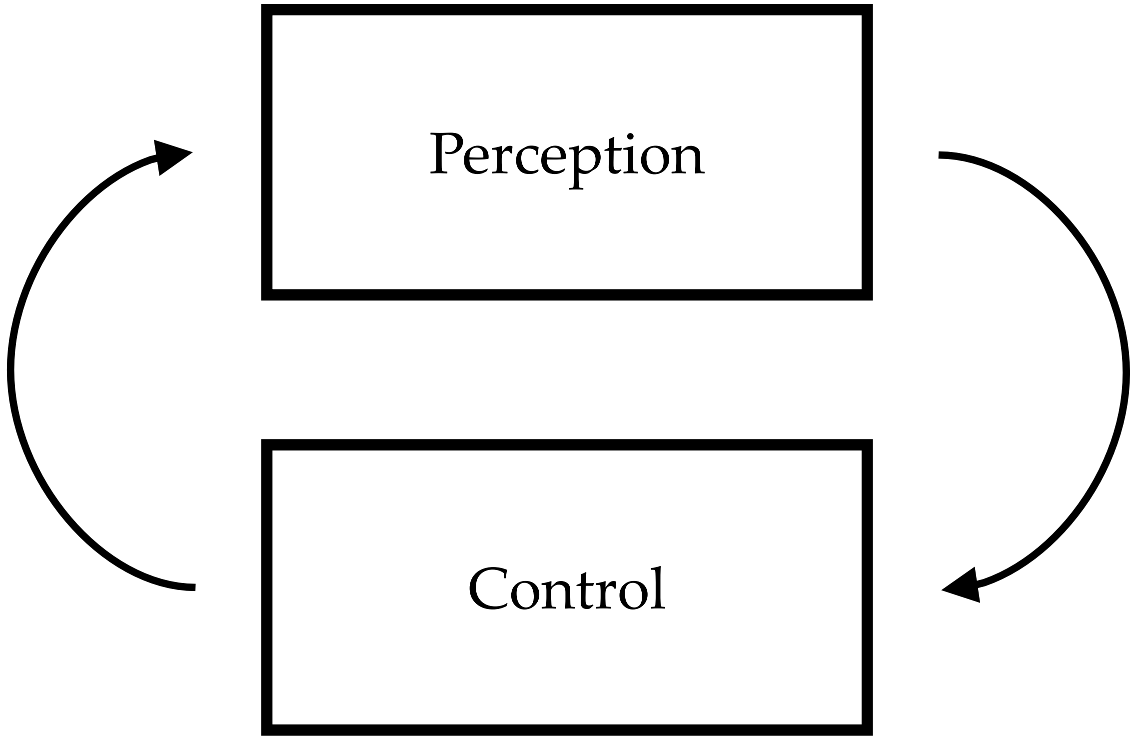
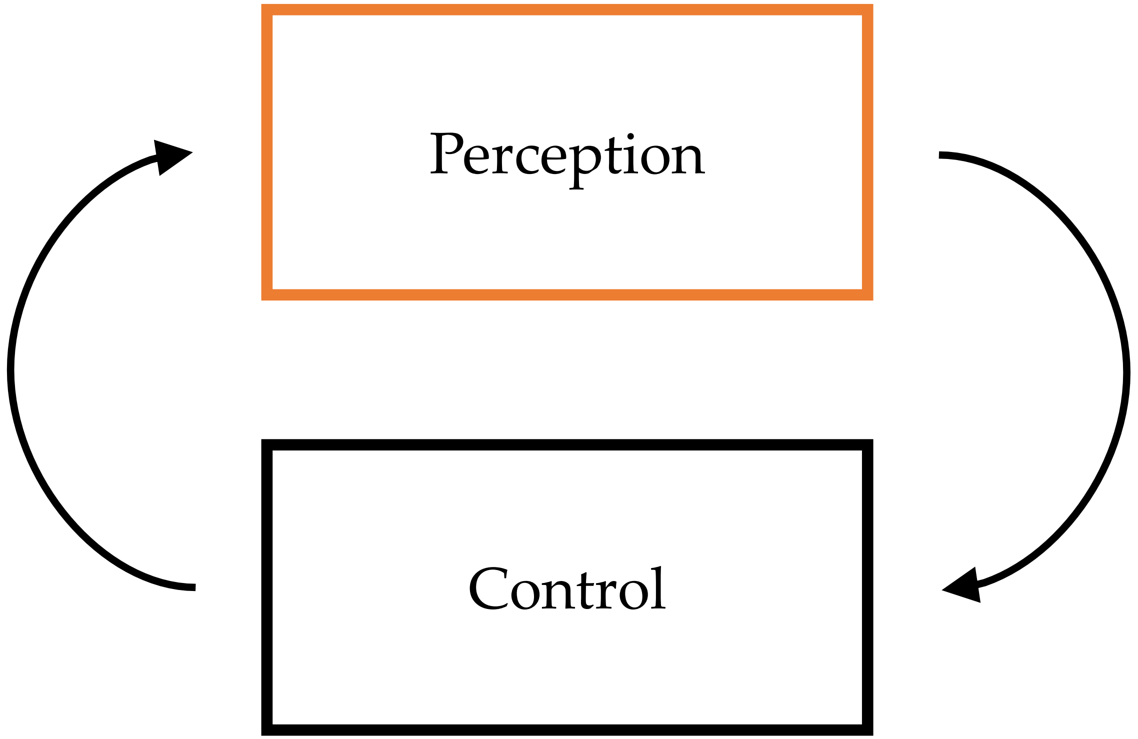
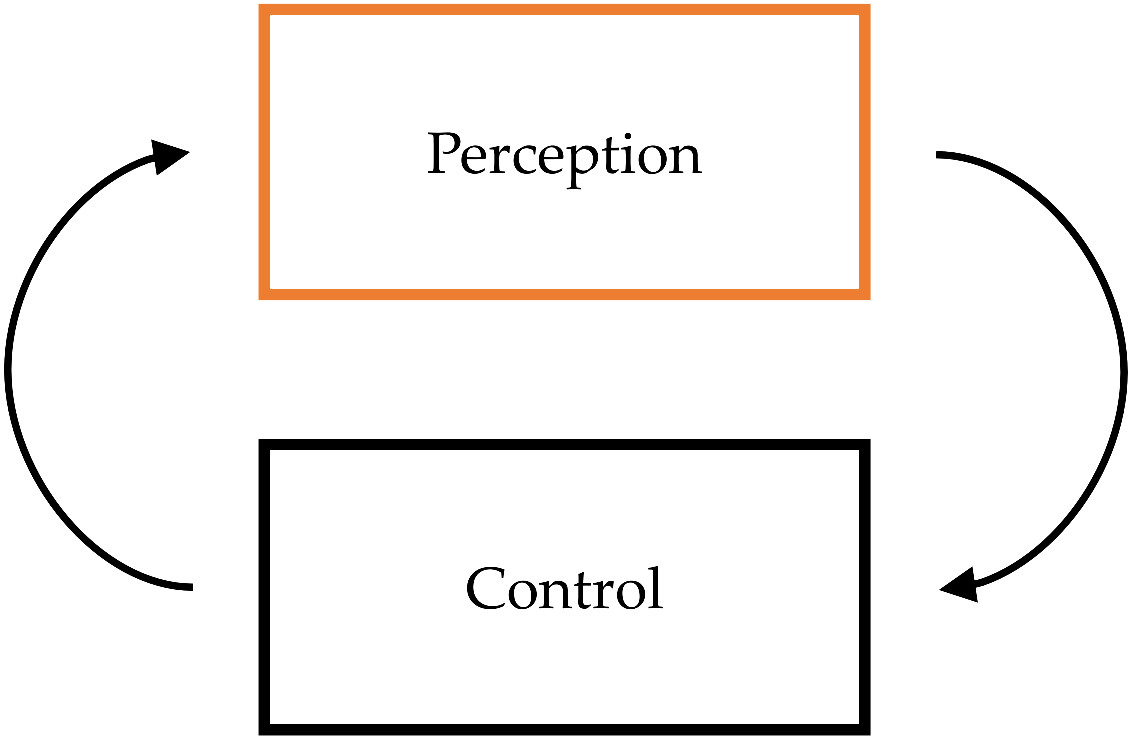
Class Overview

CSCI 545 Introduction to Robotics
Instructor: Stefanos Nikolaidis


Perception
Control


Perception
Control

Bayesian Filtering



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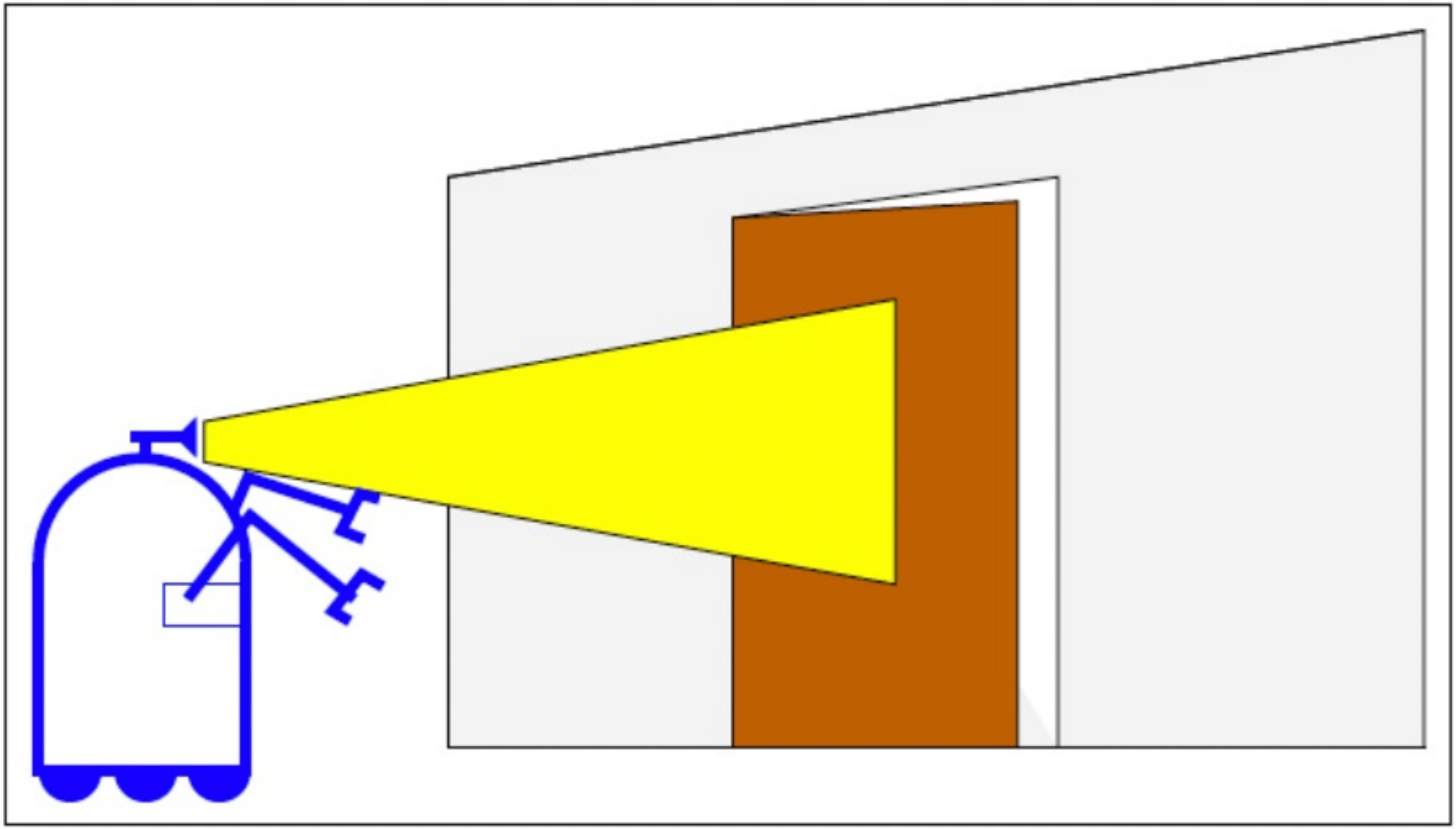
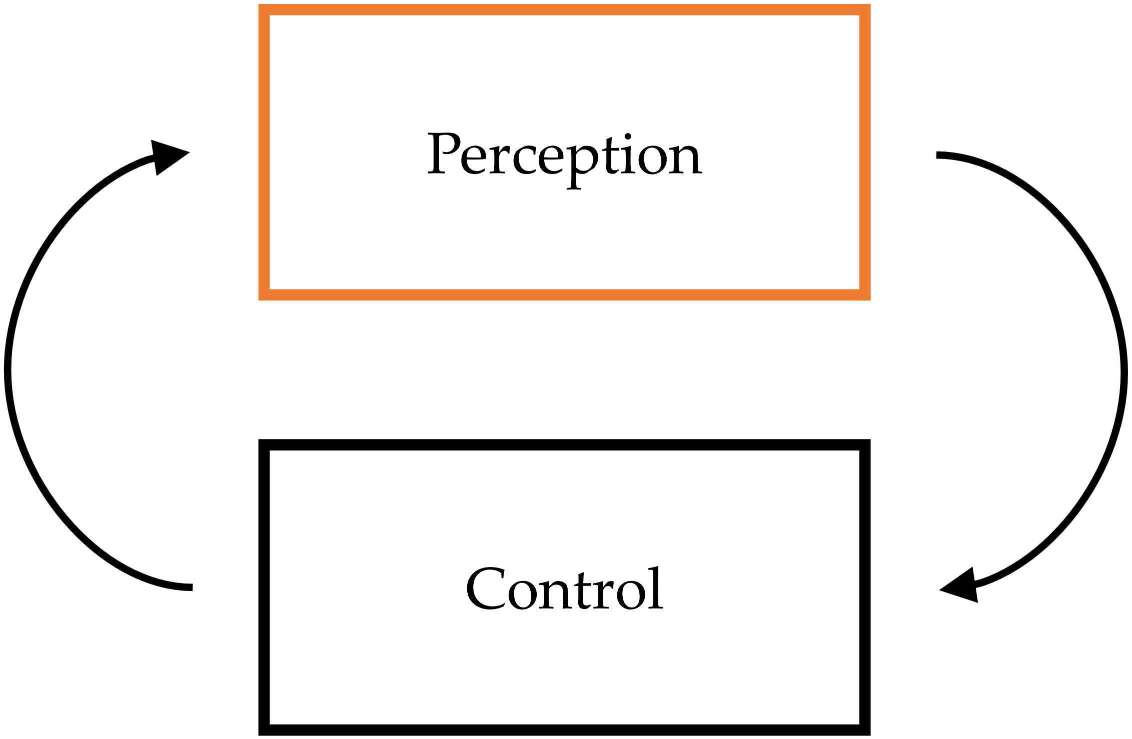
Bayesian Filtering



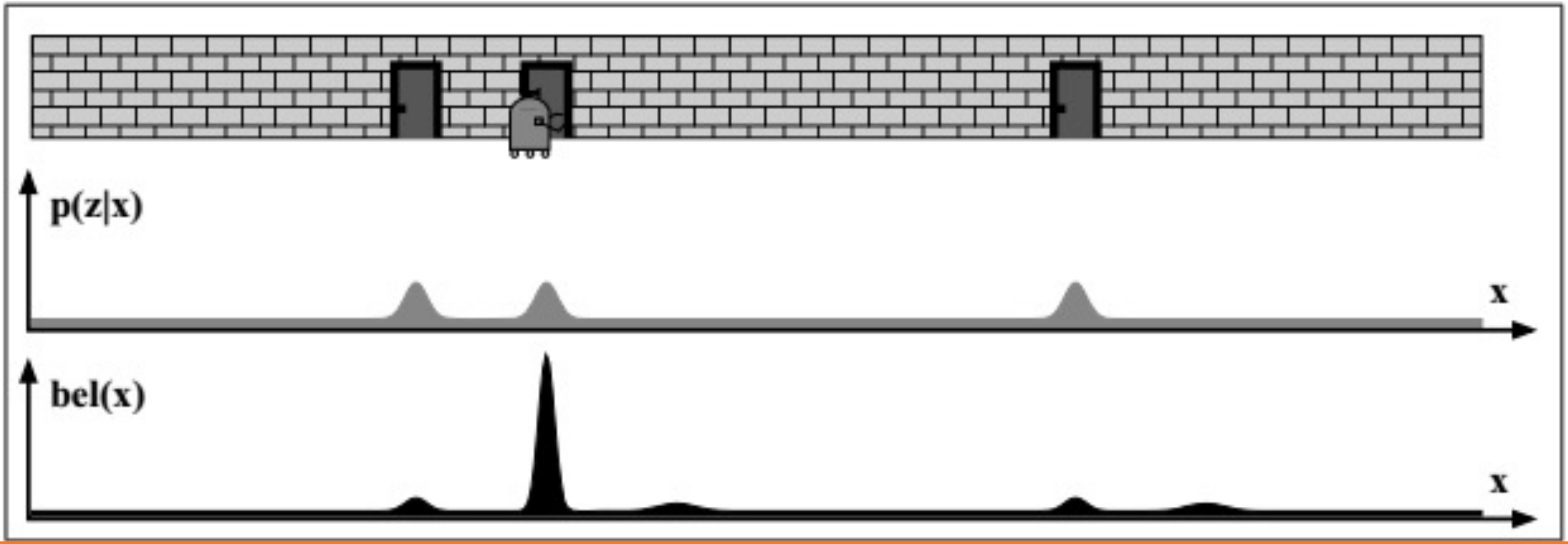
Bayesian Filtering

KF / EKF



Bayesian Filtering

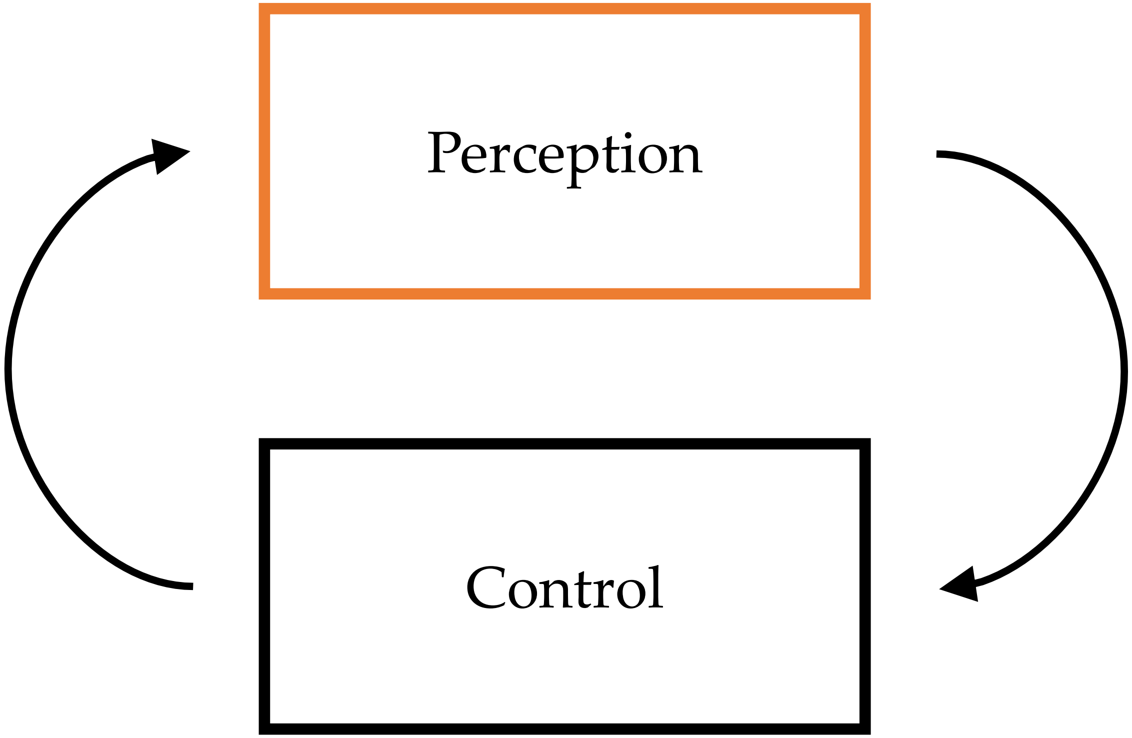
KF / EKF



Bayesian Filtering

Particle Filtering

KF / EKF



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Bayesian Filtering **Particle Filtering**

KF / EKF **Localization**

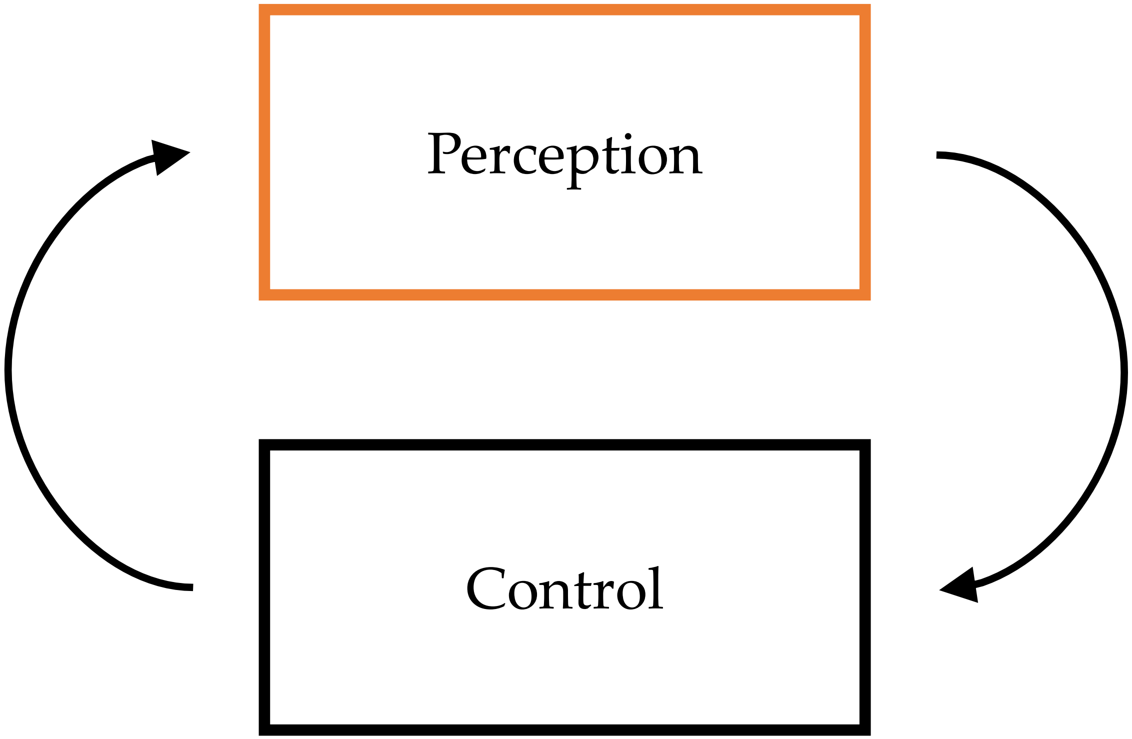
Bayesian Filtering

KF / EKF

Particle Filtering

Mapping

Localization



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Bayesian Filtering

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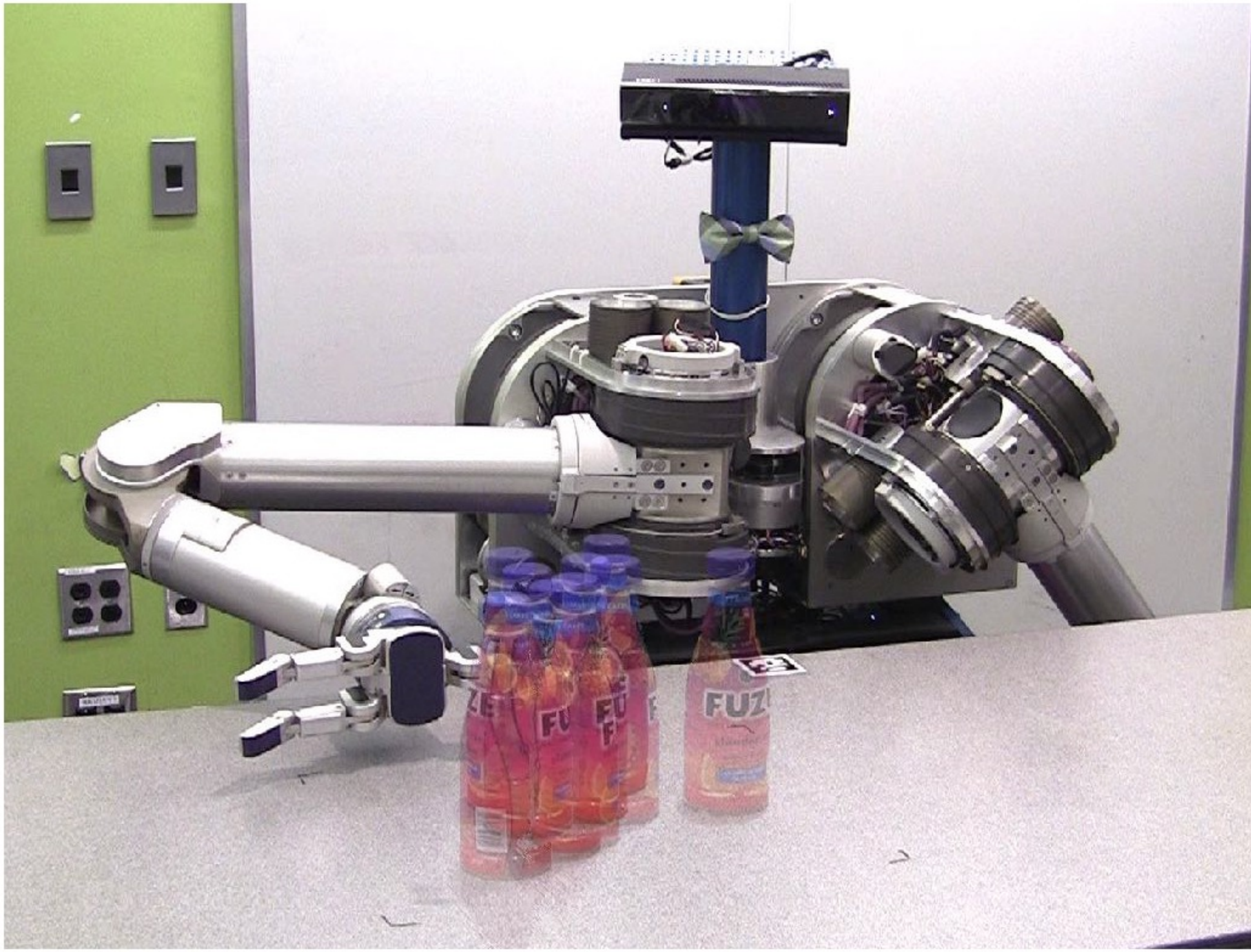

Bayesian Filtering

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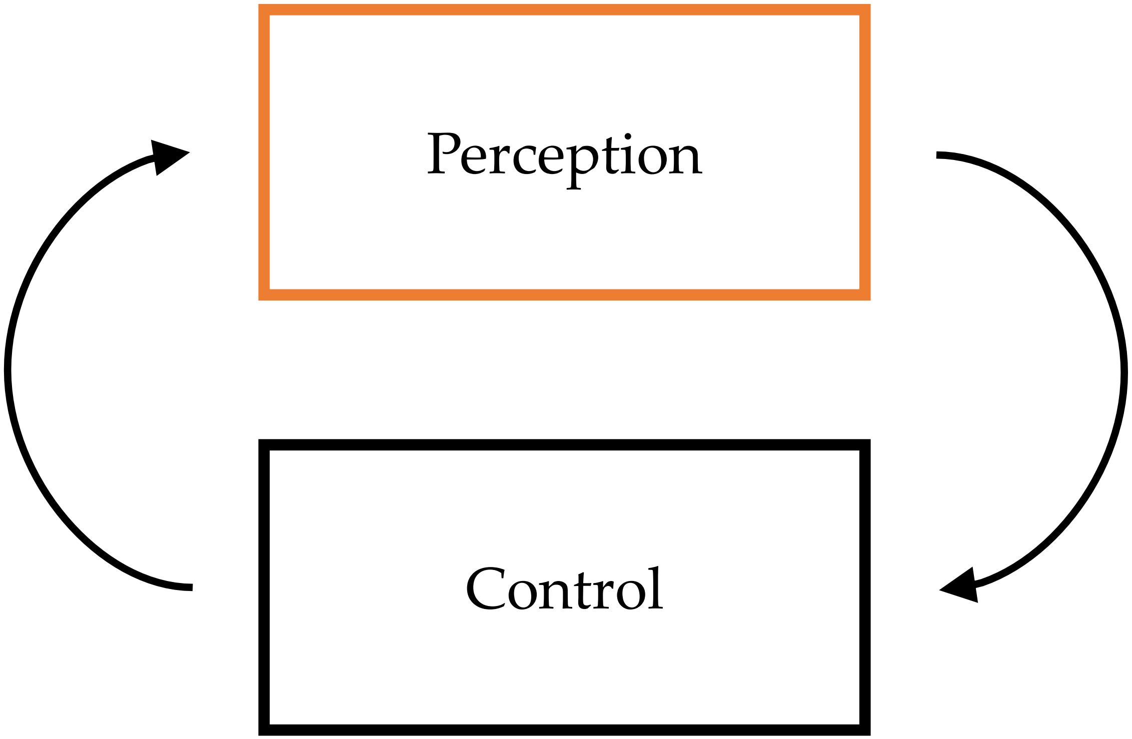

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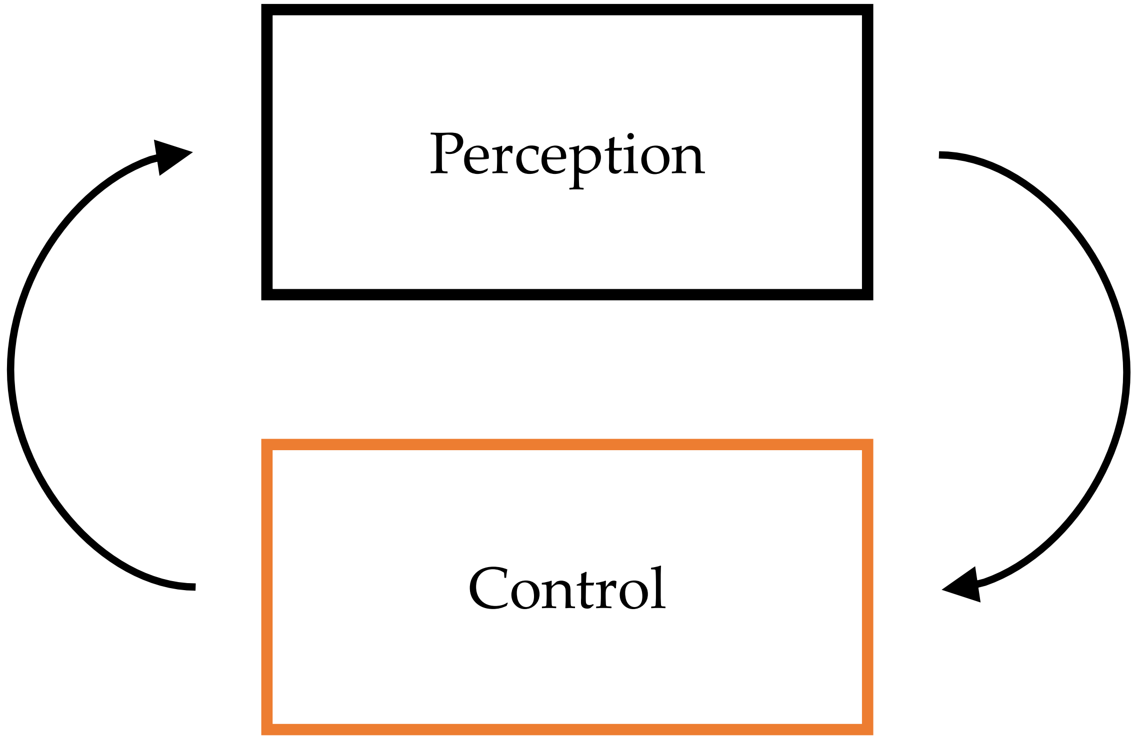
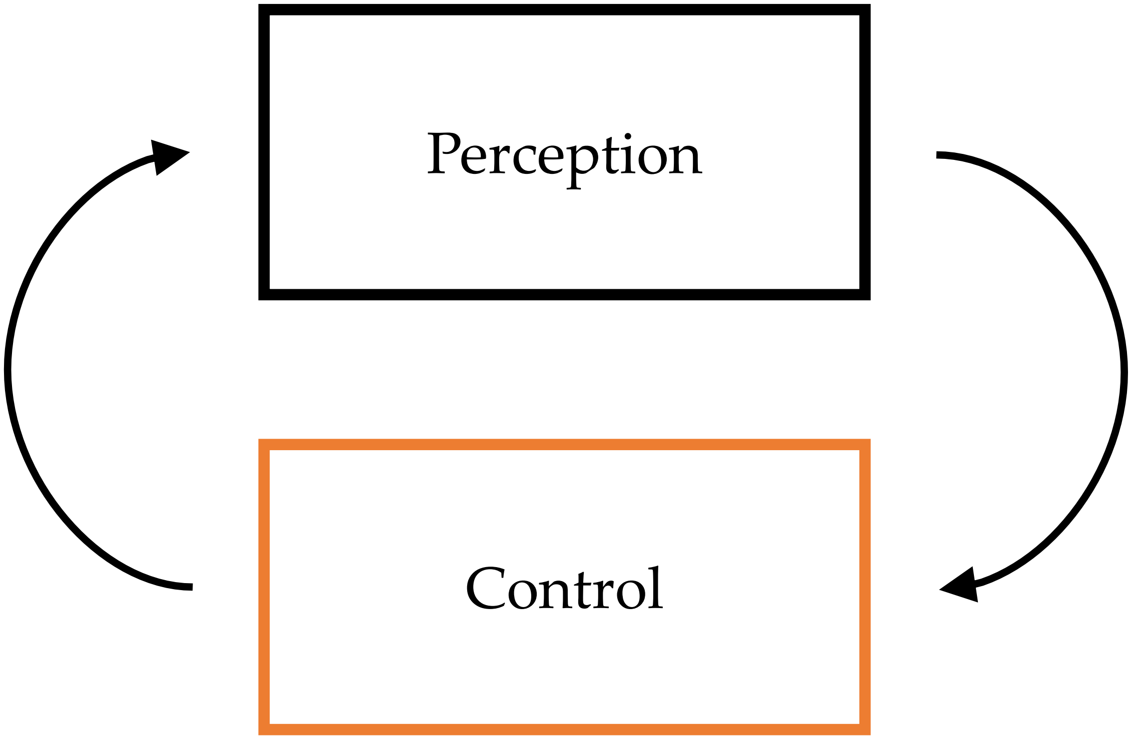


Bayesian Filtering **Particle Filtering** **Mapping**

KF / EKF **Localization**



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Bayesian Filtering **Particle Filtering** **Mapping**

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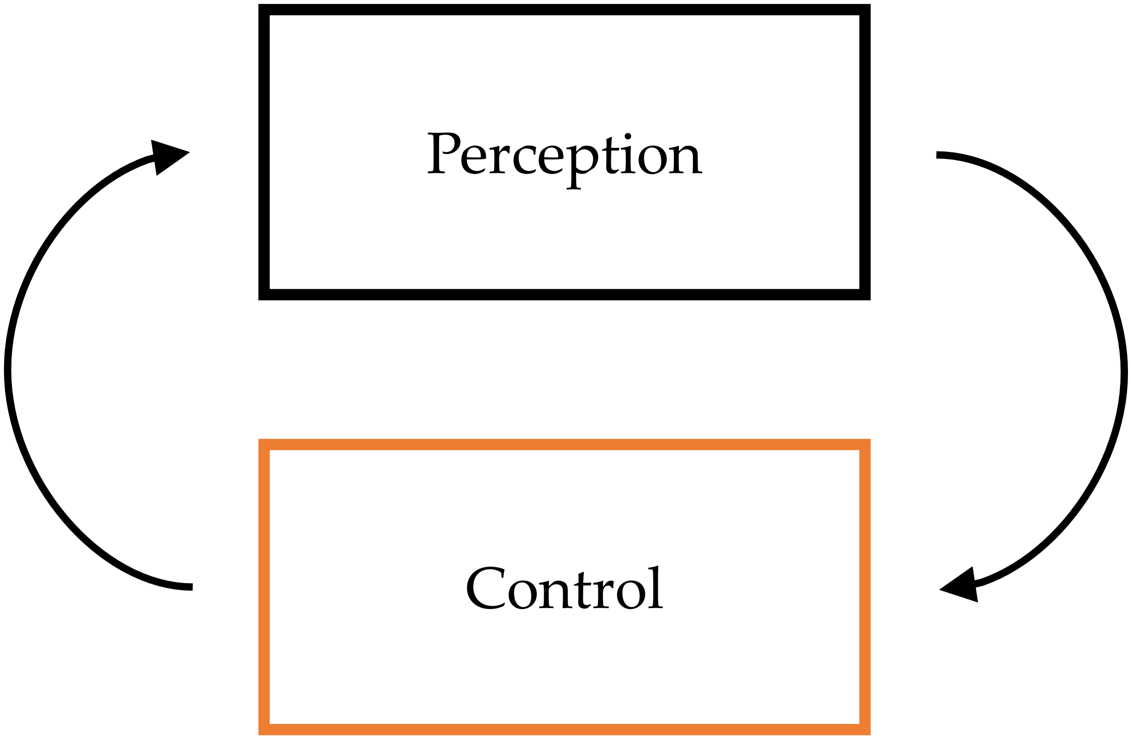

Inverse Kinematics

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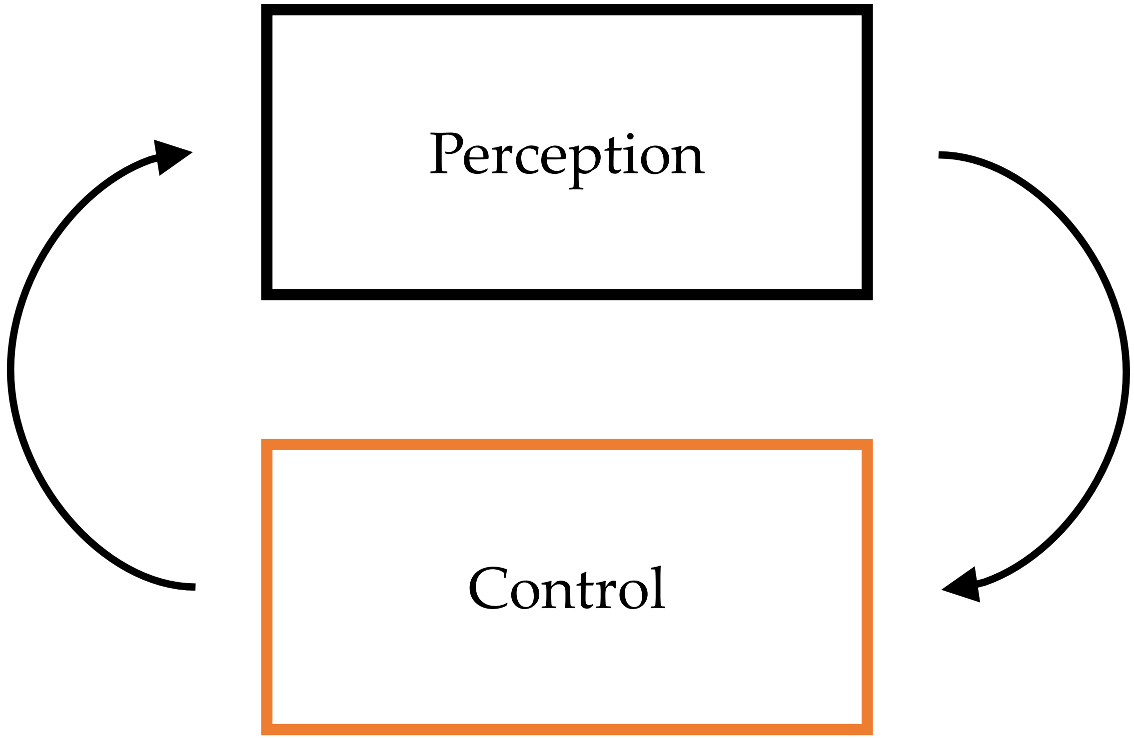
Bayesian Filtering

KF / EKF

Particle Filtering **Mapping**

Localization



Inverse Kinematics

Task Space Regions

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Bayesian Filtering

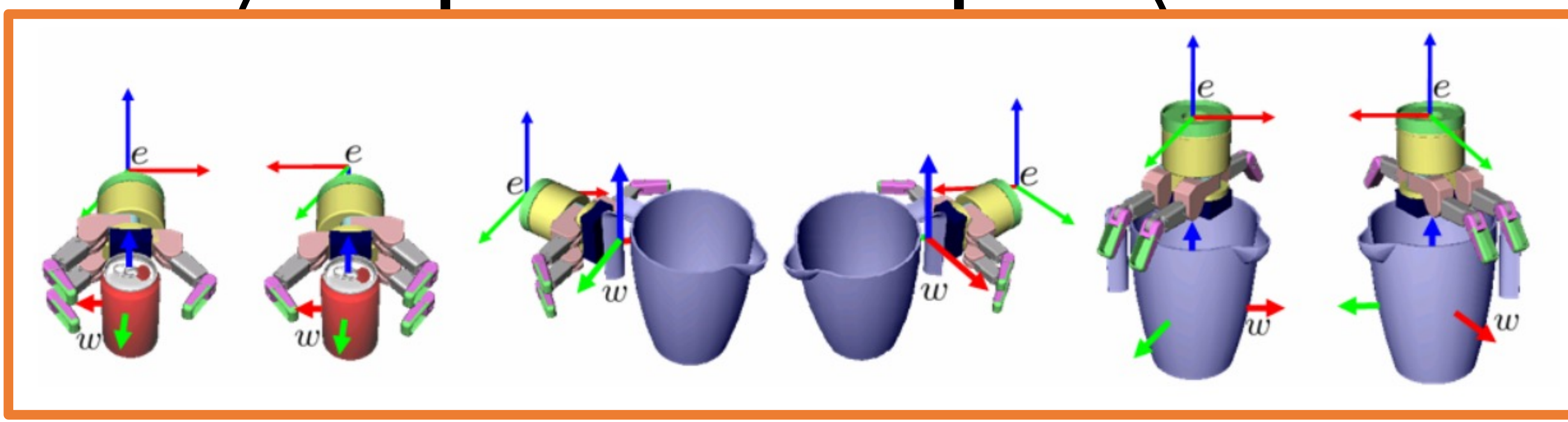
KF / EKF

Particle Filtering

Mapping

Localization

Perception



Inverse Kinematics

Task Space Regions

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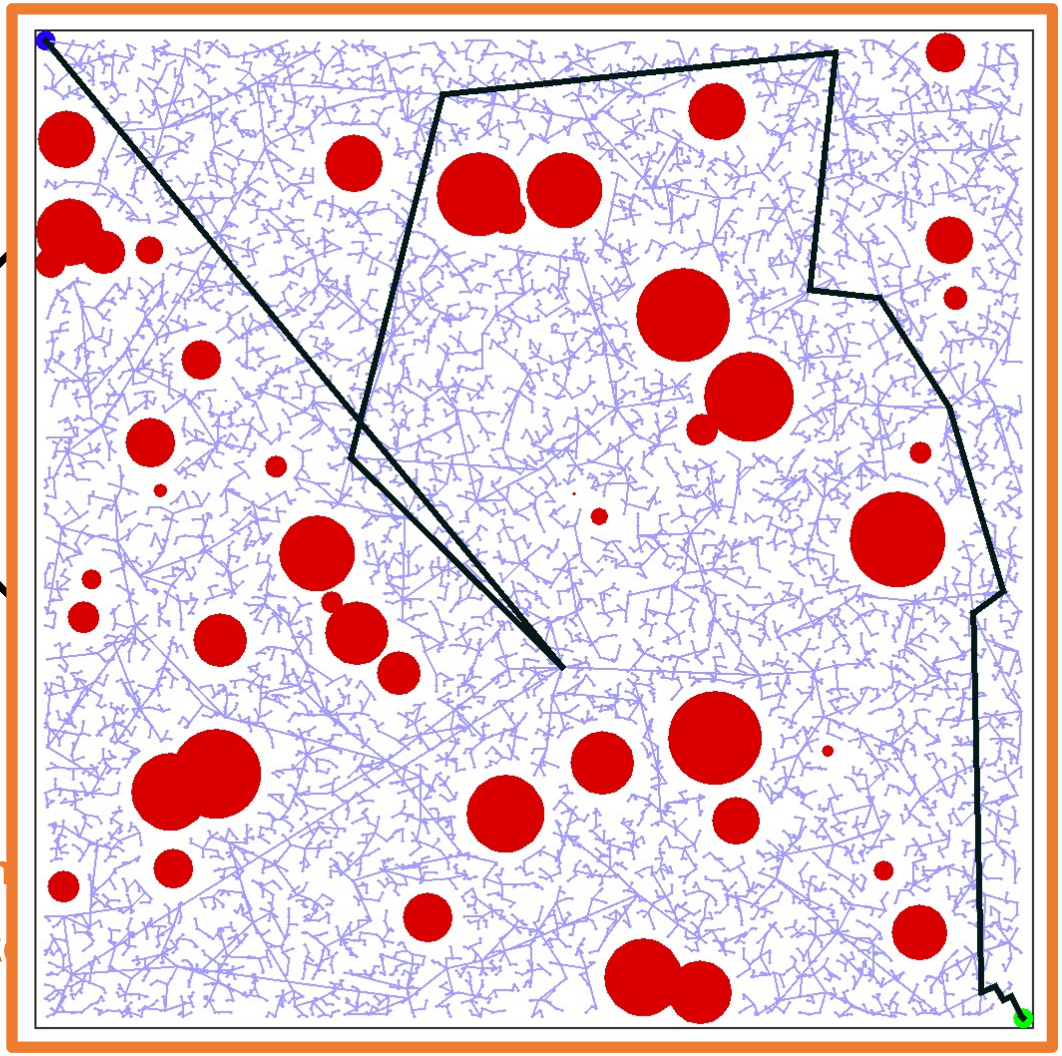

Inverse Kinen

Task Space R

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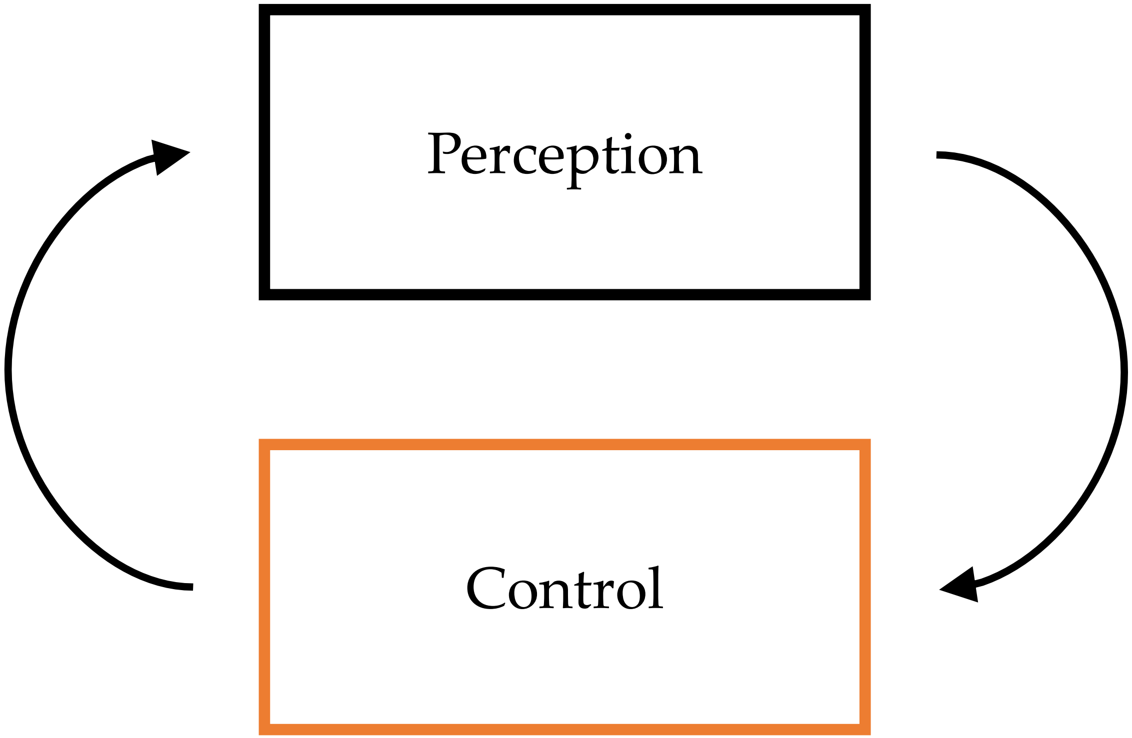
Bayesian Filtering **Particle Filtering** **Mapping**

KF / EKF **Localization**



Inverse Kinematics **Motion Planning**

Task Space Regions **Dynamics**

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Bayesian Filtering

KF / EKF

Particle Filtering

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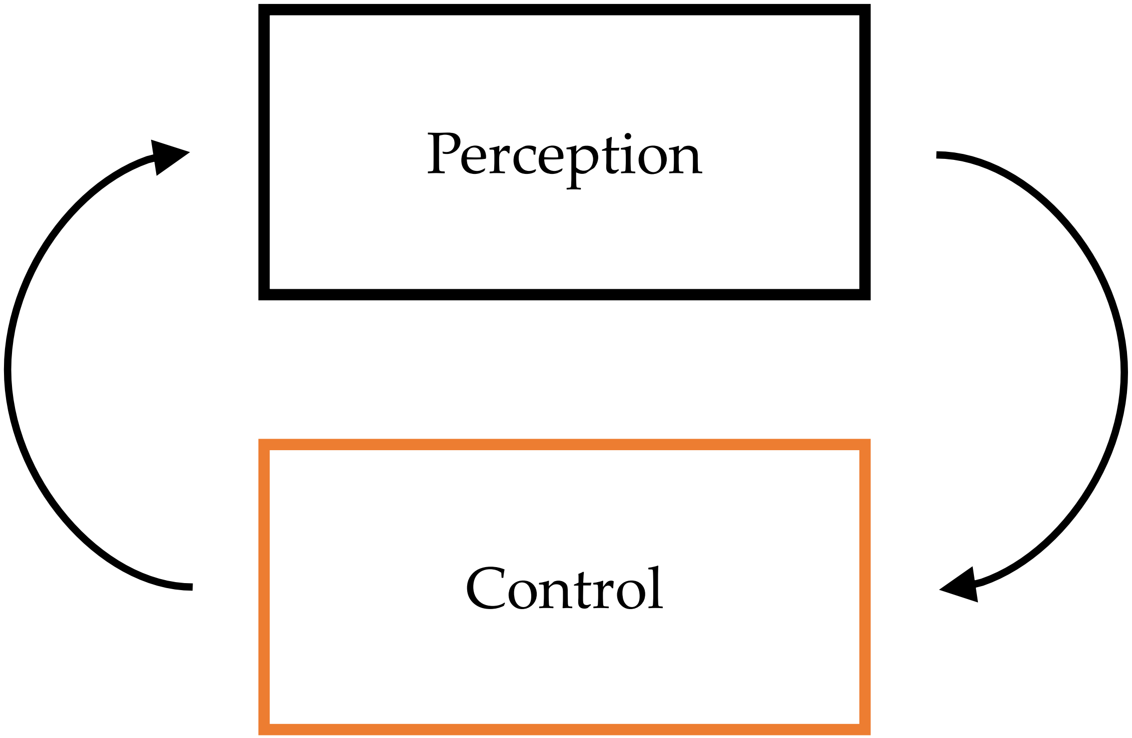

Inverse Kinematics

Task Space Regions

Motion Planning

Dynamics

Linear / Non-Linear Control

Bayesian Filtering

Particle Filtering

Mapping

KF / EKF

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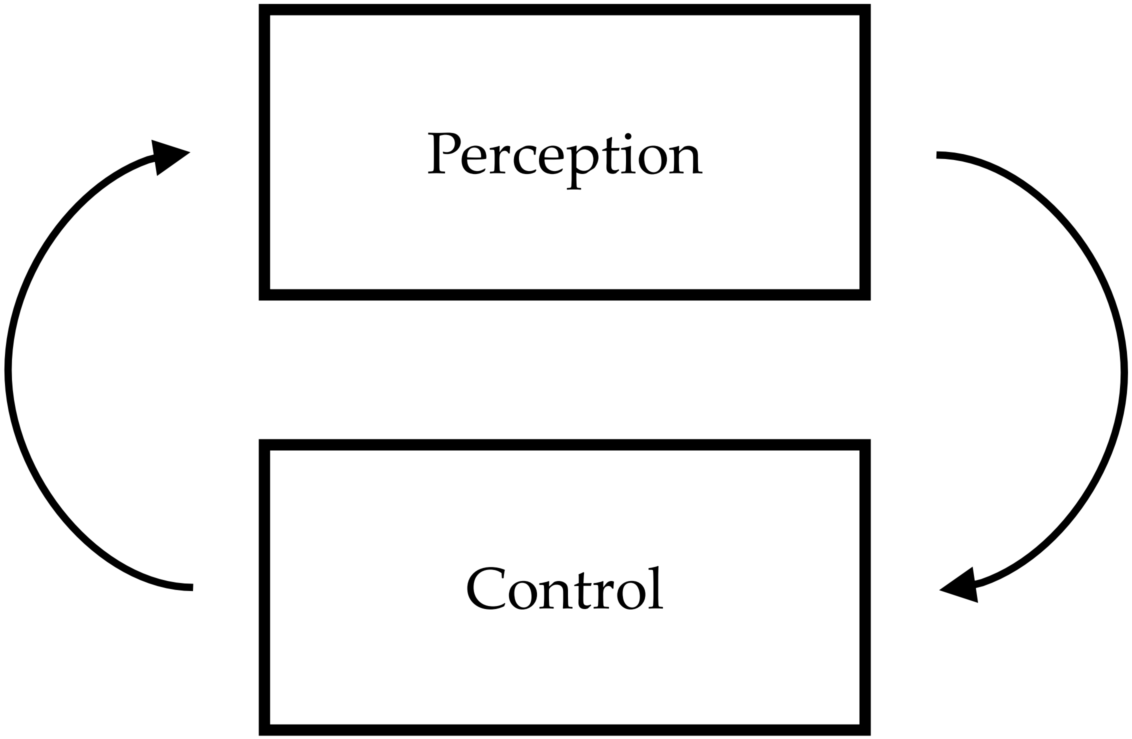

Inverse Kinematics

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Bayesian Filtering **Particle Filtering** **Mapping**

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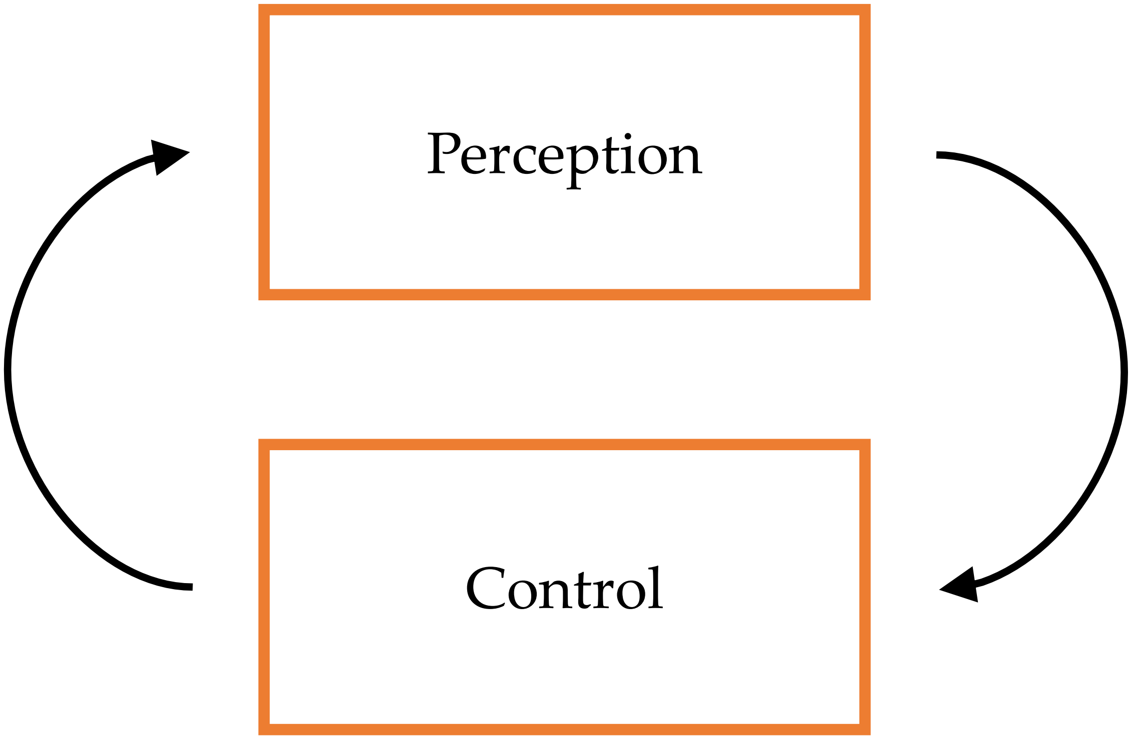

Decision Theory
Planning Under Uncertainty

Inverse Kinematics **Motion Planning** **Linear / Non-Linear Control**

Task Space Regions **Dynamics**

Bayesian Filtering   Particle Filtering   Mapping

KF / EKF

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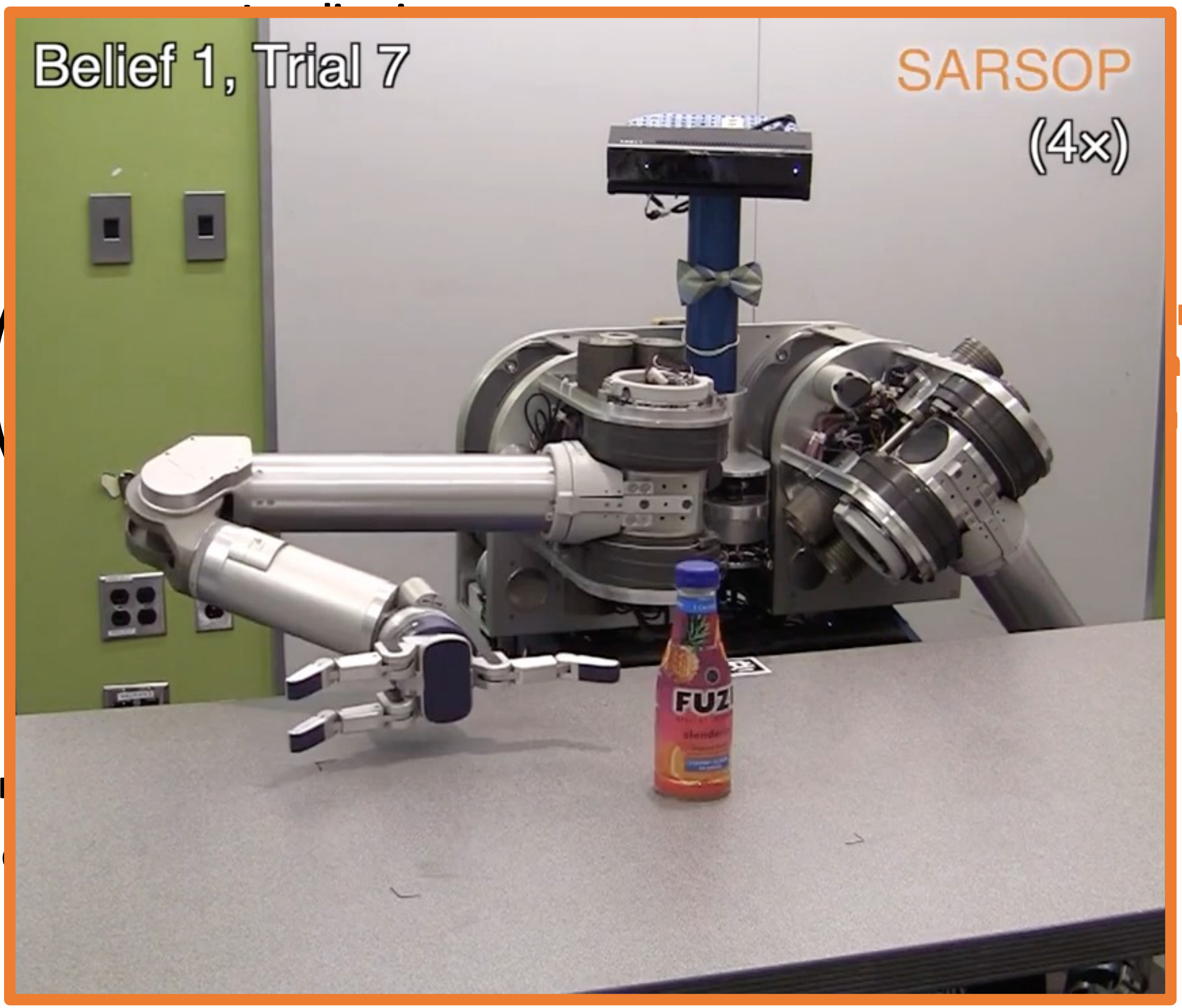

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Non-linear Control

CSCI 545 Introduction to Robotics
Instructor: Stefanos Nikolaidis

Resources: CSCI 545 Lecture Notes by Prof. Stefan Schaal

Dynamics Equations

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau$$

These equations are non-linear.

Non-linear Control

Joint-Space Control: Create motor commands in joint space

Operational Space Control: Create motor commands in end-effector space

Non-linear Control

Joint-Space Control: Create motor commands in joint space

Operational Space Control: Create motor commands in end-effector space

Joint-Space Control

- Independent (Decentralized) Joint-Space Control
  - Appropriate when coupling terms are negligible, e.g., robot is decoupled (two 1DOF robots)
  - I focus on the diagonal elements of B,C and I treat the rest as disturbances
  - I create a PD / PID controller for each joint-space independently

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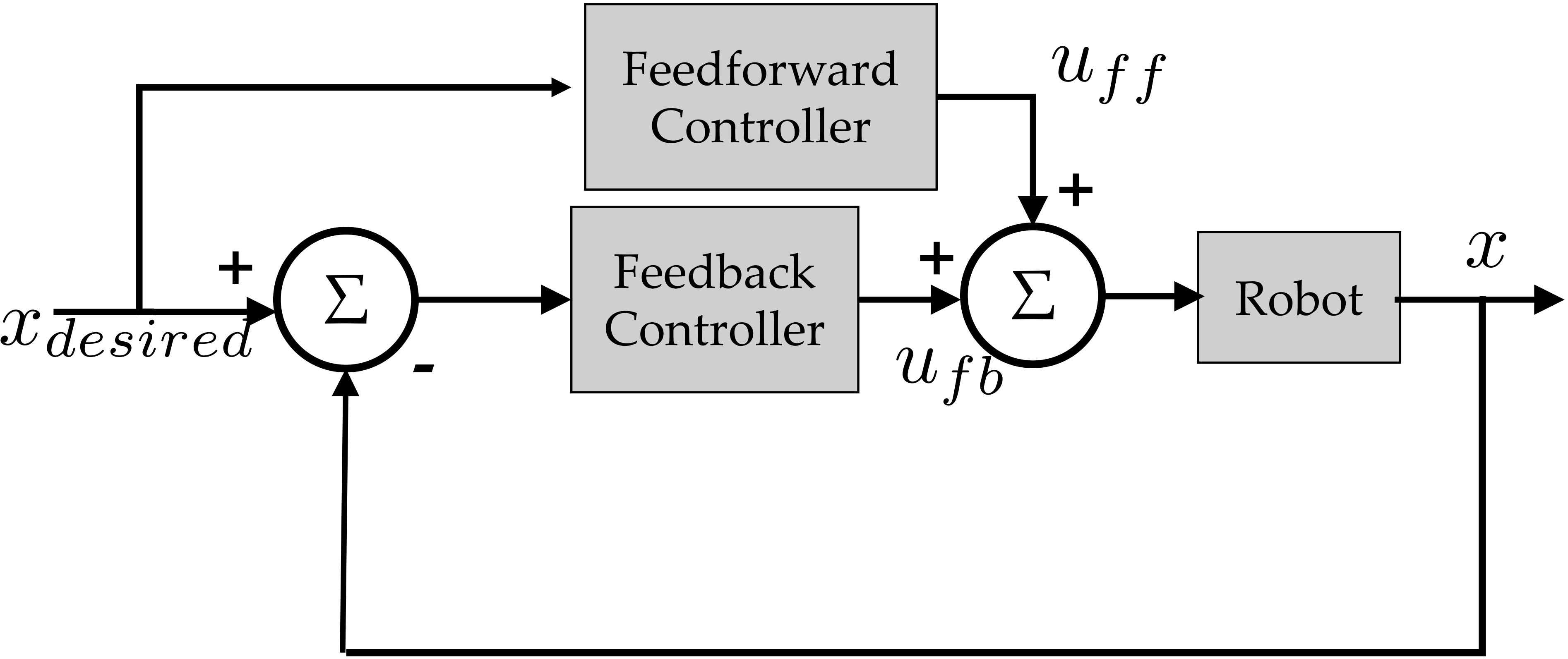
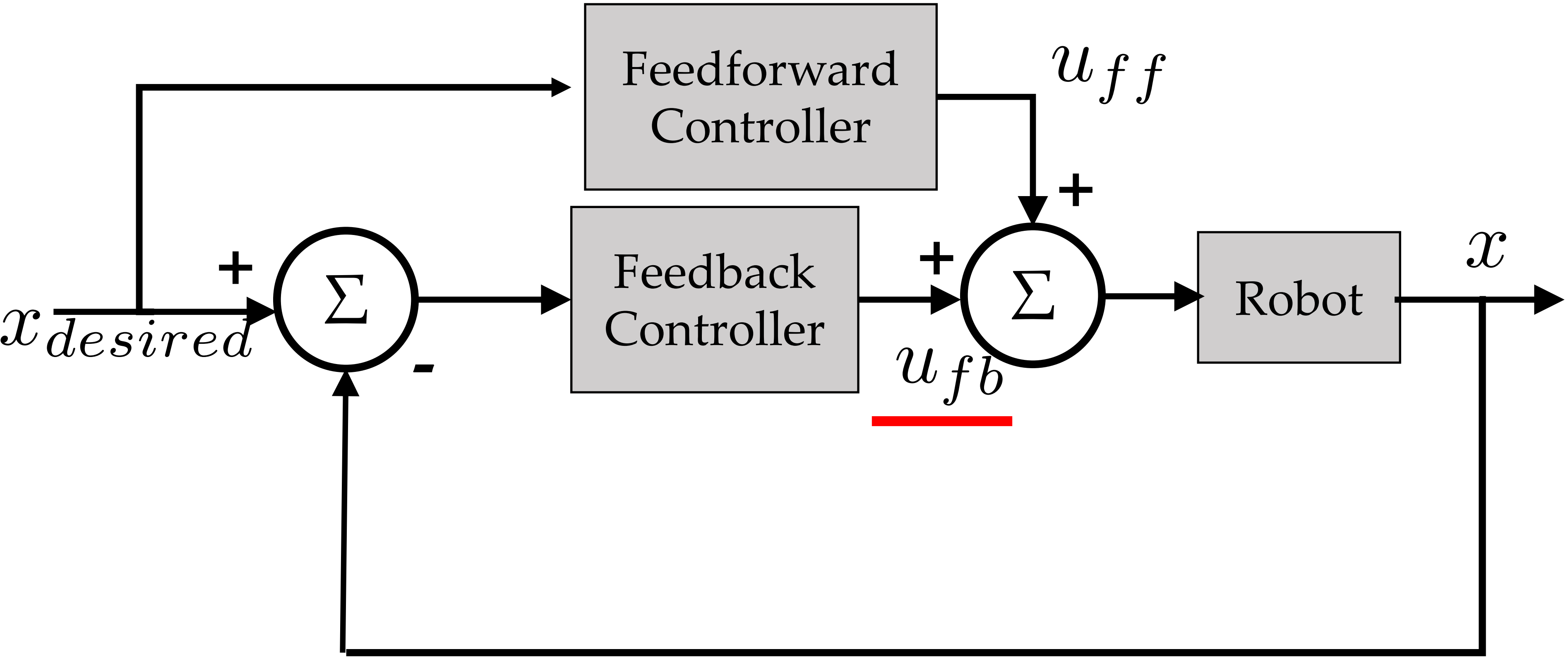
  - I focus on the diagonal elements of B,C and I treat the rest as disturbances
  - I create a PD / PID controller for each joint-space independently
- Dependent Joint-Space Control
  - I cannot ignore the coupling terms

Negative Feedback & Feedforward Control



Negative Feedback & Feedforward Control



Negative Feedback Control

- Based on linear control:
  - Proportional Control ("Position Error")

$$u_P = \pi(x - x_{des}, \alpha, t) = K_P(x_{des}(t) - x(t))$$

  - Derivative Control ("Damping")

$$u_D = \pi(x - x_{des}, \alpha, t) = K_D(\dot{x}_{des}(t) - \dot{x}(t))$$

  - Integral Control ("Steady State Error")

$$u_I(t) = K_I \int_{\tau=0}^{\tau=t} (x_{des}(t) - x(t))dt$$

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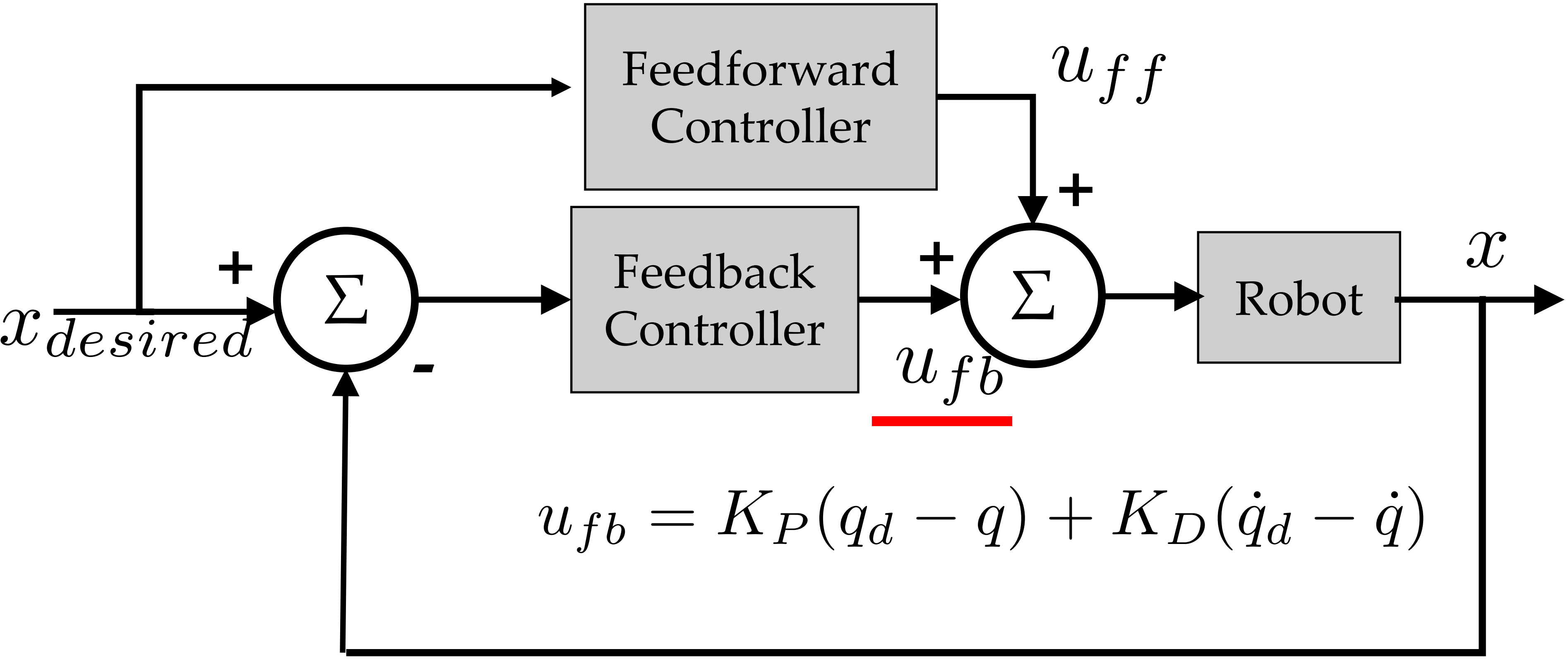
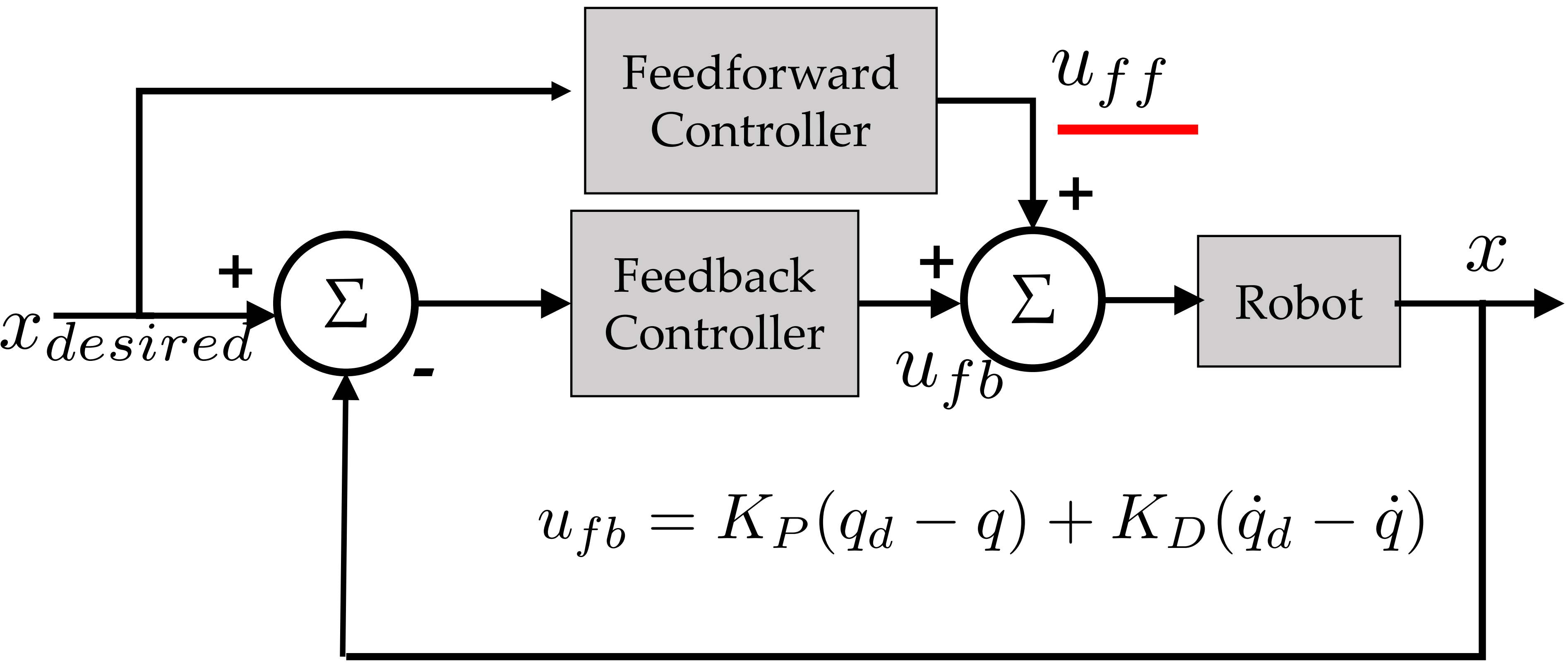


$$u_{fb} = K_P(q_d - q) + K_D(\dot{q}_d - \dot{q})$$

Negative Feedback & Feedforward Control



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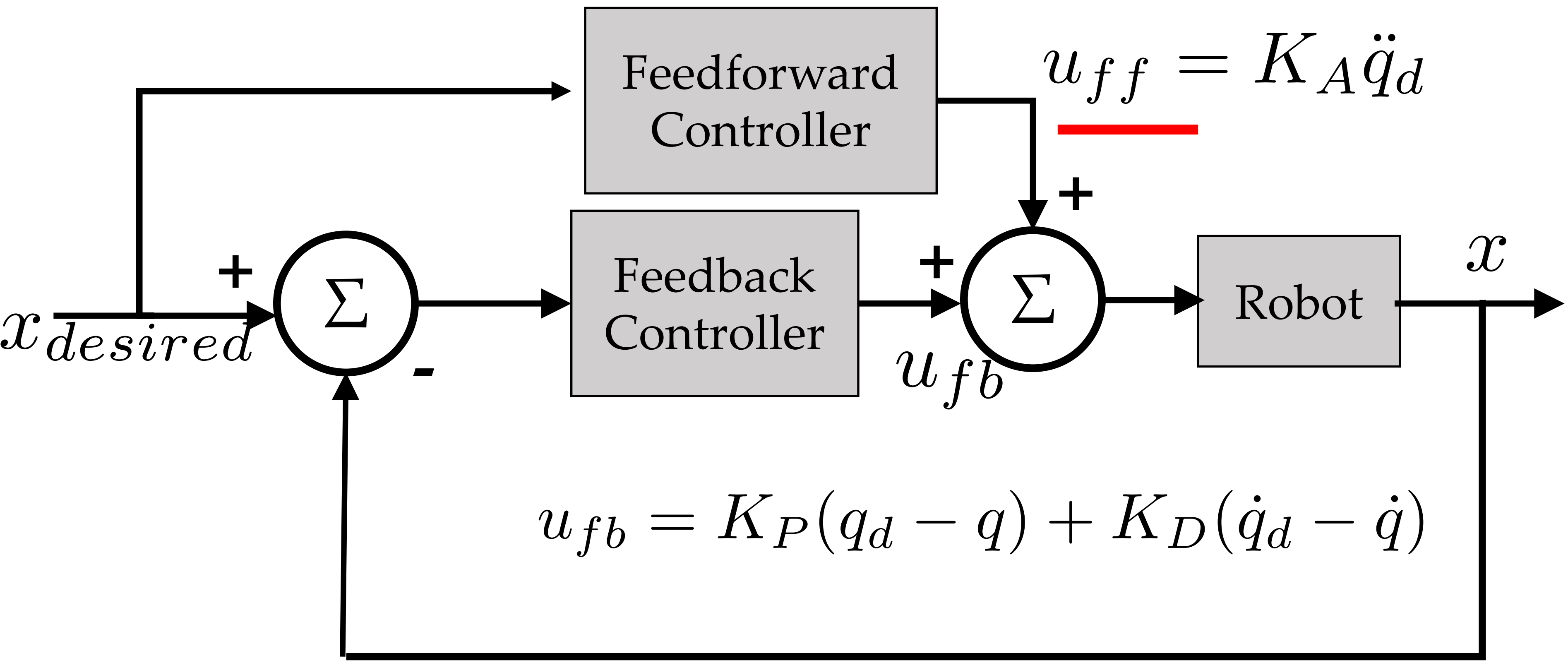

Independent Joint-Space Control

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = K_A\ddot{q}_d + K_P(q_d - q) + K_D(\dot{q}_d - \dot{q})$$

Independent Joint-Space Control

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = K_A\ddot{q}_d + K_P(q_d - q) + K_D(\dot{q}_d - \dot{q})$$

We compensate for the coupling terms by treating them as noise

Hybrid Joint-Space Control

- We can take into account the dynamics of the system

$$B(q)\ddot{q} + C(q,\dot{q}) + G(q) = \tau$$

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = B(q_d)\ddot{q}_d + C(q_d,\dot{q}_d)\dot{q}_d + G(q_d) + K_P(q_d - q) + K_D(\dot{q}_d - \dot{q})$$

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$$B(q) \approx B(q_d), C(q,\dot{q}) \approx C(q_d,\dot{q}_d)$$

$$B(q)\ddot{\tilde{q}} + (C(q,\dot{q}) + K_D)\dot{\tilde{q}} + K_P\tilde{q} = 0$$

Hybrid Joint-Space Control

- Not accurate if the system deviates too much from the desired trajectory

- They can be computed offline

Proper Inverse Dynamics Control

$$B(q)\ddot{q} + C(q,\dot{q}) + G(q) = \tau$$

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = B(q)[\ddot{q}_d + K_P(q_d - q) + K_D(\dot{q}_d - \dot{q})] + C(q,\dot{q})\dot{q} + G(q)$$

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$$\ddot{\tilde{q}} + K_D\dot{\tilde{q}} + K_P\tilde{q} = 0$$

- Requires online computation of the dynamics

- Requires perfect knowledge of the system

A simpler example: PD with Gravity Compensation

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau$$

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = G(q) + K_P(q_d - q) + K_D(\dot{q}_d - \dot{q})$$

Assume $\dot{q}_d = 0$

Controller Stability

- We can use Lyapunov's method for proving the stability of the system
- It is a function of energy of the system.
- To prove stability, we need:
  - V(x) = 0 iff x = 0
  - V(x) > 0 iff x ≠ 0
  - $\dot{V}(x) = \frac{d}{dt}V(x) = \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} f_i(x) = \nabla V \cdot f(x) \leq 0, \forall x \neq 0$

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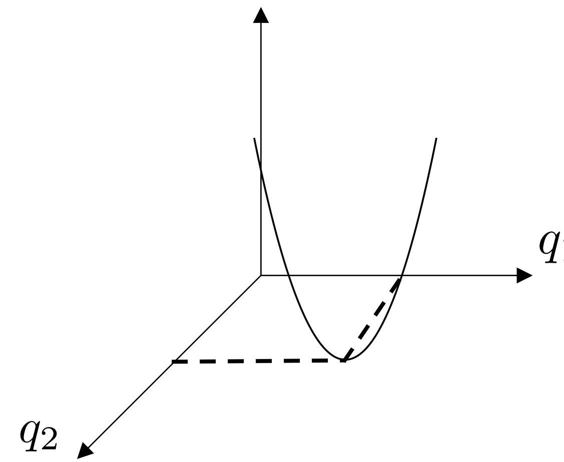

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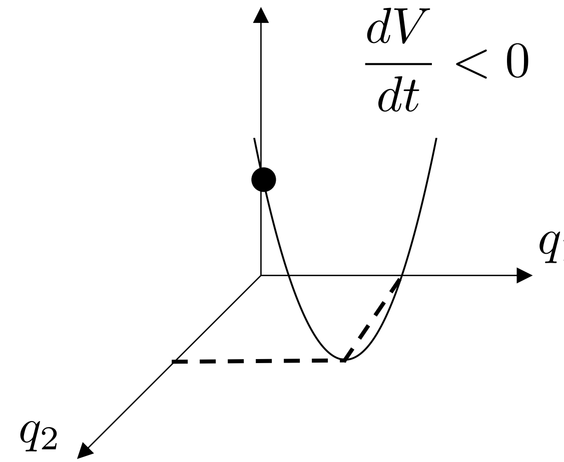

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Stability of PD+Gravity Compensation Controller

- We consider the following Lyapunov function:

$$V(\dot{q}, \tilde{q}) = \frac{1}{2}\dot{q}^T B(q)\dot{q} + \frac{1}{2}\tilde{q}^T K_P \tilde{q} > 0, \forall \dot{q}, \tilde{q} \neq 0, \tilde{q} = q_d - q$$

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$$\dot{V} = \dot{q}^T B(q)\ddot{q} + \frac{1}{2}\dot{q}^T \dot{B}(q)\dot{q} - \dot{q}^T K_p \tilde{q}$$

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$$(\dot{\tilde{q}} = -\dot{q})$$

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We can substitute: $$B(q)\ddot{q} = \tau - C(q, \dot{q})\dot{q} - G(q)$$

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Stability of PD+Gravity Compensation Controller

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$$\dot{q}^T(\dot{B} - 2C)\dot{q} = 0$$

(see SS, "7.2 Notable Properties of Dynamic Model")

Stability of PD+Gravity Compensation Controller

$$\dot{V} = \dot{q}^T \tau - \dot{q}^T C(q, \dot{q})\dot{q} - \dot{q}^T G(q) + \frac{1}{2}\dot{q}^T \dot{B}(q)\dot{q} - \dot{q}^T K_p \tilde{q}$$

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$$\dot{V} = \dot{q}^T \tau - \dot{q}^T \cancel{C(q, \dot{q})\dot{q}} - \dot{q}^T G(q) + \frac{1}{2}\dot{q}^T \cancel{\dot{B}(q)\dot{q}} - \dot{q}^T K_p \tilde{q}$$

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Stability of PD+Gravity Compensation Controller

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$$\dot{V} = \dot{q}^T\tau - \dot{q}^T G(q) - \dot{q}^T K_p \tilde{q}$$

When is $\dot{V} < 0$?

Stability of PD+Gravity Compensation Controller

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When is $\dot{V} < 0$?

We can let $\tau = G(q) + K_p \tilde{q} - K_d \dot{q}$

Stability of PD+Gravity Compensation Controller

$$\dot{V} = \dot{q}^T \tau - \dot{q}^T C(q, \dot{q})\dot{q} - \dot{q}^T G(q) + \frac{1}{2}\dot{q}^T \dot{B}(q)\dot{q} - \dot{q}^T K_p \tilde{q}$$

$\dot{B} - 2C = 0$ (see SS, "7.2 Notable Properties of Dynamic Model")

$$\dot{V} = \dot{q}^T \tau - \dot{q}^T G(q) - \dot{q}^T K_p \tilde{q}$$

When is $\dot{V} < 0$?

We can let $\tau = G(q) + K_p \tilde{q} - K_d \dot{q}$

Feedback Linearization

A simpler example: PD with Gravity Compensation

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau$$

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = G(q) + K_P(q_d - q) + K_D(\dot{q}_d - \dot{q})$$

Assume $\dot{q}_d = 0$