USC Viterbi School of Engineering

Bayesian Filtering

CSCI 545 Introduction to Robotics
Instructor: Stefanos Nikolaidis

Recap

- Stationary Dynamics

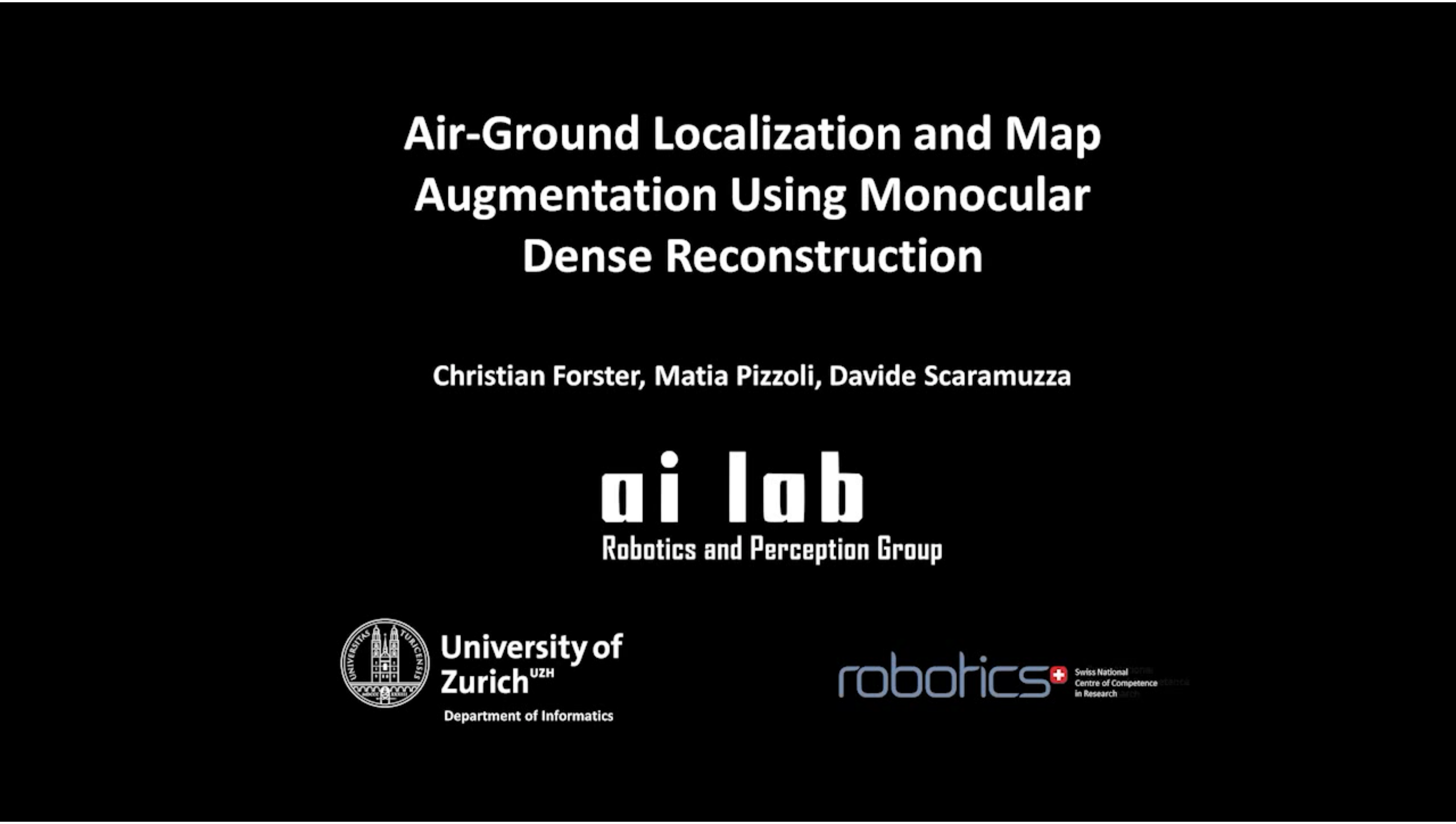
$$P(X_t|X_{0:t-1}, U_{0:t}) = P(X_t|X_{t-1}, U_t)$$

- Markov Assumption

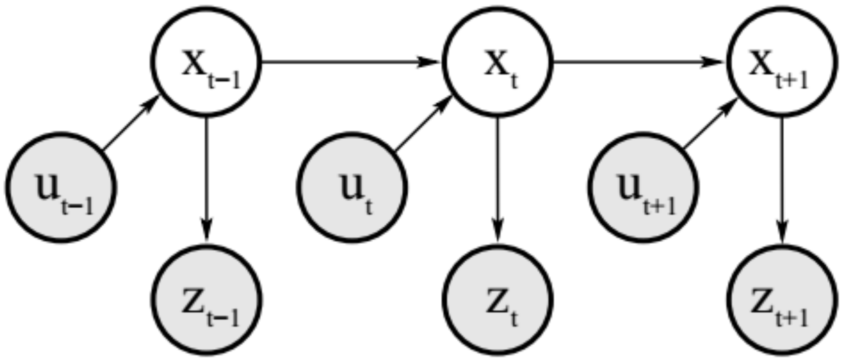
$$P(X_t|X_{0:t-1}, U_{0:t}) = P(X_t|X_{t-1}, U_t)$$

$$P(Z_t|X_{0:t}, U_{0:t}) = P(Z_t|X_t)$$

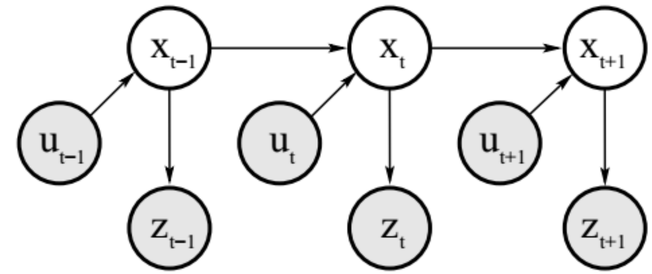
State Estimation



Bayesian Filtering

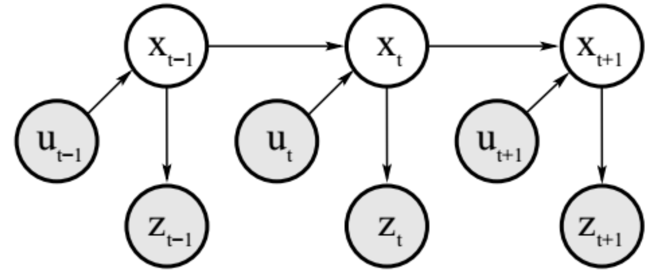


Bayesian Filtering



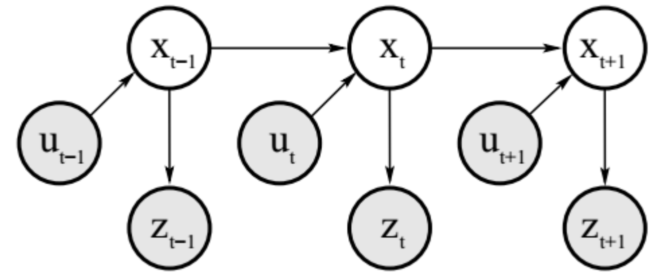
$$P(x_t | z_{1:t}, u_{1:t})$$

Bayesian Filtering



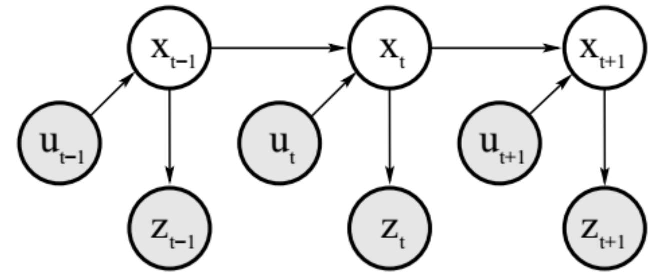
$$P(x_t|z_{1:t}, u_{1:t}) = P(x_t|z_t, z_{1:t-1}, u_{1:t})$$

Bayesian Filtering



$$
\begin{aligned}
P(x_t|z_{1:t},u_{1:t}) &= P(x_t|z_t,z_{1:t-1},u_{1:t}) \\
&= \frac{P(z_t|x_t,z_{1:t-1},u_{1:t})P(x_t|z_{1:t-1},u_{1:t})}{P(z_t|z_{1:t-1},u_{1:t})}
\end{aligned}
$$

Bayesian Filtering



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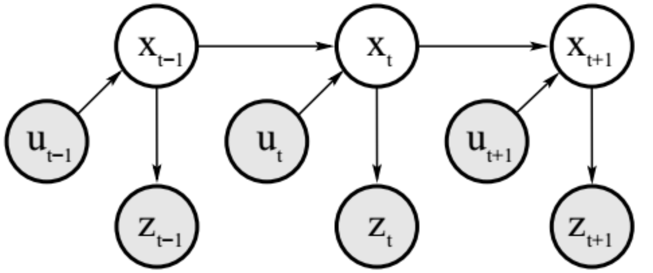
&= \frac{P(z_t|x_t, z_{1:t-1}, u_{1:t})P(x_t|z_{1:t-1}, u_{1:t})}{P(z_t|z_{1:t-1}, u_{1:t})} \\
&= \eta P(z_t|x_t, z_{1:t-1}, u_{1:t})P(x_t|z_{1:t-1}, u_{1:t})
\end{aligned}
$$

Bayesian Filtering



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\begin{aligned}
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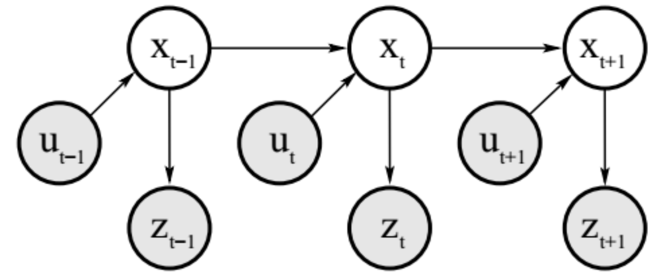
&= \eta P(z_t|x_t, z_{1:t-1}, u_{1:t})P(x_t|z_{1:t-1}, u_{1:t}) \\
&= \eta P(z_t|x_t) \sum_{x_{t-1}} P(x_t, x_{t-1}|z_{1:t-1}, u_{1:t})
\end{aligned}
$$

Bayesian Filtering



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\begin{aligned}
P(x_t|z_{1:t}, u_{1:t}) &= P(x_t|z_t, z_{1:t-1}, u_{1:t}) \\
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&= \eta P(z_t|x_t, z_{1:t-1}, u_{1:t}) P(x_t|z_{1:t-1}, u_{1:t}) \\
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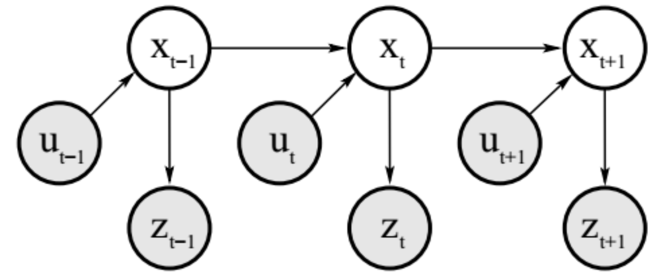
Bayesian Filtering



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P(x_t|z_{1:t}, u_{1:t}) &= P(x_t|z_t, z_{1:t-1}, u_{1:t}) \\
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&= \eta P(z_t|x_t) \sum_{x_{t-1}} P(x_t, x_{t-1}|z_{1:t-1}, u_{1:t}) \\
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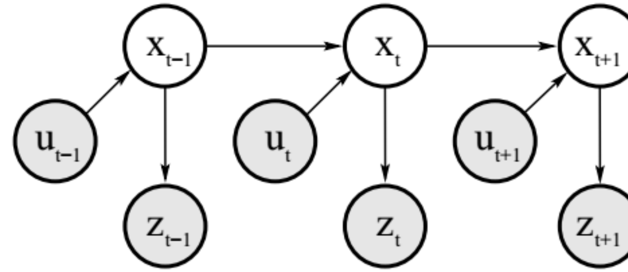
&= \eta P(z_t|x_t) \sum_{x_{t-1}} P(x_t|x_{t-1}, u_t)P(x_{t-1}|z_{1:t-1}, u_{1:t})
\end{aligned}
$$

Bayesian Filtering



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\begin{aligned}
P(x_t|z_{1:t}, u_{1:t}) &= P(x_t|z_t, z_{1:t-1}, u_{1:t}) \\
&= \frac{P(z_t|x_t, z_{1:t-1}, u_{1:t})P(x_t|z_{1:t-1}, u_{1:t})}{P(z_t|z_{1:t-1}, u_{1:t})} \\
&= \eta P(z_t|x_t, z_{1:t-1}, u_{1:t})P(x_t|z_{1:t-1}, u_{1:t}) \\
&= \eta P(z_t|x_t) \sum_{x_{t-1}} P(x_t, x_{t-1}|z_{1:t-1}, u_{1:t}) \\
&= \eta P(z_t|x_t) \sum_{x_{t-1}} P(x_t|x_{t-1}, z_{1:t-1}, u_{1:t})P(x_{t-1}|z_{1:t-1}, u_{1:t}) \\
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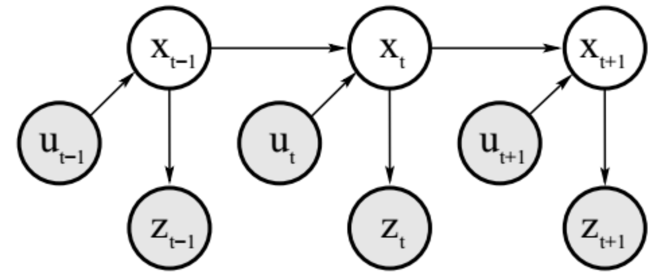
&= \eta P(z_t|x_t) \sum_{x_{t-1}} P(x_t|x_{t-1}, z_{1:t-1}, u_{1:t})P(x_{t-1}|z_{1:t-1}, u_{1:t-1})
\end{aligned}
$$

Bayesian Filtering



$$\underline{P(x_t|z_{1:t}, u_{1:t})} = P(x_t|z_t, z_{1:t-1}, u_{1:t})$$

$$= \frac{P(z_t|x_t, z_{1:t-1}, u_{1:t})P(x_t|z_{1:t-1}, u_{1:t})}{P(z_t|z_{1:t-1}, u_{1:t})}$$

$$= \eta P(z_t|x_t, z_{1:t-1}, u_{1:t})P(x_t|z_{1:t-1}, u_{1:t})$$

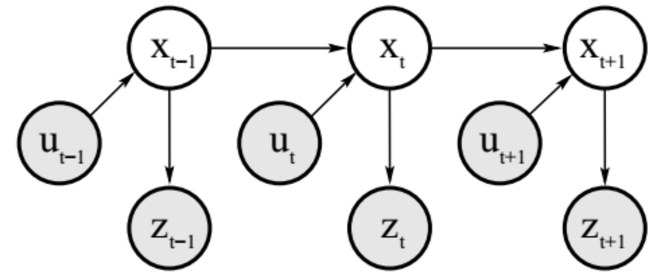
$$= \eta P(z_t|x_t) \sum_{x_{t-1}} P(x_t, x_{t-1}|z_{1:t-1}, u_{1:t})$$

$$= \eta P(z_t|x_t) \sum_{x_{t-1}} P(x_t|x_{t-1}, z_{1:t-1}, u_{1:t})P(x_{t-1}|z_{1:t-1},$$

$$= \eta P(z_t|x_t) \sum_{x_{t-1}} P(x_t|x_{t-1}, u_t)P(x_{t-1}|z_{1:t-1}, u_{1:t})$$

$$= \eta P(z_t|x_t) \sum_{x_{t-1}} P(x_t|x_{t-1}, u_t)\underline{P(x_{t-1}|z_{1:t-1}, u_{1:t-1})}$$

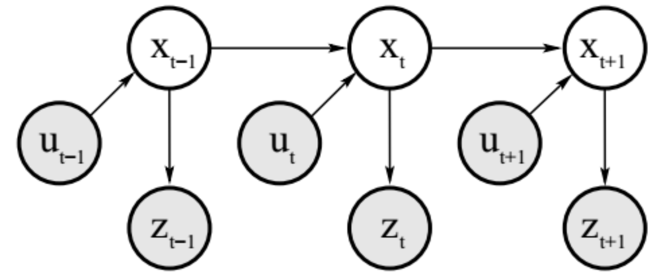
Bayesian Filtering



$$P(x_t|z_{1:t}, u_{1:t}) = \eta P(z_t|x_t) \sum_{x_{t-1}} P(x_t|x_{t-1}, u_t) P(x_{t-1}|z_{1:t-1}, u_{1:t-1})$$

$$b(x_t) = \eta P(z_t|x_t) \sum_{x_{t-1}} P(x_t|x_{t-1}, u_t) b(x_{t-1})$$

Bayesian Filtering



$$b(x_t) = \eta P(z_t|x_t) \sum_{x_{t-1}} P(x_t|x_{t-1}, z_{1:t-1}, u_{1:t}) b(x_{t-1})$$

Algorithm **Bayes Filter**($b(x_{t-1})$, u_t , z_t)

For all x_t do:

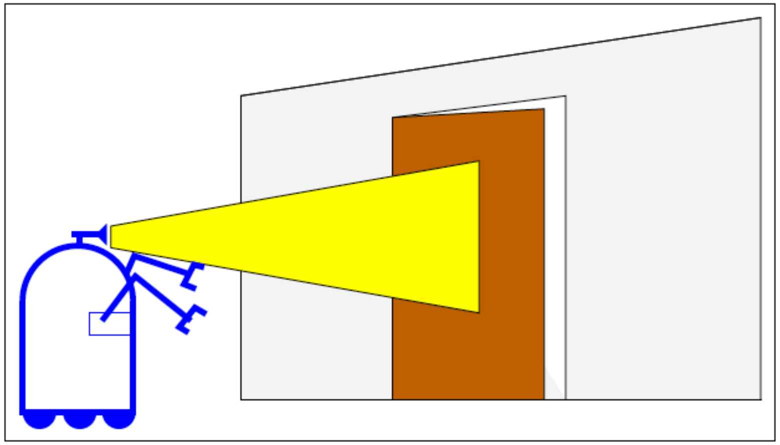
$$\bar{b}(x_t) = \sum_{x_{t-1}} P(x_t|x_{t-1}, u_t) b(x_{t-1})$$

$$b(x_t) = \eta P(z_t|x_t) \bar{b}(x_t)$$

Endfor

Return $b(x_t)$

Example

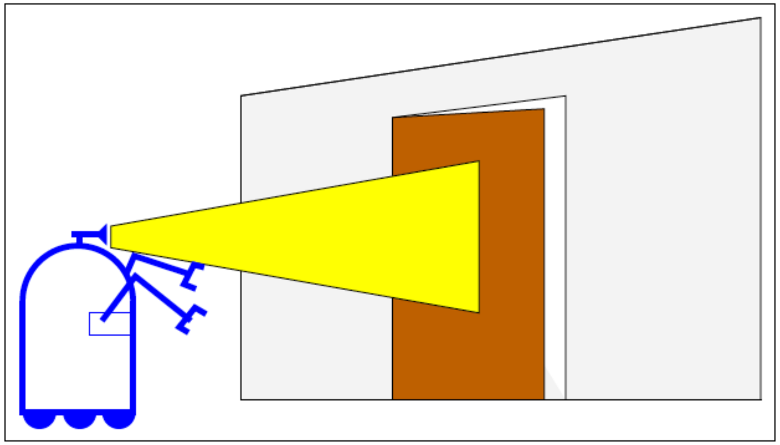


States: $x = \{open, closed\}$

Observations: $z = \{sense - open, sense - closed\}$

Actions: $u = \{push, do - nothing\}$

Example



States: $x = \{open, closed\}$

Observations: $z = \{sense - open, sense - closed\}$

Actions: $u = \{push, do - nothing\}$

Sensor Model:

$$P(z_t = sense - open | x_t = open) = 0.6$$
$$P(z_t = sense - closed | x_t = open) = 0.4$$
$$P(z_t = sense - open | x_t = closed) = 0.2$$
$$P(z_t = sense - closed | x_t = closed) = 0.8$$

Example

States: $x = \{open, closed\}$

Observations: $z = \{sense - open, sense - closed\}$

Actions: $u = \{push, do - nothing\}$

Sensor Model: $P(z_t = sense - open | x_t = open) = 0.6$

$$P(z_t = sense - closed | x_t = open) = 0.4$$

$$P(z_t = sense - open | x_t = closed) = 0.2$$

$$P(z_t = sense - closed | x_t = closed) = 0.8$$

Action Model:

$$P(x_t = open | x_{t-1} = open, u_t = push) = 1.0$$

$$P(x_t = closed | x_{t-1} = open, u_t = push) = 0.0$$

$$P(x_t = open | x_{t-1} = closed, u_t = push) = 0.8$$

$$P(x_t = closed | x_{t-1} = closed, u_t = push) = 0.2$$

Example

States: $x = \{open, closed\}$

Observations: $z = \{sense - open, sense - closed\}$

Actions: $u = \{push, do - nothing\}$

Sensor Model:

$$P(z_t = sense - open | x_t = open) = 0.6$$
$$P(z_t = sense - closed | x_t = open) = 0.4$$
$$P(z_t = sense - open | x_t = closed) = 0.2$$
$$P(z_t = sense - closed | x_t = closed) = 0.8$$

Action Model:

$$P(x_t = open | x_{t-1} = open, u_t = push) = 1.0$$
$$P(x_t = closed | x_{t-1} = open, u_t = push) = 0.0$$
$$P(x_t = open | x_{t-1} = closed, u_t = push) = 0.8$$
$$P(x_t = closed | x_{t-1} = closed, u_t = push) = 0.2$$

$$P(x_t = open | x_{t-1} = open, u_t = do - nothing) = 1.0$$
$$P(x_t = closed | x_{t-1} = open, u_t = do - nothing) = 0.0$$
$$P(x_t = open | x_{t-1} = closed, u_t = do - nothing) = 0.0$$
$$P(x_t = closed | x_{t-1} = closed, u_t = do - nothing) = 1.0$$

Example

States: $x = \{open, closed\}$

Observations: $z = \{sense - open, sense - closed\}$

Actions: $u = \{push, do - nothing\}$

Sensor Model: $P(z_t = sense - open | x_t = open) = 0.6$

$$P(z_t = sense - closed | x_t = open) = 0.4$$
$$P(z_t = sense - open | x_t = closed) = 0.2$$
$$P(z_t = sense - closed | x_t = closed) = 0.8$$

Action Model: $P(x_t = open | x_{t-1} = open, u_t = push) = 1.0$

$$P(x_t = closed | x_{t-1} = open, u_t = push) = 0.0$$
$$P(x_t = open | x_{t-1} = closed, u_t = push) = 0.8$$
$$P(x_t = closed | x_{t-1} = closed, u_t = push) = 0.2$$

Example

- In the beginning, we don’t know if the door is open or closed.
- How can we model the initial belief?

$t = 0$

States: $x = \{open, closed\}$

Observations: $z = \{sense - open, sense - closed\}$

Actions: $u = \{push, do - nothing\}$

Sensor Model: $P(z_t = sense - open | x_t = open) = 0.6$

$P(z_t = sense - closed | x_t = open) = 0.4$

$P(z_t = sense - open | x_t = closed) = 0.2$

$P(z_t = sense - closed | x_t = closed) = 0.8$

Action Model: $P(x_t = open | x_{t-1} = open, u_t = push) = 1.0$

$P(x_t = closed | x_{t-1} = open, u_t = push) = 0.0$

$P(x_t = open | x_{t-1} = closed, u_t = push) = 0.8$

$P(x_t = closed | x_{t-1} = closed, u_t = push) = 0.2$

$$b(x_0 = open) = 0.5$$

$$b(x_0 = closed) = 0.5$$

$t = 1$

States: $x = \{open, closed\}$

Observations: $z = \{sense - open, sense - closed\}$

Actions: $u = \{push, do - nothing\}$

Sensor Model: $P(z_t = sense - open | x_t = open) = 0.6$

$P(z_t = sense - closed | x_t = open) = 0.4$

$P(z_t = sense - open | x_t = closed) = 0.2$

$P(z_t = sense - closed | x_t = closed) = 0.8$

Action Model: $P(x_t = open | x_{t-1} = open, u_t = push) = 1.0$

$P(x_t = closed | x_{t-1} = open, u_t = push) = 0.0$

$P(x_t = open | x_{t-1} = closed, u_t = push) = 0.8$

$P(x_t = closed | x_{t-1} = closed, u_t = push) = 0.2$

$$b(x_0 = open) = 0.5$$

$$b(x_0 = closed) = 0.5$$

Assume that the robot does nothing in the first time-step
and then senses an open door!

$t = 1$

States: $x = \{open, closed\}$

Observations: $z = \{sense - open, sense - closed\}$

Actions: $u = \{push, do - nothing\}$

Sensor Model: $P(z_t = sense - open | x_t = open) = 0.6$

$$P(z_t = sense - closed | x_t = open) = 0.4$$

$$P(z_t = sense - open | x_t = closed) = 0.2$$

$$P(z_t = sense - closed | x_t = closed) = 0.8$$

Action Model: $P(x_t = open | x_{t-1} = open, u_t = push) = 1.0$

$$P(x_t = closed | x_{t-1} = open, u_t = push) = 0.0$$

$$P(x_t = open | x_{t-1} = closed, u_t = push) = 0.8$$

$$P(x_t = closed | x_{t-1} = closed, u_t = push) = 0.2$$

$$b(x_0 = open) = 0.5$$

$$b(x_0 = closed) = 0.5$$

Assume that the robot does nothing in the first time-step and then senses an open door!

$$u_1 = do - nothing$$

$$z_1 = sense - open$$

$t = 1$

States: $x = \{open, closed\}$

Observations: $z = \{sense - open, sense - closed\}$

Actions: $u = \{push, do - nothing\}$

Sensor Model: $P(z_t = sense - open | x_t = open) = 0.6$

$$P(z_t = sense - closed | x_t = open) = 0.4$$

$$P(z_t = sense - open | x_t = closed) = 0.2$$

$$P(z_t = sense - closed | x_t = closed) = 0.8$$

Action Model: $P(x_t = open | x_{t-1} = open, u_t = push) = 1.0$

$$P(x_t = closed | x_{t-1} = open, u_t = push) = 0.0$$

$$P(x_t = open | x_{t-1} = closed, u_t = push) = 0.8$$

$$P(x_t = closed | x_{t-1} = closed, u_t = push) = 0.2$$

$$b(x_0 = open) = 0.5$$

$$b(x_0 = closed) = 0.5$$

Assume that the robot does nothing in the first time-step and then senses an open door!

$$u_1 = do - nothing$$

$$z_1 = sense - open$$

Belief update equation: $b(x_t) = \eta P(z_t | x_t) \sum_{x_{t-1}} P(x_t | x_{t-1}, u_t) b(x_{t-1})$

$t = 1$

States: $x = \{open, closed\}$

Observations: $z = \{sense - open, sense - closed\}$

Actions: $u = \{push, do - nothing\}$

Sensor Model: $P(z_t = sense - open | x_t = open) = 0.6$

$$P(z_t = sense - closed | x_t = open) = 0.4$$

$$P(z_t = sense - open | x_t = closed) = 0.2$$

$$P(z_t = sense - closed | x_t = closed) = 0.8$$

Action Model: $P(x_t = open | x_{t-1} = open, u_t = push) = 1.0$

$$P(x_t = closed | x_{t-1} = open, u_t = push) = 0.0$$

$$P(x_t = open | x_{t-1} = closed, u_t = push) = 0.8$$

$$P(x_t = closed | x_{t-1} = closed, u_t = push) = 0.2$$

$$b(x_0 = open) = 0.5$$

$$b(x_0 = closed) = 0.5$$

Assume that the robot does nothing in the first time-step and then senses an open door!

$$b(x_1) = \eta P(z_1 | x_1) \sum_{x_0} P(x_1 | x_0, u_1) b(x_0)$$

$t = 1$

States: $x = \{open, closed\}$

Observations: $z = \{sense - open, sense - closed\}$

Actions: $u = \{push, do - nothing\}$

Sensor Model: $P(z_t = sense - open | x_t = open) = 0.6$
$P(z_t = sense - closed | x_t = open) = 0.4$
$P(z_t = sense - open | x_t = closed) = 0.2$
$P(z_t = sense - closed | x_t = closed) = 0.8$

Action Model: $P(x_t = open | x_{t-1} = open, u_t = push) = 1.0$
$P(x_t = closed | x_{t-1} = open, u_t = push) = 0.0$
$P(x_t = open | x_{t-1} = closed, u_t = push) = 0.8$
$P(x_t = closed | x_{t-1} = closed, u_t = push) = 0.2$

$$b(x_0 = open) = 0.5$$
$$b(x_0 = closed) = 0.5$$

Assume that the robot does nothing in the first time-step and then senses an open door!

$$b(x_1) = \eta P(z_1 | x_1) \sum_{x_0} P(x_1 | x_0, u_1) b(x_0)$$

Bayes Filter

$$\bar{b}(x_1) = \sum_{x_0} P(x_1 | x_0, u_1) b(x_0)$$

$$b(x_1) = \eta P(z_1 | x_1) \bar{b}(x_1)$$

$t = 1$

States: $x = \{open, closed\}$

Observations: $z = \{sense - open, sense - closed\}$

Actions: $u = \{push, do - nothing\}$

Sensor Model: $P(z_t = sense - open|x_t = open) = 0.6$

$P(z_t = sense - closed|x_t = open) = 0.4$

$P(z_t = sense - open|x_t = closed) = 0.2$

$P(z_t = sense - closed|x_t = closed) = 0.8$

Action Model: $P(x_t = open|x_{t-1} = open, u_t = push) = 1.0$

$P(x_t = closed|x_{t-1} = open, u_t = push) = 0.0$

$P(x_t = open|x_{t-1} = closed, u_t = push) = 0.8$

$P(x_t = closed|x_{t-1} = closed, u_t = push) = 0.2$

Bayes Filter

$$\bar{b}(x_1) = \sum_{x_0} P(x_1|x_0, u_1)b(x_0)$$

$$b(x_1) = \eta P(z_1|x_1)\bar{b}(x_1)$$

$t = 1$

States: $x = \{open, closed\}$

Observations: $z = \{sense - open, sense - closed\}$

Actions: $u = \{push, do - nothing\}$

Sensor Model: $P(z_t = sense - open | x_t = open) = 0.6$

$P(z_t = sense - closed | x_t = open) = 0.4$

$P(z_t = sense - open | x_t = closed) = 0.2$

$P(z_t = sense - closed | x_t = closed) = 0.8$

Action Model: $P(x_t = open | x_{t-1} = open, u_t = push) = 1.0$

$P(x_t = closed | x_{t-1} = open, u_t = push) = 0.0$

$P(x_t = open | x_{t-1} = closed, u_t = push) = 0.8$

$P(x_t = closed | x_{t-1} = closed, u_t = push) = 0.2$

Bayes Filter

$$\bar{b}(x_1) = \sum_{x_0} P(x_1 | x_0, u_1) b(x_0)$$

$$b(x_1) = \eta P(z_1 | x_1) \bar{b}(x_1)$$

$$\begin{aligned}\bar{b}(x_1 = open) = & P(x_1 = open | x_0 = open, u_1 = do - nothing) b(x_0 = open) \\ & + P(x_1 = open | x_0 = closed, u_1 = do - nothing) b(x_0 = closed)\end{aligned}$$

$$\bar{b}(x_1 = open) = 1.0 * 0.5 + 0.0 * 0.5$$

$t = 1$

States: $x = \{open, closed\}$

Observations: $z = \{sense - open, sense - closed\}$

Actions: $u = \{push, do - nothing\}$

Sensor Model: $P(z_t = sense - open | x_t = open) = 0.6$

$P(z_t = sense - closed | x_t = open) = 0.4$

$P(z_t = sense - open | x_t = closed) = 0.2$

$P(z_t = sense - closed | x_t = closed) = 0.8$

Action Model: $P(x_t = open | x_{t-1} = open, u_t = push) = 1.0$

$P(x_t = closed | x_{t-1} = open, u_t = push) = 0.0$

$P(x_t = open | x_{t-1} = closed, u_t = push) = 0.8$

$P(x_t = closed | x_{t-1} = closed, u_t = push) = 0.2$

Bayes Filter

$$\bar{b}(x_1) = \sum_{x_0} P(x_1 | x_0, u_1) b(x_0)$$

$$b(x_1) = \eta P(z_1 | x_1) \bar{b}(x_1)$$

$$\begin{aligned}\bar{b}(x_1 = open) = {} & P(x_1 = open | x_0 = open, u_1 = do - nothing) b(x_0 = open) \\ & + P(x_1 = open | x_0 = closed, u_1 = do - nothing) b(x_0 = closed)\end{aligned}$$

$$\bar{b}(x_1 = open) = 1.0 * 0.5 + 0.0 * 0.5$$

$$\begin{aligned}\bar{b}(x_1 = closed) = {} & P(x_1 = closed | x_0 = open, u_1 = do - nothing) b(x_0 = open) \\ & + P(x_1 = closed | x_0 = closed, u_1 = do - nothing) b(x_0 = closed)\end{aligned}$$

$$\bar{b}(x_1 = closed) = 0.0 * 0.5 + 1.0 * 0.5$$

$t = 1$

States: $x = \{open, closed\}$

Observations: $z = \{sense - open, sense - closed\}$

Actions: $u = \{push, do - nothing\}$

Sensor Model: $P(z_t = sense - open | x_t = open) = 0.6$

$$P(z_t = sense - closed | x_t = open) = 0.4$$

$$P(z_t = sense - open | x_t = closed) = 0.2$$

$$P(z_t = sense - closed | x_t = closed) = 0.8$$

Action Model: $P(x_t = open | x_{t-1} = open, u_t = push) = 1.0$

$$P(x_t = closed | x_{t-1} = open, u_t = push) = 0.0$$

$$P(x_t = open | x_{t-1} = closed, u_t = push) = 0.8$$

$$P(x_t = closed | x_{t-1} = closed, u_t = push) = 0.2$$

Bayes Filter

$$\bar{b}(x_1) = \sum_{x_0} P(x_1 | x_0, u_1) b(x_0)$$

$$b(x_1) = \eta P(z_1 | x_1) \bar{b}(x_1)$$

Why did the belief not change?

$t = 1$

States: $x = \{open, closed\}$

Observations: $z = \{sense - open, sense - closed\}$

Actions: $u = \{push, do - nothing\}$

Sensor Model: $P(z_t = sense - open|x_t = open) = 0.6$

$P(z_t = sense - closed|x_t = open) = 0.4$

$P(z_t = sense - open|x_t = closed) = 0.2$

$P(z_t = sense - closed|x_t = closed) = 0.8$

Action Model: $P(x_t = open|x_{t-1} = open, u_t = push) = 1.0$

$P(x_t = closed|x_{t-1} = open, u_t = push) = 0.0$

$P(x_t = open|x_{t-1} = closed, u_t = push) = 0.8$

$P(x_t = closed|x_{t-1} = closed, u_t = push) = 0.2$

Bayes Filter

$$\bar{b}(x_1) = \sum_{x_0} P(x_1|x_0, u_1)b(x_0)$$

$$b(x_1) = \eta P(z_1|x_1)\bar{b}(x_1)$$

$$\begin{aligned} b(x_1 = open) &= \eta P(z_1 = sense - open|x_1 = open)\bar{b}(x_1 = open) \\ &= \eta 0.6 * 0.5 \\ &= \eta 0.3 \end{aligned}$$

$t = 1$

States: $x = \{open, closed\}$

Observations: $z = \{sense - open, sense - closed\}$

Actions: $u = \{push, do - nothing\}$

Sensor Model: $P(z_t = sense - open | x_t = open) = 0.6$

$P(z_t = sense - closed | x_t = open) = 0.4$

$P(z_t = sense - open | x_t = closed) = 0.2$

$P(z_t = sense - closed | x_t = closed) = 0.8$

Action Model: $P(x_t = open | x_{t-1} = open, u_t = push) = 1.0$

$P(x_t = closed | x_{t-1} = open, u_t = push) = 0.0$

$P(x_t = open | x_{t-1} = closed, u_t = push) = 0.8$

$P(x_t = closed | x_{t-1} = closed, u_t = push) = 0.2$

Bayes Filter

$$\bar{b}(x_1) = \sum_{x_0} P(x_1 | x_0, u_1) b(x_0)$$

$$b(x_1) = \eta P(z_1 | x_1) \bar{b}(x_1)$$

$$\begin{aligned} b(x_1 = open) &= \eta P(z_1 = sense - open | x_1 = open) \bar{b}(x_1 = open) \\ &= \eta 0.6 * 0.5 \\ &= \eta 0.3 \end{aligned}$$

$$\begin{aligned} b(x_1 = closed) &= \eta P(z_1 = sense - open | x_1 = closed) \bar{b}(x_1 = closed) \\ &= \eta 0.2 * 0.5 \\ &= \eta 0.1 \end{aligned}$$

$t = 1$

States: $x = \{open, closed\}$

Observations: $z = \{sense - open, sense - closed\}$

Actions: $u = \{push, do - nothing\}$

Sensor Model: $P(z_t = sense - open|x_t = open) = 0.6$

$P(z_t = sense - closed|x_t = open) = 0.4$

$P(z_t = sense - open|x_t = closed) = 0.2$

$P(z_t = sense - closed|x_t = closed) = 0.8$

Action Model: $P(x_t = open|x_{t-1} = open, u_t = push) = 1.0$

$P(x_t = closed|x_{t-1} = open, u_t = push) = 0.0$

$P(x_t = open|x_{t-1} = closed, u_t = push) = 0.8$

$P(x_t = closed|x_{t-1} = closed, u_t = push) = 0.2$

Bayes Filter

$$\bar{b}(x_1) = \sum_{x_0} P(x_1|x_0, u_1)b(x_0)$$

$$b(x_1) = \eta P(z_1|x_1)\bar{b}(x_1)$$

$$b(x_1 = open) + b(x_1 = closed) = 1.0$$

$$\eta * 0.3 + \eta * 0.1 = 1.0$$

$$\eta = 2.5$$

$t = 1$

Sensor Model: $P(z_t = sense-open|x_t = open) = 0.6$

$P(z_t = sense-closed|x_t = open) = 0.4$

$P(z_t = sense-open|x_t = closed) = 0.2$

$P(z_t = sense-closed|x_t = closed) = 0.8$

States: $x = \{open, closed\}$

Observations: $z = \{sense-open, sense-closed\}$

Actions: $u = \{push, do-nothing\}$

Action Model: $P(x_t = open|x_{t-1} = open, u_t = push) = 1.0$

$P(x_t = closed|x_{t-1} = open, u_t = push) = 0.0$

$P(x_t = open|x_{t-1} = closed, u_t = push) = 0.8$

$P(x_t = closed|x_{t-1} = closed, u_t = push) = 0.2$

Bayes Filter

$$\bar{b}(x_1) = \sum_{x_0} P(x_1|x_0, u_1)b(x_0)$$

$$b(x_1) = \eta P(z_1|x_1)\bar{b}(x_1)$$

$$\begin{aligned} b(x_1 = open) &= 2.5 * 0.3 \\ &= 0.75 \\ b(x_1 = closed) &= 2.5 * 0.1 \\ &= 0.25 \end{aligned}$$

$t = 2$

States: $x = \{open, closed\}$

Observations: $z = \{sense - open, sense - closed\}$

Actions: $u = \{push, do - nothing\}$

Sensor Model: $P(z_t = sense - open | x_t = open) = 0.6$

$P(z_t = sense - closed | x_t = open) = 0.4$

$P(z_t = sense - open | x_t = closed) = 0.2$

$P(z_t = sense - closed | x_t = closed) = 0.8$

Action Model: $P(x_t = open | x_{t-1} = open, u_t = push) = 1.0$

$P(x_t = closed | x_{t-1} = open, u_t = push) = 0.0$

$P(x_t = open | x_{t-1} = closed, u_t = push) = 0.8$

$P(x_t = closed | x_{t-1} = closed, u_t = push) = 0.2$

What if the robot pushes the door and sense-open at $t = 2$?

$t = 2$

States: $x = \{open, closed\}$

Observations: $z = \{sense - open, sense - closed\}$

Actions: $u = \{push, do - nothing\}$

Sensor Model: $P(z_t = sense - open | x_t = open) = 0.6$

$P(z_t = sense - closed | x_t = open) = 0.4$

$P(z_t = sense - open | x_t = closed) = 0.2$

$P(z_t = sense - closed | x_t = closed) = 0.8$

Action Model: $P(x_t = open | x_{t-1} = open, u_t = push) = 1.0$

$P(x_t = closed | x_{t-1} = open, u_t = push) = 0.0$

$P(x_t = open | x_{t-1} = closed, u_t = push) = 0.8$

$P(x_t = closed | x_{t-1} = closed, u_t = push) = 0.2$

What if the robot pushes the door and sense-open at $t = 2$?

$$\bar{b}(x_2 = open) = P(x_2 = open | x_1 = open, u_2 = push) b(x_1 = open) + P(x_2 = open | x_1 = closed, u_2 = push) b(x_1 = closed)$$

$$\bar{b}(x_2 = open) = 1.0 * 0.75 + 0.8 * 0.25 = 0.95$$

$t = 2$

States: $x = \{open, closed\}$

Observations: $z = \{sense - open, sense - closed\}$

Actions: $u = \{push, do - nothing\}$

Sensor Model: $P(z_t = sense - open | x_t = open) = 0.6$

$P(z_t = sense - closed | x_t = open) = 0.4$

$P(z_t = sense - open | x_t = closed) = 0.2$

$P(z_t = sense - closed | x_t = closed) = 0.8$

Action Model: $P(x_t = open | x_{t-1} = open, u_t = push) = 1.0$

$P(x_t = closed | x_{t-1} = open, u_t = push) = 0.0$

$P(x_t = open | x_{t-1} = closed, u_t = push) = 0.8$

$P(x_t = closed | x_{t-1} = closed, u_t = push) = 0.2$

What if the robot pushes the door and sense-open at $t = 2$?

$$\bar{b}(x_2 = open) = P(x_2 = open | x_1 = open, u_2 = push) b(x_1 = open) + P(x_2 = open | x_1 = closed, u_2 = push) b(x_1 = closed)$$

$$\bar{b}(x_2 = open) = 1.0 * 0.75 + 0.8 * 0.25 = 0.95$$

$$\bar{b}(x_2 = closed) = P(x_2 = closed | x_1 = open, u_2 = push) b(x_1 = open) + P(x_2 = closed | x_1 = closed, u_2 = push) b(x_1 = closed)$$

$$\bar{b}(x_2 = closed) = 0.0 * 0.75 + 0.2 * 0.25 = 0.05$$

$t = 2$

States: $x = \{open, closed\}$

Observations: $z = \{sense - open, sense - closed\}$

Actions: $u = \{push, do - nothing\}$

Sensor Model: $P(z_t = sense - open | x_t = open) = 0.6$
$P(z_t = sense - closed | x_t = open) = 0.4$
$P(z_t = sense - open | x_t = closed) = 0.2$
$P(z_t = sense - closed | x_t = closed) = 0.8$

Action Model: $P(x_t = open | x_{t-1} = open, u_t = push) = 1.0$
$P(x_t = closed | x_{t-1} = open, u_t = push) = 0.0$
$P(x_t = open | x_{t-1} = closed, u_t = push) = 0.8$
$P(x_t = closed | x_{t-1} = closed, u_t = push) = 0.2$

What if the robot pushes the door and sense-open at $t = 2$?

$$
\begin{aligned}
b(x_2 = open) &= \eta P(z_2 = sense - open | x_2 = open)\bar{b}(x_2 = open) \\
&= \eta 0.6 * 0.95 \\
&= \eta 0.57
\end{aligned}
$$

$$
\begin{aligned}
b(x_2 = closed) &= \eta P(z_2 = sense - open | x_2 = closed)\bar{b}(x_2 = closed) \\
&= \eta 0.2 * 0.05 \\
&= \eta 0.01
\end{aligned}
$$

$t = 2$

States: $x = \{open, closed\}$

Observations: $z = \{sense - open, sense - closed\}$

Actions: $u = \{push, do - nothing\}$

Sensor Model: $P(z_t = sense - open | x_t = open) = 0.6$
$P(z_t = sense - closed | x_t = open) = 0.4$
$P(z_t = sense - open | x_t = closed) = 0.2$
$P(z_t = sense - closed | x_t = closed) = 0.8$

Action Model: $P(x_t = open | x_{t-1} = open, u_t = push) = 1.0$
$P(x_t = closed | x_{t-1} = open, u_t = push) = 0.0$
$P(x_t = open | x_{t-1} = closed, u_t = push) = 0.8$
$P(x_t = closed | x_{t-1} = closed, u_t = push) = 0.2$

What if the robot pushes the door and sense-open at $t = 2$?

$$b(x_2 = open) + b(x_2 = closed) = 1.0$$
$$\eta * 0.57 + \eta * 0.01 = 1.0$$
$$\eta = 1.72$$

$t = 2$

States: $x = \{open, closed\}$

Observations: $z = \{sense-open, sense-closed\}$

Actions: $u = \{push, do-nothing\}$

Sensor Model: $P(z_t = sense-open|x_t = open) = 0.6$

$P(z_t = sense-closed|x_t = open) = 0.4$

$P(z_t = sense-open|x_t = closed) = 0.2$

$P(z_t = sense-closed|x_t = closed) = 0.8$

Action Model: $P(x_t = open|x_{t-1} = open, u_t = push) = 1.0$

$P(x_t = closed|x_{t-1} = open, u_t = push) = 0.0$

$P(x_t = open|x_{t-1} = closed, u_t = push) = 0.8$

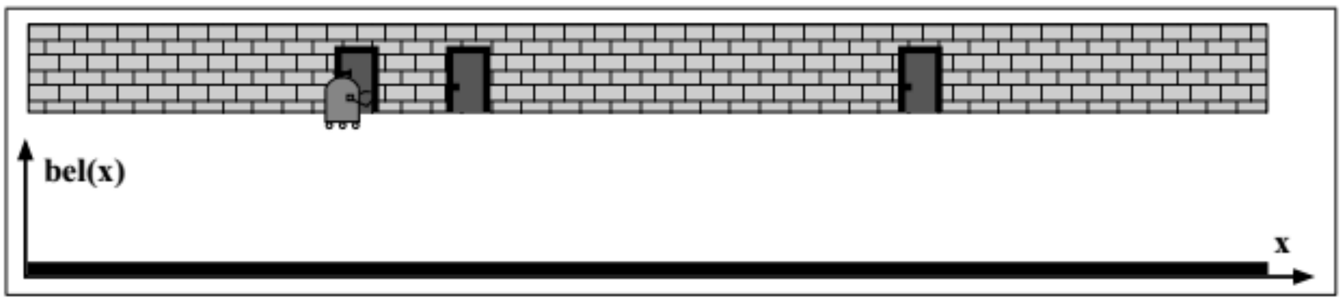
$P(x_t = closed|x_{t-1} = closed, u_t = push) = 0.2$

What if the robot pushes the door and sense-open at $t = 2$?

$$\begin{aligned} b(x_2 = open) &= 0.57 * 1.72 \\ &= 0.98 \\ b(x_2 = closed) &= 0.01 * 1.72 \\ &= 0.02 \end{aligned}$$

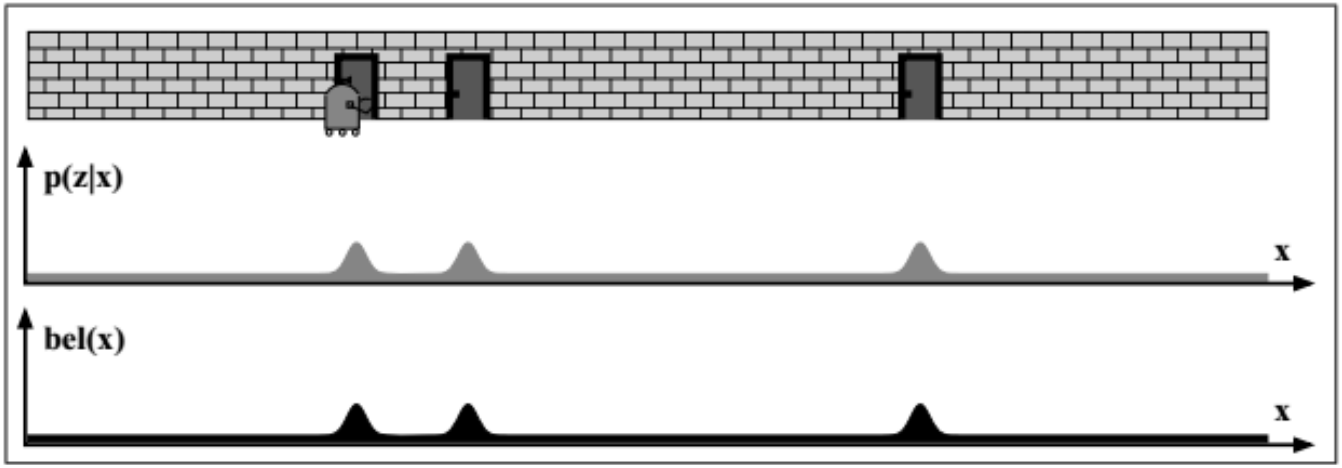
Localization Example

(a)



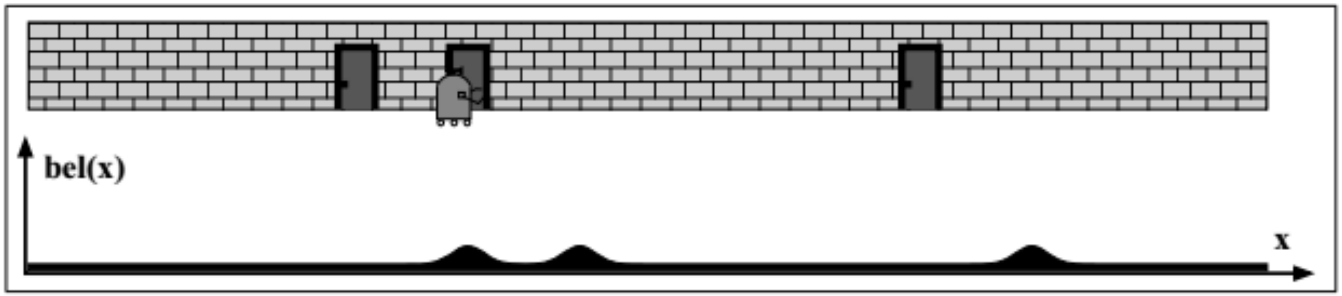
Localization Example

(b)



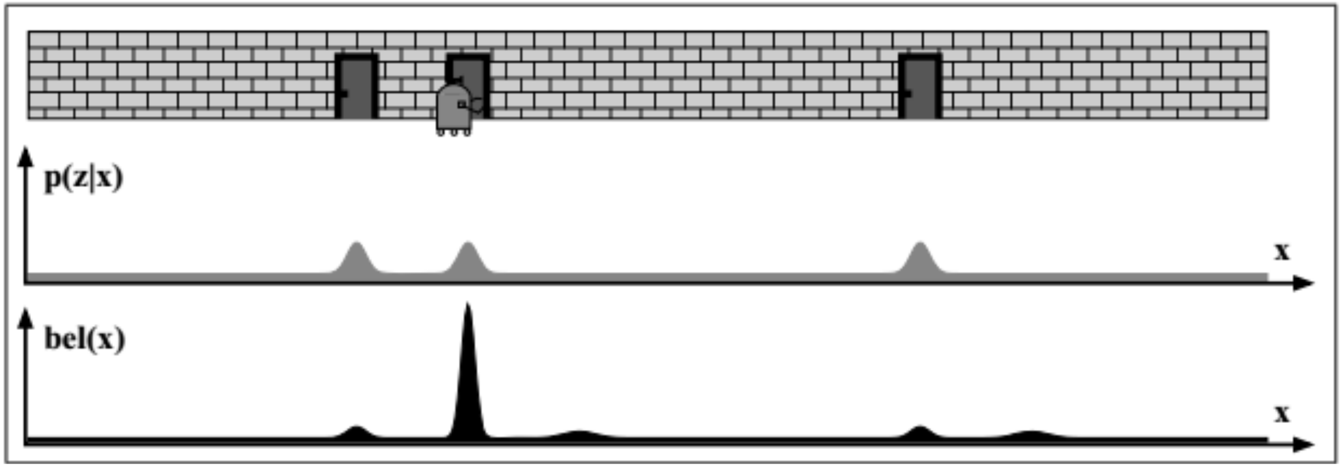
Localization Example

(c)



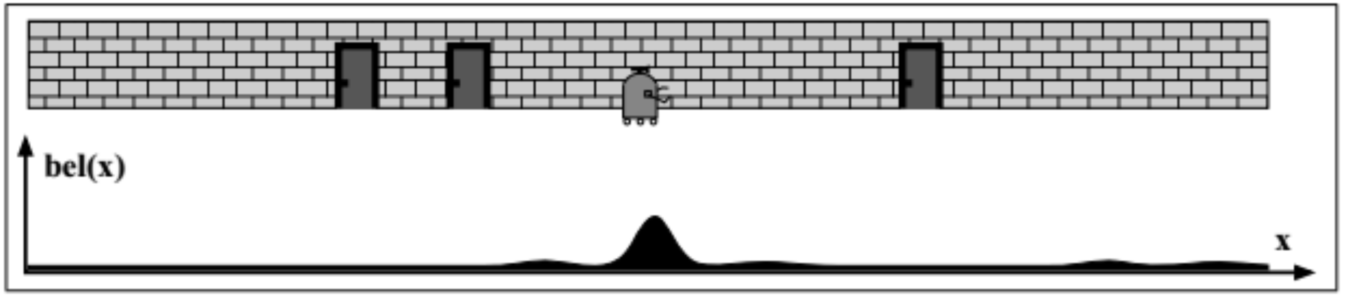
Localization Example

(d)



Localization Example

(e)



Question

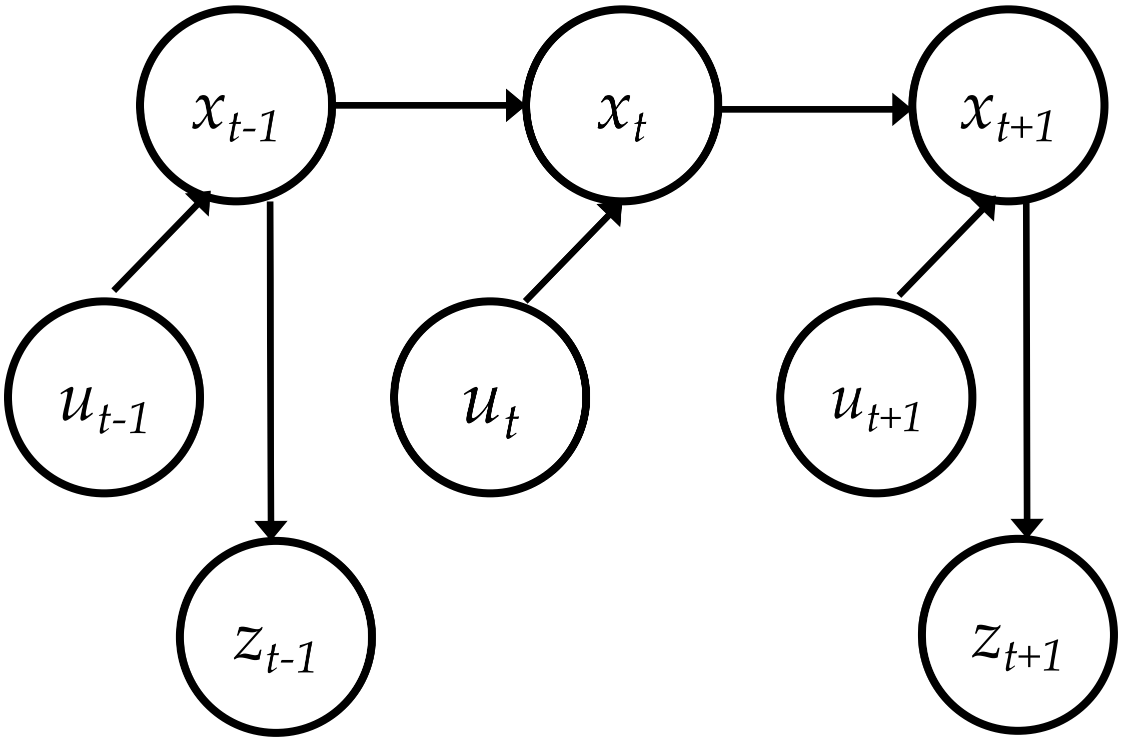
- Do actions always increase uncertainty?

Question

- What if you receive an observation every 2 timesteps?

Question

- What if you miss an observation?



Question

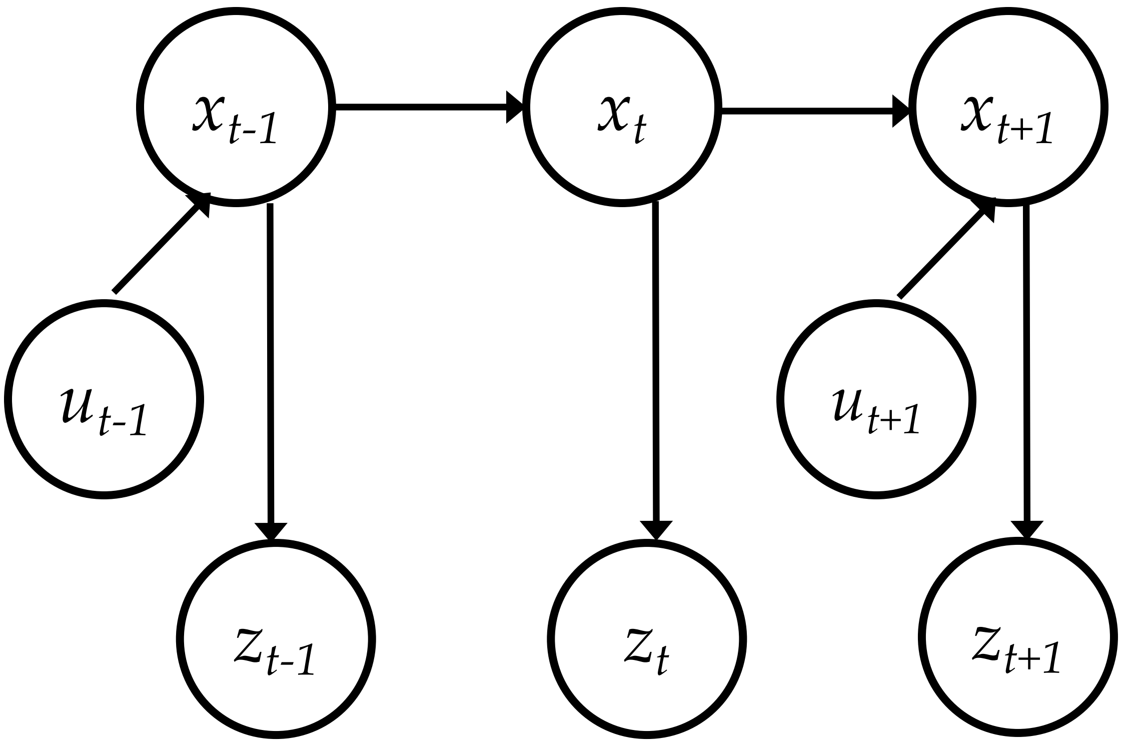
- What if you miss an observation?

$$P(x_t|z_{1:t}, u_{1:t}) = \eta \sum_{x_{t-1}} P(x_t|x_{t-1}, u_t) P(x_{t-1}|z_{1:t-1}, u_{1:t-1})$$

$$b(x_t) = \eta \sum_{x_{t-1}} P(x_t|x_{t-1}, u_t) b(x_{t-1})$$

Question

- What if you do not receive an input (assume state changes only if there is an input)?



Question

- What if you do not receive an input (assume state changes only if there is an input)?

$$P(x_t|z_{1:t}, u_{1:t}) = \eta P(z_t|x_t) \sum_{x_{t-1}} P(x_t|x_{t-1}, u_t) P(x_{t-1}|z_{1:t-1}, u_{1:t-1})$$

$$b(x_t) = \eta P(z_t|x_t) \sum_{x_{t-1}} P(x_t|x_{t-1}, u_t) b(x_{t-1})$$

Question

- What if you do not receive an input (assume state changes only if there is an input)?

$$P(x_t|z_{1:t}, u_{1:t}) = \eta P(z_t|x_t) \sum_{x_{t-1}} P(x_t|x_{t-1}, u_t) P(x_{t-1}|z_{1:t-1}, u_{1:t-1})$$

$$b(x_t) = \eta P(z_t|x_t) \sum_{x_{t-1}} \mathbf{1}_{\{x_t\}}(x_{t-1}) b(x_{t-1})$$

Question

- What if $b(x_t)$ becomes 0 for some x_t ?