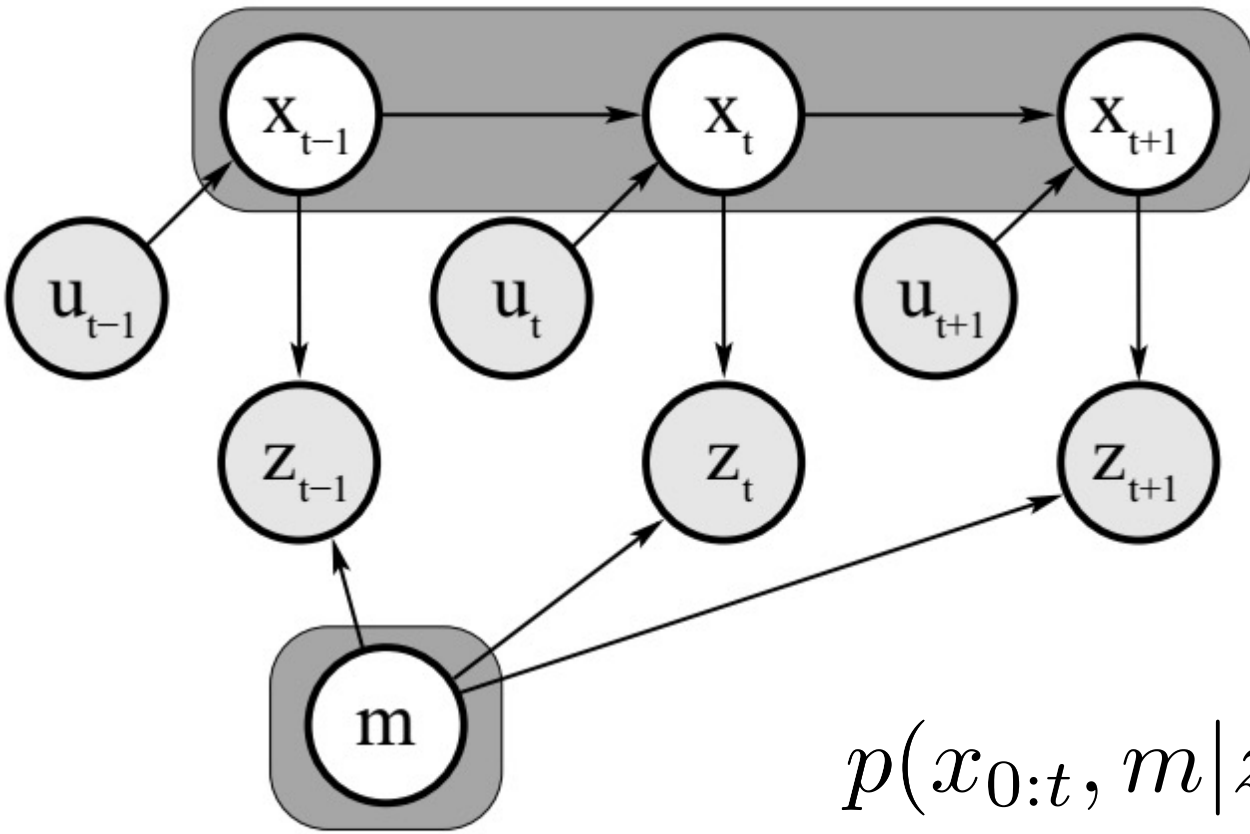


Mapping



$$p(m | x_{0:t}, z_{0:t})$$

Simultaneous Localization and Mapping



$$p(x_{0:t}, m | z_{0:t}, u_{0:t})$$

Simultaneous Localization and Mapping

Simultaneous Localization and Mapping

EKF - SLAM

$$\begin{aligned} y_t &= \begin{pmatrix} x_t \\ m \end{pmatrix} \\ &= \begin{pmatrix} x & y & \theta & m_{1,x} & m_{1,y} & s_1 & m_{2,x} & m_{2,y} & s_2 & \ldots & m_{N,x} & m_{N,y} & s_N \end{pmatrix}^T \end{aligned}$$

Assumptions:

- Static map
- Known set of landmarks
- Known correspondences

EKF

Algorithm Extended_Kalman_filter($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$)**:**

$\bar{\mu}_t = g(u_t, \mu_{t-1})$

$\bar{\Sigma}_t = G_t \, \Sigma_{t-1} \, G_t^T + R_t$

$K_t = \bar{\Sigma}_t \, H_t^T (H_t \, \bar{\Sigma}_t \, H_t^T + Q_t)^{-1}$

$\mu_t = \bar{\mu}_t + K_t(z_t - h(\bar{\mu}_t))$

$\Sigma_t = (I - K_t \, H_t) \, \bar{\Sigma}_t$

return μ_t, Σ_t

Initial Mean and Covariance

$$\mu_0 = \begin{pmatrix} 0 & 0 & 0 & \ldots & 0 \end{pmatrix}^T$$

$$\Sigma_0 = \begin{pmatrix} 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \infty & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \infty \end{pmatrix}$$

Dynamics Model

$$y_t = y_{t-1} + \begin{pmatrix} -\frac{v_t}{\omega_t}\sin\theta + \frac{v_t}{\omega_t}\ \sin(\theta + \omega_t \Delta t) \\ \frac{v_t}{\omega_t}\cos\theta - \frac{v_t}{\omega_t}\ \cos(\theta + \omega_t \Delta t) \\ \omega_t \Delta t + \gamma_t \Delta t \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

Dynamics Model

$$y_t = y_{t-1} + \begin{pmatrix} -\frac{v_t}{\omega_t}\sin\theta + \frac{v_t}{\omega_t}\ \sin(\theta + \omega_t \Delta t) \\ \frac{v_t}{\omega_t}\cos\theta - \frac{v_t}{\omega_t}\ \cos(\theta + \omega_t \Delta t) \\ \omega_t \Delta t + \gamma_t \Delta t \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$y_t = y_{t-1} + F_x^T \begin{pmatrix} -\frac{v_t}{\omega_t}\sin\theta + \frac{v_t}{\omega_t}\ \sin(\theta + \omega_t \Delta t) \\ \frac{v_t}{\omega_t}\cos\theta - \frac{v_t}{\omega_t}\ \cos(\theta + \omega_t \Delta t) \\ \omega_t \Delta t + \gamma_t \Delta t \end{pmatrix}$$

$$F_x = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & \underbrace{0 \cdots 0}_{3N \text{ columns}} \end{pmatrix}$$

Dynamics Model

$$y_t = y_{t-1} + \begin{pmatrix} -\frac{v_t}{\omega_t}\sin\theta + \frac{v_t}{\omega_t}\ \sin(\theta+\omega_t\Delta t) \\ \frac{v_t}{\omega_t}\cos\theta - \frac{v_t}{\omega_t}\ \cos(\theta+\omega_t\Delta t) \\ \omega_t\Delta t + \gamma_t\Delta t \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$y_t = \underbrace{y_{t-1} + F_x^T \begin{pmatrix} -\frac{v_t}{\omega_t}\sin\theta + \frac{v_t}{\omega_t}\ \sin(\theta+\omega_t\Delta t) \\ \frac{v_t}{\omega_t}\cos\theta - \frac{v_t}{\omega_t}\ \cos(\theta+\omega_t\Delta t) \\ \omega\Delta t \end{pmatrix}}_{g(u_t, y_{t-1})} + \mathcal{N}(0, F_x^T\ R_t\ F_x)$$

$$F_x = \begin{pmatrix} 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & \underbrace{0 \cdots 0}_{3N \text{ columns}} \end{pmatrix}$$

Dynamics Model

$$y_t = y_{t-1} + \begin{pmatrix} -\frac{v_t}{\omega_t}\sin\theta + \frac{v_t}{\omega_t}\ \sin(\theta + \omega_t \Delta t) \\ \frac{v_t}{\omega_t}\cos\theta - \frac{v_t}{\omega_t}\ \cos(\theta + \omega_t \Delta t) \\ \omega_t \Delta t + \gamma_t \Delta t \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$g(u_t, y_{t-1}) \approx g(u_t, \mu_{t-1}) + G_t\ (y_{t-1} - \mu_{t-1})$$

EKF-SLAM

Algorithm EKF_SLAM_known_correspondences($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, c_t$):

$$F_x = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & \underbrace{0 \cdots 0}_{3N} \end{pmatrix}$$

$$\bar{\mu}_t = \mu_{t-1} + F_x^T \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \mu_{t-1,\theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1,\theta} + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \mu_{t-1,\theta} - \frac{v_t}{\omega_t} \cos(\mu_{t-1,\theta} + \omega_t \Delta t) \\ \omega_t \Delta t \end{pmatrix}$$

$$G_t = I + F_x^T \begin{pmatrix} 0 & 0 & -\frac{v_t}{\omega_t} \cos \mu_{t-1,\theta} + \frac{v_t}{\omega_t} \cos(\mu_{t-1,\theta} + \omega_t \Delta t) \\ 0 & 0 & -\frac{v_t}{\omega_t} \sin \mu_{t-1,\theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1,\theta} + \omega_t \Delta t) \\ 0 & 0 & 0 \end{pmatrix} F_x$$

$$\bar{\Sigma}_t = G_t \, \Sigma_{t-1} \, G_t^T + F_x^T \, R_t \, F_x$$

$$Q_t = \begin{pmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & \sigma_s^2 \end{pmatrix}$$

for all observed features $z_t^i = (r_t^i \;\; \phi_t^i \;\; s_t^i)^T$ *do*

$j = c_t^i$

if landmark j *never seen before*

$$\begin{pmatrix} \bar{\mu}_{j,x} \\ \bar{\mu}_{j,y} \\ \bar{\mu}_{j,s} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \\ s_t^i \end{pmatrix} + \begin{pmatrix} r_t^i \cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i \sin(\phi_t^i + \bar{\mu}_{t,\theta}) \\ 0 \end{pmatrix}$$

endif

$$\delta = \begin{pmatrix} \delta_x \\ \delta_y \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{j,x} - \bar{\mu}_{t,x} \\ \bar{\mu}_{j,y} - \bar{\mu}_{t,y} \end{pmatrix}$$

$q = \delta^T \delta$

$$\hat{z}_t^i = \begin{pmatrix} \sqrt{q} \\ \operatorname{atan2}(\delta_y, \delta_x) - \bar{\mu}_{t,\theta} \\ \bar{\mu}_{j,s} \end{pmatrix}$$

$$F_{x,j} = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 1 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & \underbrace{0 \cdots 0}_{3j-3} & 0 & 0 & 1 & \underbrace{0 \cdots 0}_{3N-3j} \end{pmatrix}$$

$$H_t^i = \frac{1}{q} \begin{pmatrix} -\sqrt{q}\delta_x & -\sqrt{q}\delta_y & 0 & +\sqrt{q}\delta_x & \sqrt{q}\delta_y & 0 \\ \delta_y & -\delta_x & -q & -\delta_y & +\delta_x & 0 \\ 0 & 0 & 0 & 0 & 0 & q \end{pmatrix} F_{x,j}$$

$K_t^i = \bar{\Sigma}_t \, H_t^{iT} (H_t^i \, \bar{\Sigma}_t \, H_t^{iT} + Q_t)^{-1}$

$\bar{\mu}_t = \bar{\mu}_t + K_t^i (z_t^i - \hat{z}_t^i)$

$\bar{\Sigma}_t = (I - K_t^i \, H_t^i) \, \bar{\Sigma}_t$

endfor

$\mu_t = \bar{\mu}_t$

$\Sigma_t = \bar{\Sigma}_t$

return μ_t, Σ_t

Table 10.1 The EKF algorithm for the SLAM problem (with known correspondences).

Measurement Model

$$z_t^i = \underbrace{\begin{pmatrix} \sqrt{(m_{j,x}-x)^2 + (m_{j,y}-y)^2} \\ \text{atan2}(m_{j,y}-y, m_{j,x}-x) - \theta \\ m_{j,s} \end{pmatrix}}_{h(y_t,j)} + \mathcal{N}(0, \underbrace{\begin{pmatrix} \sigma_r & 0 & 0 \\ 0 & \sigma_\phi & 0 \\ 0 & 0 & \sigma_s \end{pmatrix}}_{Q_t})$$

$$h(y_t, j) \approx h(\bar{\mu}_t, j) + H_t^i \, (y_t - \bar{\mu}_t)$$

Measurement Model

$$z_t^i \;=\; \underbrace{\begin{pmatrix} \sqrt{(m_{j,x}-x)^2+(m_{j,y}-y)^2} \\ \operatorname{atan2}(m_{j,y}-y, m_{j,x}-x)-\theta \\ m_{j,s} \end{pmatrix}}_{h(y_t,j)} + \mathcal{N}(0, \underbrace{\begin{pmatrix} \sigma_r & 0 & 0 \\ 0 & \sigma_\phi & 0 \\ 0 & 0 & \sigma_s \end{pmatrix}}_{Q_t})$$

$$h(y_t, j) \;\approx\; h(\bar{\mu}_t, j) \;+\; H_t^i\,(y_t - \bar{\mu}_t)$$

What if landmark is seen for the first time?

Measurement Model

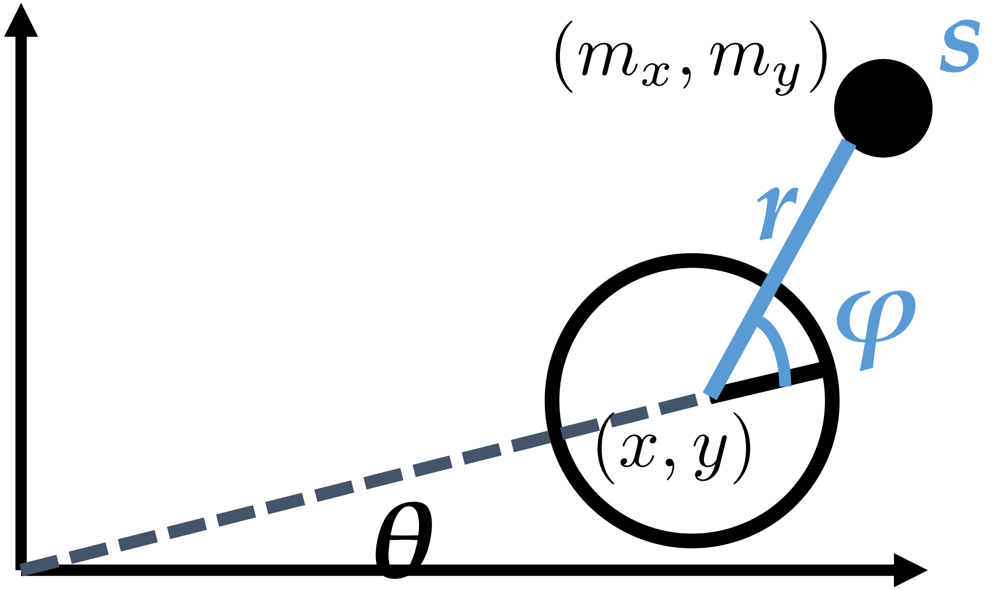
$$z_t^i = \underbrace{\begin{pmatrix} \sqrt{(m_{j,x}-x)^2+(m_{j,y}-y)^2} \\ \operatorname{atan2}(m_{j,y}-y, m_{j,x}-x)-\theta \\ m_{j,s} \end{pmatrix}}_{h(y_t,j)} + \mathcal{N}(0, \underbrace{\begin{pmatrix} \sigma_r & 0 & 0 \\ 0 & \sigma_\phi & 0 \\ 0 & 0 & \sigma_s \end{pmatrix}}_{Q_t})$$

$$h(y_t, j) \approx h(\bar{\mu}_t, j) + H_t^i \, (y_t - \bar{\mu}_t)$$

What if landmark is seen for the first time?

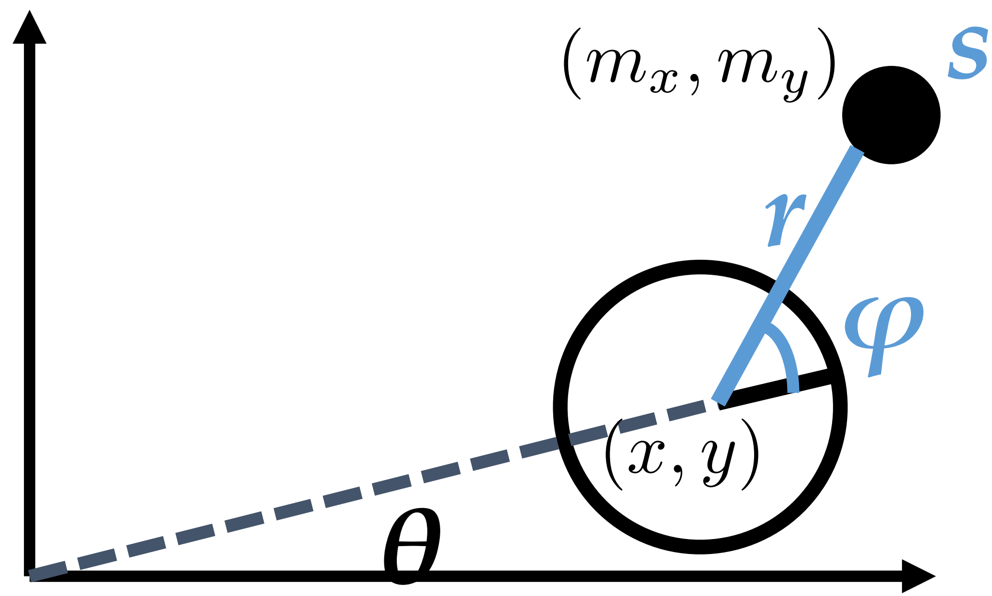
Estimate it based on the robot's position!

Measurement Model



Measurement Model

$$\begin{pmatrix} \bar{\mu}_{j,x} \\ \bar{\mu}_{j,y} \\ \bar{\mu}_{j,s} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \\ s_t^i \end{pmatrix} + \begin{pmatrix} r_t^i \cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i \sin(\phi_t^i + \bar{\mu}_{t,\theta}) \\ 0 \end{pmatrix}$$



Measurement Model

$$z_t^i \;=\; \underbrace{\begin{pmatrix} \sqrt{(m_{j,x}-x)^2+(m_{j,y}-y)^2} \\ \text{atan2}(m_{j,y}-y, m_{j,x}-x)-\theta \\ m_{j,s} \end{pmatrix}}_{h(y_t,j)} + \mathcal{N}(0, \underbrace{\begin{pmatrix} \sigma_r & 0 & 0 \\ 0 & \sigma_\phi & 0 \\ 0 & 0 & \sigma_s \end{pmatrix}}_{Q_t})$$

$$h(y_t, j) \;\approx\; h(\bar{\mu}_t, j) \;+\; H_t^i\,(y_t - \bar{\mu}_t)$$

What if landmark is seen for the first time?

Estimate it based on the robot's position!

Otherwise use previous estimate

EKF-SLAM

Algorithm EKF_SLAM_known_correspondences($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, c_t$):

$$F_x = \begin{pmatrix} 1 & 0 & 0 & 0\cdots 0 \\ 0 & 1 & 0 & 0\cdots 0 \\ 0 & 0 & 1 & \underbrace{0\cdots 0}_{3N} \end{pmatrix}$$

$$\bar{\mu}_t = \mu_{t-1} + F_x^T \begin{pmatrix} -\frac{v_t}{\omega_t}\sin\mu_{t-1,\theta} + \frac{v_t}{\omega_t}\sin(\mu_{t-1,\theta}+\omega_t\Delta t) \\ \frac{v_t}{\omega_t}\cos\mu_{t-1,\theta} - \frac{v_t}{\omega_t}\cos(\mu_{t-1,\theta}+\omega_t\Delta t) \\ \omega_t\Delta t \end{pmatrix}$$

$$G_t = I + F_x^T \begin{pmatrix} 0 & 0 & -\frac{v_t}{\omega_t}\cos\mu_{t-1,\theta} + \frac{v_t}{\omega_t}\cos(\mu_{t-1,\theta}+\omega_t\Delta t) \\ 0 & 0 & -\frac{v_t}{\omega_t}\sin\mu_{t-1,\theta} + \frac{v_t}{\omega_t}\sin(\mu_{t-1,\theta}+\omega_t\Delta t) \\ 0 & 0 & 0 \end{pmatrix} F_x$$

$$\bar{\Sigma}_t = G_t\,\Sigma_{t-1}\,G_t^T + F_x^T\,R_t\,F_x$$

$$Q_t = \begin{pmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & \sigma_s^2 \end{pmatrix}$$

for all observed features $z_t^i = (r_t^i\ \phi_t^i\ s_t^i)^T$ *do*

$j = c_t^i$

if landmark j *never seen before*

$$\begin{pmatrix} \bar{\mu}_{j,x} \\ \bar{\mu}_{j,y} \\ \bar{\mu}_{j,s} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \\ s_t^i \end{pmatrix} + \begin{pmatrix} r_t^i\cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i\sin(\phi_t^i + \bar{\mu}_{t,\theta}) \\ 0 \end{pmatrix}$$

endif

$$\delta = \begin{pmatrix} \delta_x \\ \delta_y \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{j,x} - \bar{\mu}_{t,x} \\ \bar{\mu}_{j,y} - \bar{\mu}_{t,y} \end{pmatrix}$$

$q = \delta^T\delta$

$$\hat{z}_t^i = \begin{pmatrix} \sqrt{q} \\ \operatorname{atan2}(\delta_y,\delta_x) - \bar{\mu}_{t,\theta} \\ \bar{\mu}_{j,s} \end{pmatrix}$$

$$F_{x,j} = \begin{pmatrix} 1 & 0 & 0 & 0\cdots 0 & 0 & 0 & 0 & 0\cdots 0 \\ 0 & 1 & 0 & 0\cdots 0 & 0 & 0 & 0 & 0\cdots 0 \\ 0 & 0 & 1 & 0\cdots 0 & 0 & 0 & 0 & 0\cdots 0 \\ 0 & 0 & 0 & 0\cdots 0 & 1 & 0 & 0 & 0\cdots 0 \\ 0 & 0 & 0 & 0\cdots 0 & 0 & 1 & 0 & 0\cdots 0 \\ 0 & 0 & 0 & \underbrace{0\cdots 0}_{3j-3} & 0 & 0 & 1 & \underbrace{0\cdots 0}_{3N-3j} \end{pmatrix}$$

$$H_t^i = \frac{1}{q}\begin{pmatrix} -\sqrt{q}\delta_x & -\sqrt{q}\delta_y & 0 & +\sqrt{q}\delta_x & \sqrt{q}\delta_y & 0 \\ \delta_y & -\delta_x & -q & -\delta_y & +\delta_x & 0 \\ 0 & 0 & 0 & 0 & 0 & q \end{pmatrix} F_{x,j}$$

$K_t^i = \bar{\Sigma}_t\,H_t^{iT}(H_t^i\,\bar{\Sigma}_t\,H_t^{iT} + Q_t)^{-1}$

$\bar{\mu}_t = \bar{\mu}_t + K_t^i(z_t^i - \hat{z}_t^i)$

$\bar{\Sigma}_t = (I - K_t^i\,H_t^i)\,\bar{\Sigma}_t$

endfor

$\mu_t = \bar{\mu}_t$

$\Sigma_t = \bar{\Sigma}_t$

return μ_t, Σ_t

Table 10.1 The EKF algorithm for the SLAM problem (with known correspondences).

EKF-SLAM

Algorithm EKF_SLAM_known_correspondences($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, c_t$):

$$F_x = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & \underbrace{0 \cdots 0}_{3N} \end{pmatrix}$$

$$\bar{\mu}_t = \mu_{t-1} + F_x^T \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \mu_{t-1,\theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1,\theta} + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \mu_{t-1,\theta} - \frac{v_t}{\omega_t} \cos(\mu_{t-1,\theta} + \omega_t \Delta t) \\ \omega_t \Delta t \end{pmatrix}$$

$$G_t = I + F_x^T \begin{pmatrix} 0 & 0 & -\frac{v_t}{\omega_t} \cos \mu_{t-1,\theta} + \frac{v_t}{\omega_t} \cos(\mu_{t-1,\theta} + \omega_t \Delta t) \\ 0 & 0 & -\frac{v_t}{\omega_t} \sin \mu_{t-1,\theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1,\theta} + \omega_t \Delta t) \\ 0 & 0 & 0 \end{pmatrix} F_x$$

$$\bar{\Sigma}_t = G_t \, \Sigma_{t-1} \, G_t^T + F_x^T \, R_t \, F_x$$

$$Q_t = \begin{pmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & \sigma_s^2 \end{pmatrix}$$

for all observed features $z_t^i = (r_t^i \ \phi_t^i \ s_t^i)^T$ *do*

$j = c_t^i$

if landmark j *never seen before*

$$\begin{pmatrix} \bar{\mu}_{j,x} \\ \bar{\mu}_{j,y} \\ \bar{\mu}_{j,s} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \\ s_t^i \end{pmatrix} + \begin{pmatrix} r_t^i \cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i \sin(\phi_t^i + \bar{\mu}_{t,\theta}) \\ 0 \end{pmatrix}$$

endif

$$\delta = \begin{pmatrix} \delta_x \\ \delta_y \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{j,x} - \bar{\mu}_{t,x} \\ \bar{\mu}_{j,y} - \bar{\mu}_{t,y} \end{pmatrix}$$

$q = \delta^T \delta$

$$\hat{z}_t^i = \begin{pmatrix} \sqrt{q} \\ \operatorname{atan2}(\delta_y, \delta_x) - \bar{\mu}_{t,\theta} \\ \bar{\mu}_{j,s} \end{pmatrix}$$

$$F_{x,j} = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 1 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & \underbrace{0 \cdots 0}_{3j-3} & 0 & 0 & 1 & \underbrace{0 \cdots 0}_{3N-3j} \end{pmatrix}$$

$$H_t^i = \frac{1}{q} \begin{pmatrix} -\sqrt{q}\delta_x & -\sqrt{q}\delta_y & 0 & +\sqrt{q}\delta_x & \sqrt{q}\delta_y & 0 \\ \delta_y & -\delta_x & -q & -\delta_y & +\delta_x & 0 \\ 0 & 0 & 0 & 0 & 0 & q \end{pmatrix} F_{x,j}$$

$K_t^i = \bar{\Sigma}_t \, H_t^{iT} (H_t^i \, \bar{\Sigma}_t \, H_t^{iT} + Q_t)^{-1}$

$\bar{\mu}_t = \bar{\mu}_t + K_t^i (z_t^i - \hat{z}_t^i)$

$\bar{\Sigma}_t = (I - K_t^i \, H_t^i) \, \bar{\Sigma}_t$

endfor

$\mu_t = \bar{\mu}_t$

$\Sigma_t = \bar{\Sigma}_t$

return μ_t, Σ_t

Table 10.1 The EKF algorithm for the SLAM problem (with known correspondences).

Measurement Model

$$z_t^i = \underbrace{\begin{pmatrix} \sqrt{(m_{j,x}-x)^2+(m_{j,y}-y)^2} \\ \text{atan2}(m_{j,y}-y, m_{j,x}-x)-\theta \\ m_{j,s} \end{pmatrix}}_{h(y_t,j)} + \mathcal{N}(0, \underbrace{\begin{pmatrix} \sigma_r & 0 & 0 \\ 0 & \sigma_\phi & 0 \\ 0 & 0 & \sigma_s \end{pmatrix}}_{Q_t})$$

$$h(y_t, j) \approx h(\bar{\mu}_t, j) + H_t^i \, (y_t - \bar{\mu}_t)$$

$$H_t^i = h_t^i \, F_{x,j}$$

Measurement Model

$$z_t^i = \underbrace{\begin{pmatrix} \sqrt{(m_{j,x}-x)^2+(m_{j,y}-y)^2} \\ \operatorname{atan2}(m_{j,y}-y, m_{j,x}-x)-\theta \\ m_{j,s} \end{pmatrix}}_{h(y_t,j)} + \mathcal{N}(0, \underbrace{\begin{pmatrix} \sigma_r & 0 & 0 \\ 0 & \sigma_\phi & 0 \\ 0 & 0 & \sigma_s \end{pmatrix}}_{Q_t})$$

$$h(y_t, j) \approx h(\bar{\mu}_t, j) + H_t^i\,(y_t - \bar{\mu}_t)$$

$$H_t^i = h_t^i\,F_{x,j}$$

$$h_t^i = \begin{pmatrix} \frac{m_{j,x}-\bar{\mu}_{t,x}}{\sqrt{q_t}} & \frac{y_t-\bar{\mu}_{t,y}}{\sqrt{q_t}} & 0 & \frac{\bar{\mu}_{t,x}-m_{j,x}}{\sqrt{q_t}} & \frac{\bar{\mu}_{t,y}-y_t}{\sqrt{q_t}} & 0 \\ \frac{\bar{\mu}_{t,y}-y_t}{q_t} & \frac{m_{j,x}-\bar{\mu}_{t,x}}{q_t} & -1 & \frac{y_t-\bar{\mu}_{t,y}}{q_t} & \frac{\bar{\mu}_{t,x}-m_{j,x}}{q_t} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Measurement Model

$$z_t^i = \underbrace{\begin{pmatrix} \sqrt{(m_{j,x}-x)^2+(m_{j,y}-y)^2} \\ \mathrm{atan2}(m_{j,y}-y, m_{j,x}-x)-\theta \\ m_{j,s} \end{pmatrix}}_{h(y_t,j)} + \mathcal{N}(0, \underbrace{\begin{pmatrix} \sigma_r & 0 & 0 \\ 0 & \sigma_\phi & 0 \\ 0 & 0 & \sigma_s \end{pmatrix}}_{Q_t})$$

$$h(y_t, j) \approx h(\bar{\mu}_t, j) + H_t^i \, (y_t - \bar{\mu}_t)$$

$$H_t^i = h_t^i \, F_{x,j}$$

$$h_t^i = \begin{pmatrix} \frac{m_{j,x}-\bar{\mu}_{t,x}}{\sqrt{q_t}} & \frac{y_t-\bar{\mu}_{t,y}}{\sqrt{q_t}} & 0 & \frac{\bar{\mu}_{t,x}-m_{j,x}}{\sqrt{q_t}} & \frac{\bar{\mu}_{t,y}-y_t}{\sqrt{q_t}} & 0 \\ \frac{\bar{\mu}_{t,y}-y_t}{q_t} & \frac{m_{j,x}-\bar{\mu}_{t,x}}{q_t} & -1 & \frac{y_t-\bar{\mu}_{t,y}}{q_t} & \frac{\bar{\mu}_{t,x}-m_{j,x}}{q_t} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad F_{x,j} = \begin{pmatrix} 1 & 0 & 0 & 0\cdots0 & 0 & 0 & 0 & 0\cdots0 \\ 0 & 1 & 0 & 0\cdots0 & 0 & 0 & 0 & 0\cdots0 \\ 0 & 0 & 1 & 0\cdots0 & 0 & 0 & 0 & 0\cdots0 \\ 0 & 0 & 0 & 0\cdots0 & 1 & 0 & 0 & 0\cdots0 \\ 0 & 0 & 0 & 0\cdots0 & 0 & 1 & 0 & 0\cdots0 \\ 0 & 0 & 0 & \underbrace{0\cdots0}_{2j-2} & 0 & 0 & 1 & \underbrace{0\cdots0}_{2N-2j} \end{pmatrix}$$

EKF-SLAM

Algorithm EKF_SLAM_known_correspondences($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, c_t$):

$$F_x = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & \underbrace{0 \cdots 0}_{3N} \end{pmatrix}$$

$$\bar{\mu}_t = \mu_{t-1} + F_x^T \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \mu_{t-1,\theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1,\theta} + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \mu_{t-1,\theta} - \frac{v_t}{\omega_t} \cos(\mu_{t-1,\theta} + \omega_t \Delta t) \\ \omega_t \Delta t \end{pmatrix}$$

$$G_t = I + F_x^T \begin{pmatrix} 0 & 0 & -\frac{v_t}{\omega_t} \cos \mu_{t-1,\theta} + \frac{v_t}{\omega_t} \cos(\mu_{t-1,\theta} + \omega_t \Delta t) \\ 0 & 0 & -\frac{v_t}{\omega_t} \sin \mu_{t-1,\theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1,\theta} + \omega_t \Delta t) \\ 0 & 0 & 0 \end{pmatrix} F_x$$

$$\bar{\Sigma}_t = G_t \, \Sigma_{t-1} \, G_t^T + F_x^T \, R_t \, F_x$$

$$Q_t = \begin{pmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & \sigma_s^2 \end{pmatrix}$$

for all observed features $z_t^i = (r_t^i \;\; \phi_t^i \;\; s_t^i)^T$ *do*

$j = c_t^i$

if landmark j *never seen before*

$$\begin{pmatrix} \bar{\mu}_{j,x} \\ \bar{\mu}_{j,y} \\ \bar{\mu}_{j,s} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \\ s_t^i \end{pmatrix} + \begin{pmatrix} r_t^i \cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i \sin(\phi_t^i + \bar{\mu}_{t,\theta}) \\ 0 \end{pmatrix}$$

endif

$$\delta = \begin{pmatrix} \delta_x \\ \delta_y \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{j,x} - \bar{\mu}_{t,x} \\ \bar{\mu}_{j,y} - \bar{\mu}_{t,y} \end{pmatrix}$$

$q = \delta^T \delta$

$$\hat{z}_t^i = \begin{pmatrix} \sqrt{q} \\ \text{atan2}(\delta_y, \delta_x) - \bar{\mu}_{t,\theta} \\ \bar{\mu}_{j,s} \end{pmatrix}$$

$$F_{x,j} = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 1 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & \underbrace{0 \cdots 0}_{3j-3} & 0 & 0 & 1 & \underbrace{0 \cdots 0}_{3N-3j} \end{pmatrix}$$

$$H_t^i = \frac{1}{q} \begin{pmatrix} -\sqrt{q}\delta_x & -\sqrt{q}\delta_y & 0 & +\sqrt{q}\delta_x & \sqrt{q}\delta_y & 0 \\ \delta_y & -\delta_x & -q & -\delta_y & +\delta_x & 0 \\ 0 & 0 & 0 & 0 & 0 & q \end{pmatrix} F_{x,j}$$

$K_t^i = \bar{\Sigma}_t \, H_t^{iT} (H_t^i \, \bar{\Sigma}_t \, H_t^{iT} + Q_t)^{-1}$

$\bar{\mu}_t = \bar{\mu}_t + K_t^i (z_t^i - \hat{z}_t^i)$

$\bar{\Sigma}_t = (I - K_t^i \, H_t^i) \, \bar{\Sigma}_t$

endfor

$\mu_t = \bar{\mu}_t$

$\Sigma_t = \bar{\Sigma}_t$

return μ_t, Σ_t

Table 10.1 The EKF algorithm for the SLAM problem (with known correspondences).

EKF-SLAM

Algorithm EKF_SLAM_known_correspondences($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, c_t$):

$$F_x = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & \underbrace{0 \cdots 0}_{3N} \end{pmatrix}$$

$$\bar{\mu}_t = \mu_{t-1} + F_x^T \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \mu_{t-1,\theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1,\theta} + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \mu_{t-1,\theta} - \frac{v_t}{\omega_t} \cos(\mu_{t-1,\theta} + \omega_t \Delta t) \\ \omega_t \Delta t \end{pmatrix}$$

$$G_t = I + F_x^T \begin{pmatrix} 0 & 0 & -\frac{v_t}{\omega_t} \cos \mu_{t-1,\theta} + \frac{v_t}{\omega_t} \cos(\mu_{t-1,\theta} + \omega_t \Delta t) \\ 0 & 0 & -\frac{v_t}{\omega_t} \sin \mu_{t-1,\theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1,\theta} + \omega_t \Delta t) \\ 0 & 0 & 0 \end{pmatrix} F_x$$

$$\bar{\Sigma}_t = G_t \, \Sigma_{t-1} \, G_t^T + F_x^T \, R_t \, F_x$$

$$Q_t = \begin{pmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & \sigma_s^2 \end{pmatrix}$$

for all observed features $z_t^i = (r_t^i \ \phi_t^i \ s_t^i)^T$ *do*

$$j = c_t^i$$

if landmark j *never seen before*

$$\begin{pmatrix} \bar{\mu}_{j,x} \\ \bar{\mu}_{j,y} \\ \bar{\mu}_{j,s} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \\ s_t^i \end{pmatrix} + \begin{pmatrix} r_t^i \cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i \sin(\phi_t^i + \bar{\mu}_{t,\theta}) \\ 0 \end{pmatrix}$$

endif

$$\delta = \begin{pmatrix} \delta_x \\ \delta_y \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{j,x} - \bar{\mu}_{t,x} \\ \bar{\mu}_{j,y} - \bar{\mu}_{t,y} \end{pmatrix}$$

$$q = \delta^T \delta$$

$$\hat{z}_t^i = \begin{pmatrix} \sqrt{q} \\ \text{atan2}(\delta_y, \delta_x) - \bar{\mu}_{t,\theta} \\ \bar{\mu}_{j,s} \end{pmatrix}$$

$$F_{x,j} = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 1 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & \underbrace{0 \cdots 0}_{3j-3} & 0 & 0 & 1 & \underbrace{0 \cdots 0}_{3N-3j} \end{pmatrix}$$

$$H_t^i = \frac{1}{q} \begin{pmatrix} -\sqrt{q}\delta_x & -\sqrt{q}\delta_y & 0 & +\sqrt{q}\delta_x & \sqrt{q}\delta_y & 0 \\ \delta_y & -\delta_x & -q & -\delta_y & +\delta_x & 0 \\ 0 & 0 & 0 & 0 & 0 & q \end{pmatrix} F_{x,j}$$

$$K_t^i = \bar{\Sigma}_t \, H_t^{iT} (H_t^i \, \bar{\Sigma}_t \, H_t^{iT} + Q_t)^{-1}$$

$$\bar{\mu}_t = \bar{\mu}_t + K_t^i (z_t^i - \hat{z}_t^i)$$

$$\bar{\Sigma}_t = (I - K_t^i \, H_t^i) \, \bar{\Sigma}_t$$

endfor

$$\mu_t = \bar{\mu}_t$$

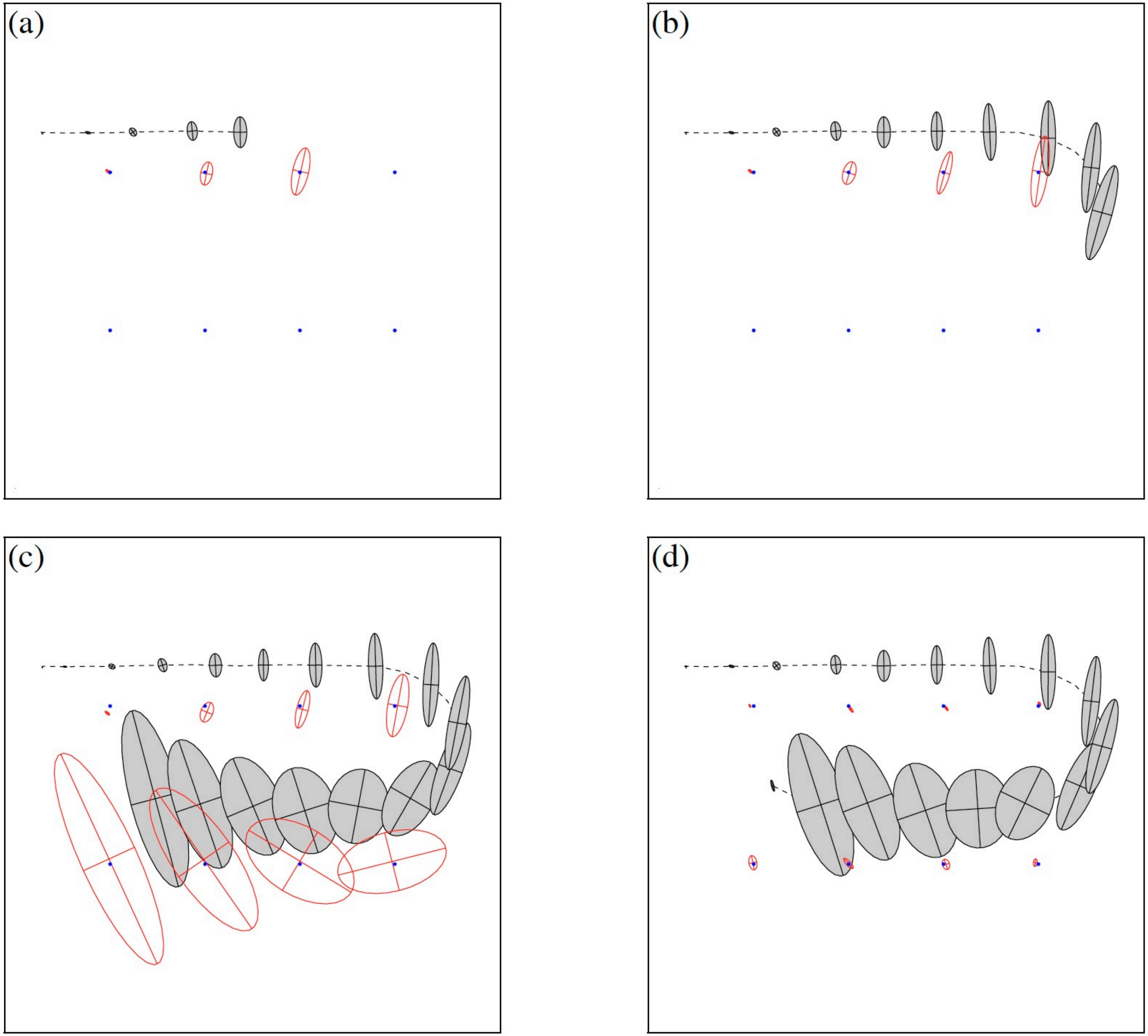
$$\Sigma_t = \bar{\Sigma}_t$$

return μ_t, Σ_t

Table 10.1 The EKF algorithm for the SLAM problem (with known correspondences).

EKF-SLAM Intuition

- Kalman gain is fully populated for all state variables.
- Observing the landmark improves the robot's pose estimate, which in turn improves the estimates of other landmarks.


(a)
(b)
(c)
(d)

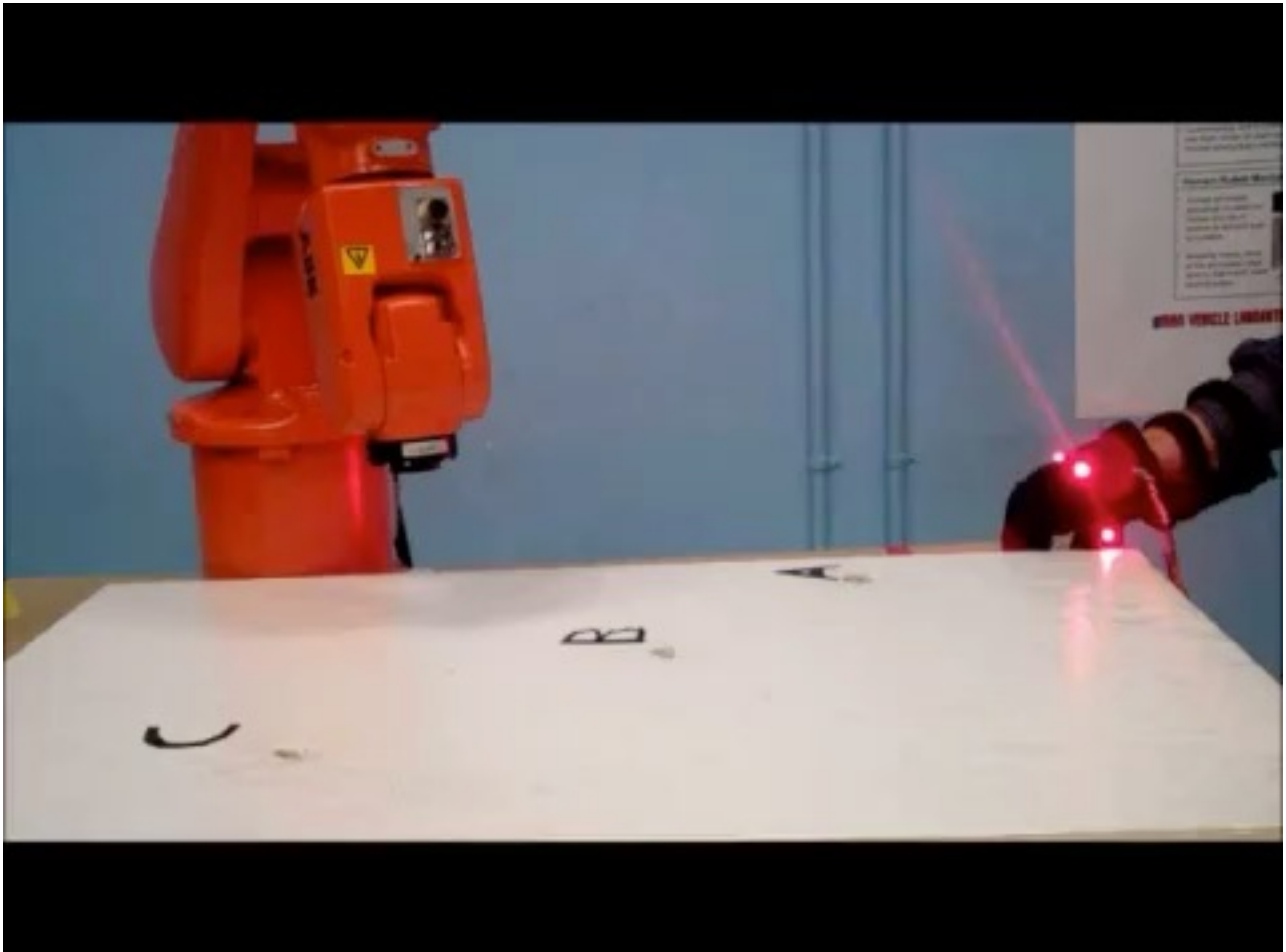
Explanation

- The position of the first landmark is well known
- Observing the first landmark again reduces uncertainty on the robot's pose
- This results in reduction of uncertainty in poses of other landmarks – from correlations represented by the non-diagonal elements of Σ_t

Explanation

- The position of the first landmark is well known
- Observing the first landmark again reduces uncertainty on the robot's pose
- This results in reduction of uncertainty in poses of other landmarks – from correlations represented by the non-diagonal elements of Σ_t
- Why does the uncertainty on previous robot poses not decrease?

Filtering Applications: Hand-Tracking for Human-Robot Interaction

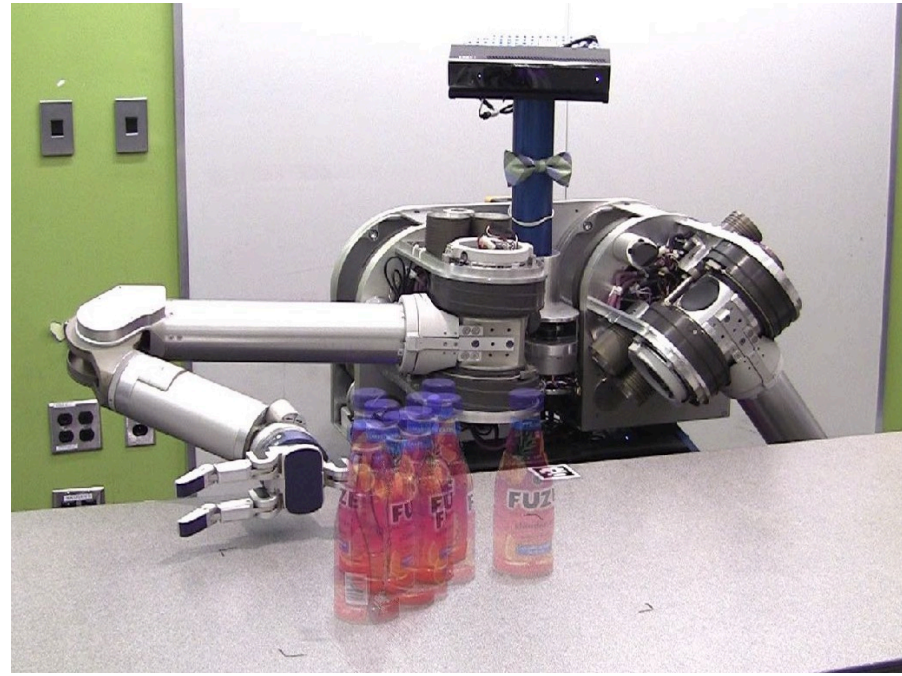


Filtering Applications: Camera Calibration

Filtering Applications: Fast Estimation

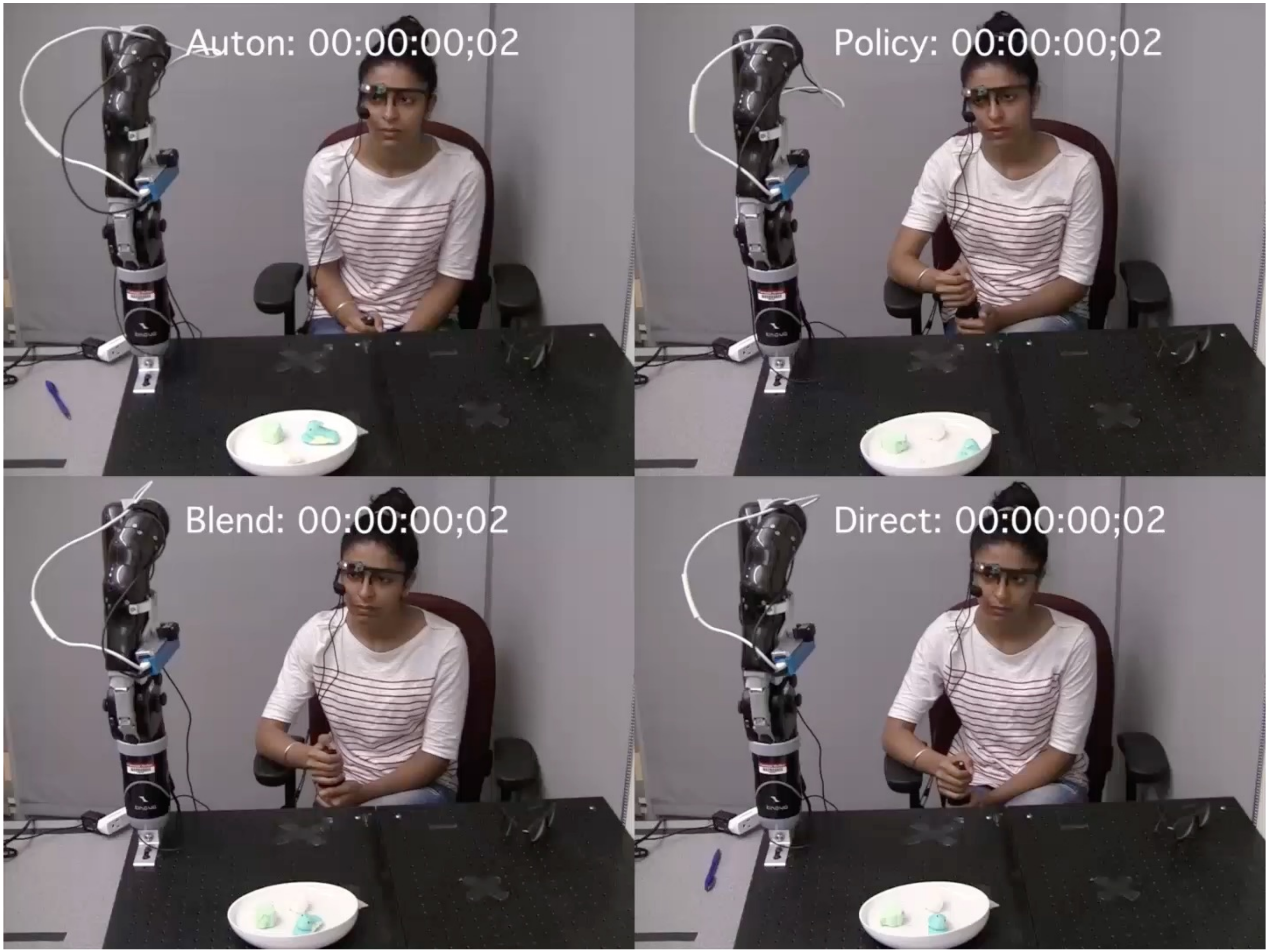
Filtering Applications: Navigation in Crowds

Filtering Applications: Object Location

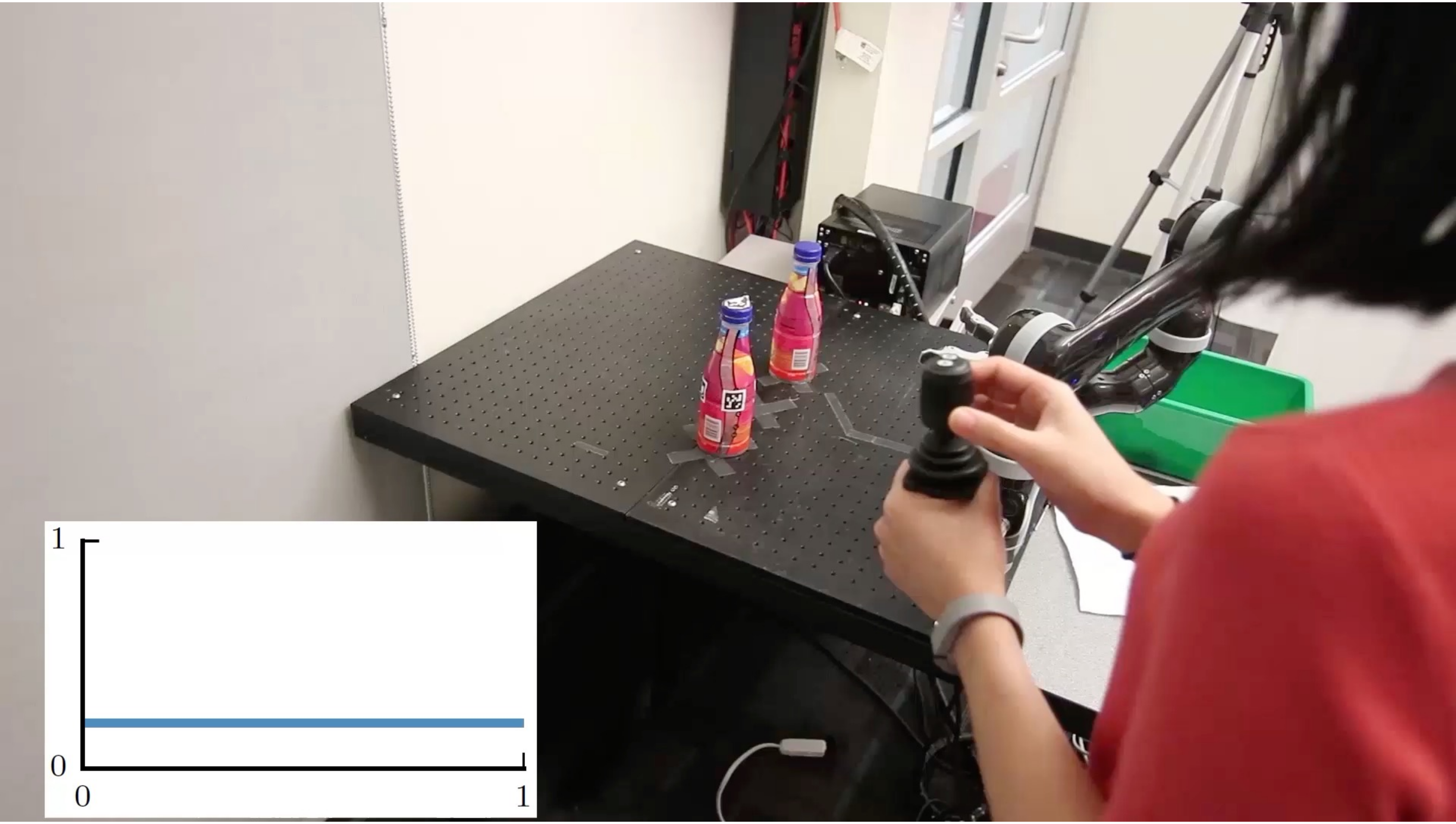


Filtering Applications: Object Properties

Filtering Applications: Human State Estimation



Filtering Applications: Human State Estimation



Extra

Mapping