

Kalman Filters

CSCI 545 Introduction to Robotics
Instructor: Stefanos Nikolaidis

State Estimation

Air-Ground Localization and Map Augmentation Using Monocular Dense Reconstruction

Christian Forster, Matia Pizzoli, Davide Scaramuzza

ai lab
Robotics and Perception Group

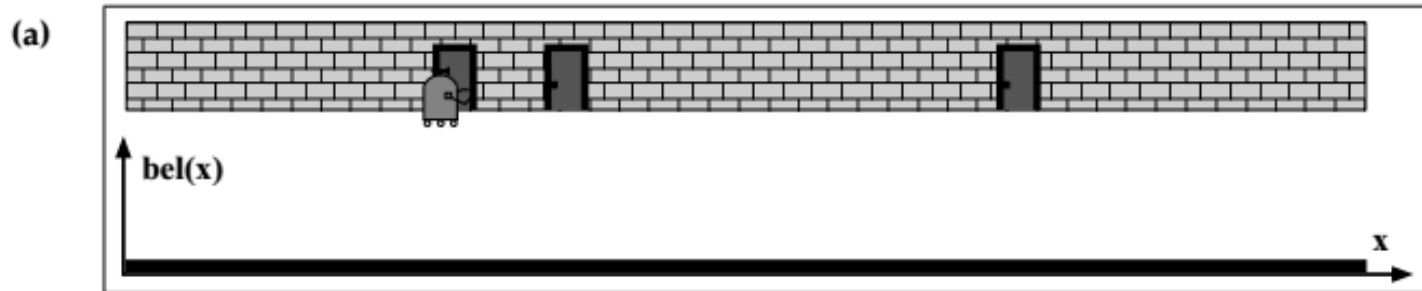


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Zurich** UZH

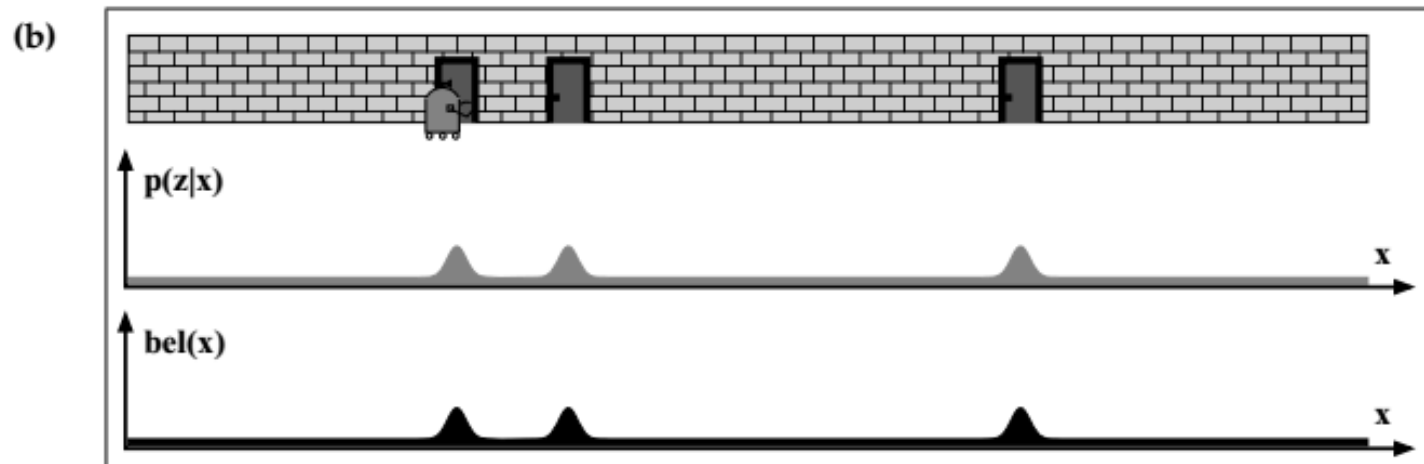
Department of Informatics

robotics  Swiss National
Centre of Competence
in Research

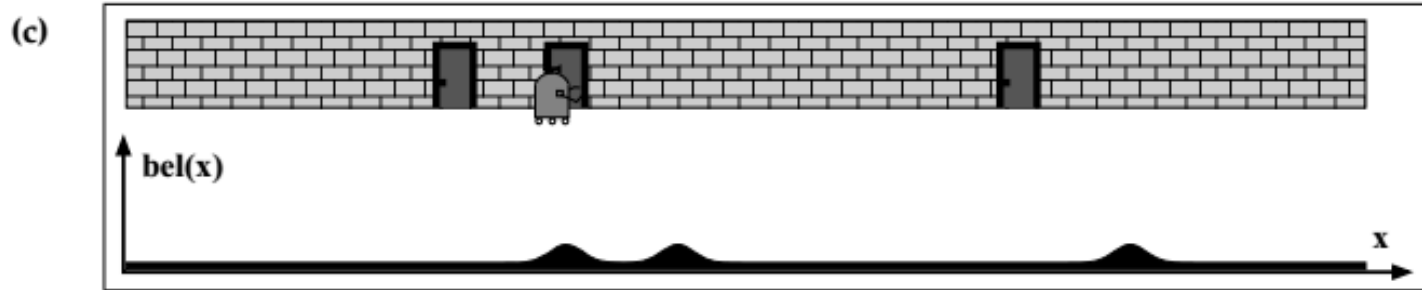
Localization Example



Localization Example

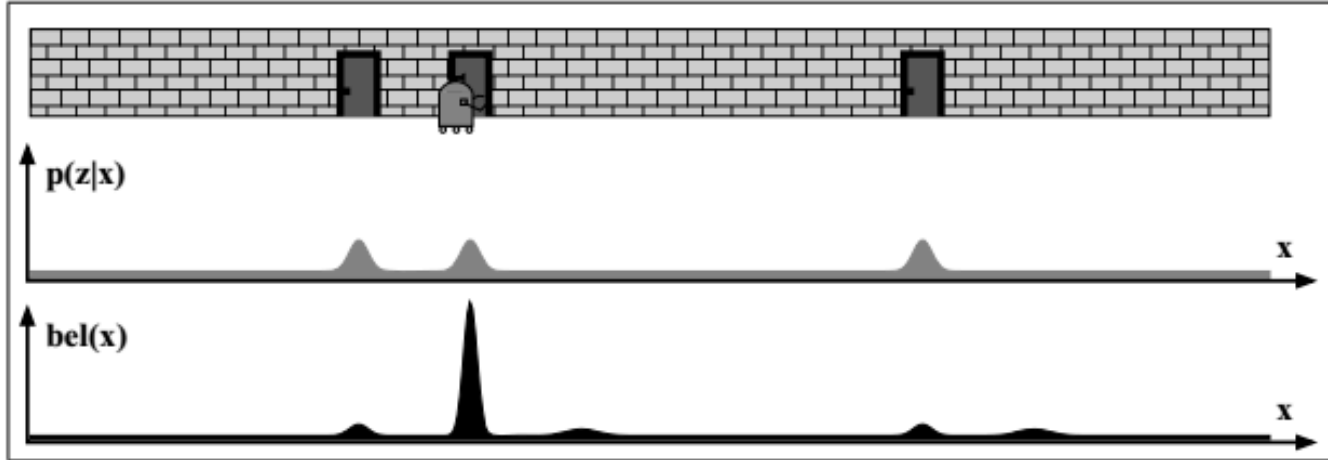


Localization Example

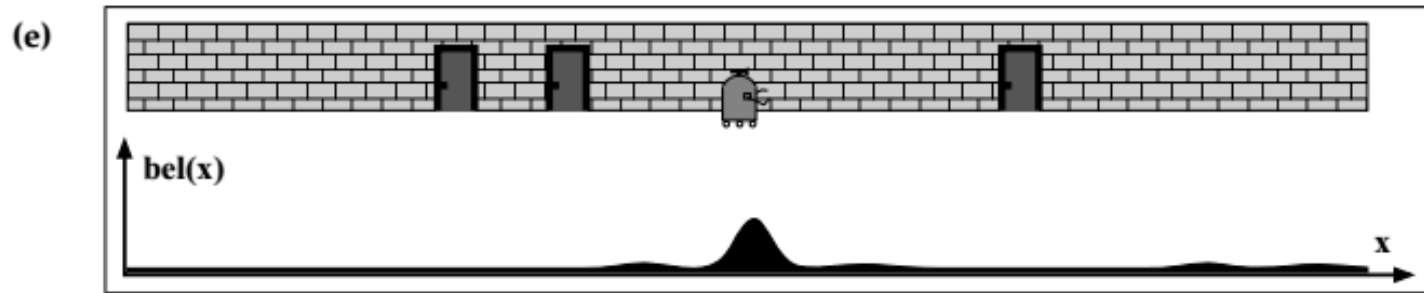


Localization Example

(d)



Localization Example



What about continuous states?

- We will assume that the dynamics model is linear.

$$x_t = Ax_{t-1} + Bu_t + \epsilon$$

$$x_t = \begin{bmatrix} x_{1,t} \\ x_{2,t} \\ \vdots \\ x_{n,t} \end{bmatrix}$$

$$u_t = \begin{bmatrix} u_{1,t} \\ u_{2,t} \\ \vdots \\ u_{m,t} \end{bmatrix}$$

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A is an $n \times n$ matrix, B is $n \times m$

$$\epsilon \sim N(0, R)$$

What about continuous states?

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$$p(x_t | u_t, x_{t-1}) =$$

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$$p(x_t|u_t, x_{t-1}) = \det(2\pi R)^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(x_t - Ax_{t-1} - Bu_t)^T R^{-1}(x_t - Ax_{t-1} - Bu_t)\right\}$$

Sensor Model

$$z_t = Cx_t + \delta$$

$$z_t = \begin{bmatrix} z_{1,t} \\ z_{2,t} \\ \vdots \\ z_{k,t} \end{bmatrix}$$

C is an $k \times n$ matrix

$$\delta \sim N(0, Q)$$

Sensor Model

$$z_t = Cx_t + \delta$$

$$z_t = \begin{bmatrix} z_{1,t} \\ z_{2,t} \\ \vdots \\ z_{k,t} \end{bmatrix}$$

$$p(z_t|x_t) = \det(2\pi Q)^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(z_t - Cx_t)^T Q^{-1}(z_t - Cx_t)\right\}$$

How is information propagated?

- Initial belief of x_0 :

$$b(x_0) = \det(2\pi\Sigma_0)^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(x_0 - \mu_0)^T \Sigma_0^{-1}(x_0 - \mu_0)\right\}$$

- Propagation of mean

$$\begin{aligned}\mu_{t+1} &= E[Ax_t + Bu_{t+1} + \epsilon] \\ &= E[Ax_t] + E[Bu_{t+1}] + E[\epsilon] \\ &= AE[x_t] + Bu_{t+1} + E[\epsilon] \\ &= A\mu_t + Bu_{t+1}\end{aligned}$$

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- Propagation of Covariance

$$\Sigma_t = E[(x_t - \mu_t)(x_t - \mu_t)^T]$$

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$$\Sigma_t = E[(x_t - \mu_t)(x_t - \mu_t)^T]$$

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Propagation of covariance

$$\begin{aligned}x_t - \mu_t &= Ax_{t-1} + Bu_t + \epsilon - \mu_t \\ &= Ax_{t-1} + Bu_t + \epsilon - A\mu_{t-1} - Bu_t \\ &= A(x_{t-1} - \mu_{t-1}) + \epsilon\end{aligned}$$

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$$\begin{aligned}\Sigma_t &= E[(x_t - \mu_t)(x_t - \mu_t)^T] \\&= E[(A(x_{t-1} - \mu_{t-1}) + \epsilon)(A(x_{t-1} - \mu_{t-1}) + \epsilon)^T] \\&= E[(A(x_{t-1} - \mu_{t-1}) + \epsilon)((x_{t-1} - \mu_{t-1})^T A^T + \epsilon^T)] \\&= E[A(x_{t-1} - \mu_{t-1})(x_{t-1} - \mu_{t-1})^T A^T \\&\quad + A(x_{t-1} - \mu_{t-1})\epsilon^T + \epsilon(x_{t-1} - \mu_{t-1})^T A^T + \epsilon\epsilon^T] \\&= E[A(x_{t-1} - \mu_{t-1})(x_{t-1} - \mu_{t-1})^T A^T] \\&\quad + E[A(x_{t-1} - \mu_{t-1})\epsilon^T] + E[\epsilon(x_{t-1} - \mu_{t-1})^T A^T] + E[\epsilon\epsilon^T]\end{aligned}$$

Propagation of covariance

$$\begin{aligned} E[A(x_{t-1} - \mu_{t-1})(x_{t-1} - \mu_{t-1})^T A^T] &= AE[(x_{t-1} - \mu_{t-1})(x_{t-1} - \mu_{t-1})^T]A^T \\ &= A\Sigma_{t-1}A^T \end{aligned}$$

$$E[A(x_{t-1} - \mu_{t-1})\epsilon^T] = 0$$

$$E[\epsilon(x_{t-1} - \mu_{t-1})^T A^T] = 0$$

$$E[\epsilon\epsilon^T] = R$$

Propagation of covariance

$$\begin{aligned}\Sigma_t &= E[(x_t - \mu_t)(x_t - \mu_t)^T] \\ &= A\Sigma_{t-1}A^T + R\end{aligned}$$

Propagation of mean and covariance

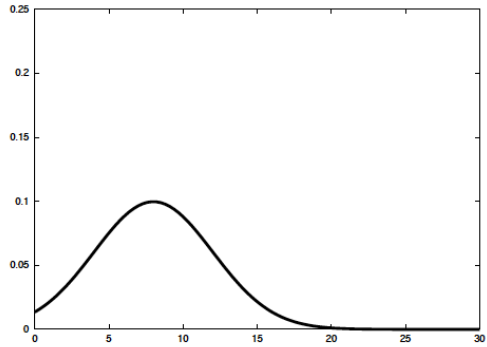
$$\mu_t = A\mu_{t-1} + Bu_t$$

$$\Sigma_t = A\Sigma_{t-1}A^T + R$$

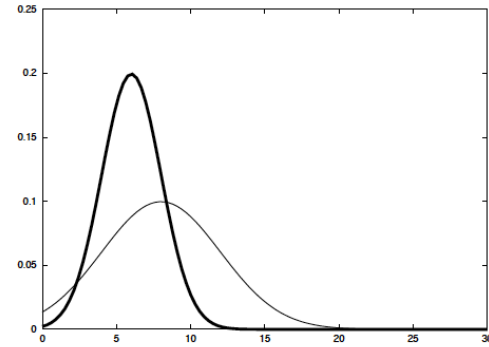
Kalman Filter

```
1:   Algorithm Kalman_filter( $\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$ ):  
2:      $\bar{\mu}_t = A_t \mu_{t-1} + B_t u_t$   
3:      $\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$   
4:      $K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$   
5:      $\mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$   
6:      $\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$   
7:     return  $\mu_t, \Sigma_t$ 
```

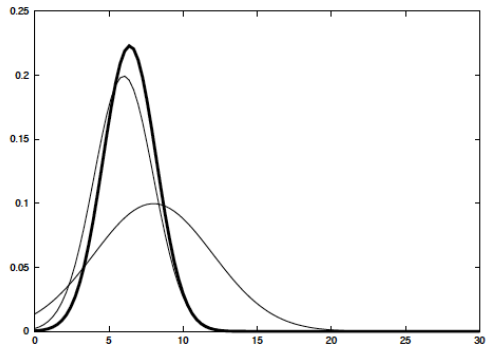
Kalman Filter



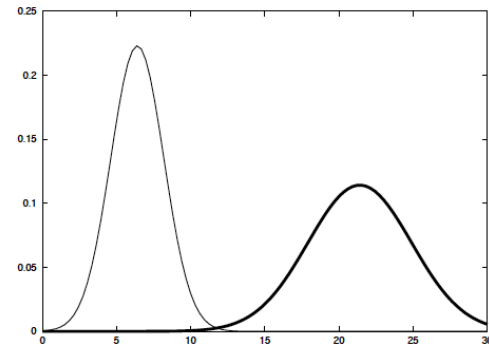
(a)



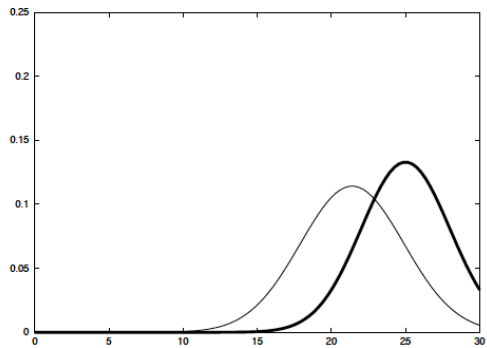
(b)



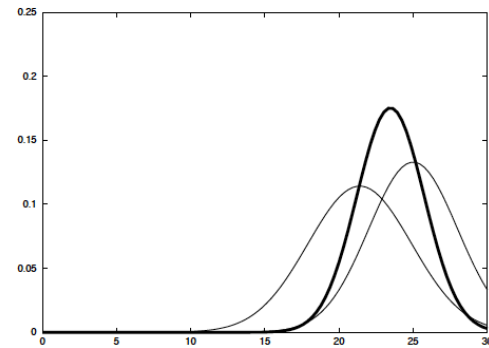
(c)



(d)



(e)



(f)

Kalman Filter

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```

K_t = Kalman Gain

Q_t = Measurement Noise (Covariance)

If $Q_t \rightarrow \infty$, how does K_t change?

Kalman Filter

```
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```

K_t = Kalman Gain

Q_t = Measurement Noise (Covariance)

What about $Q_t \rightarrow 0$?

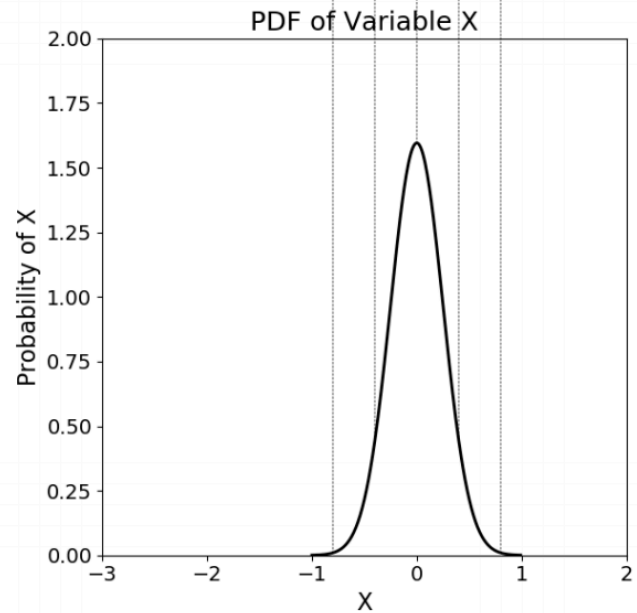
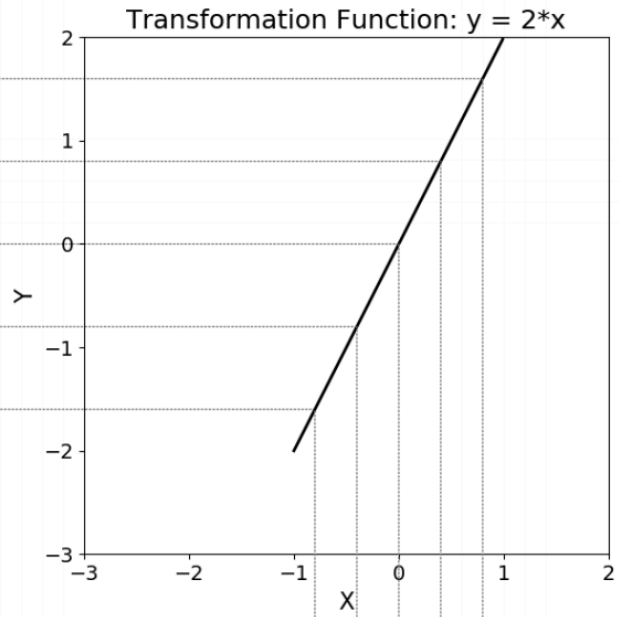
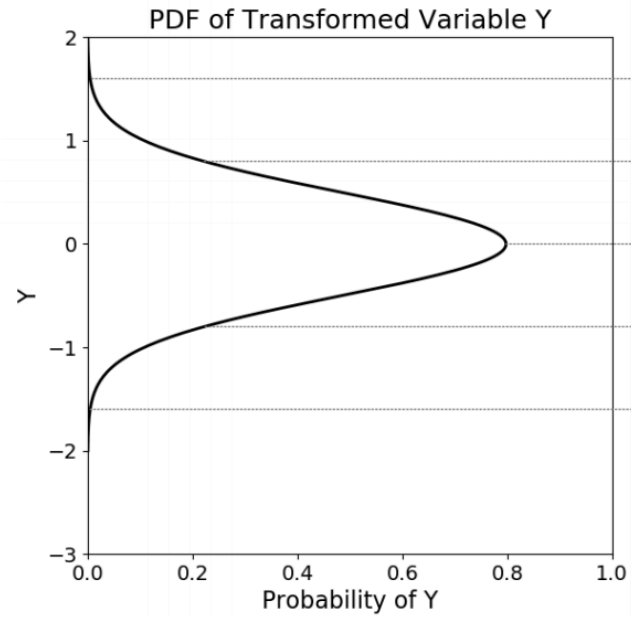
Non-Linear Systems

The linear system assumptions are rarely met in practice!

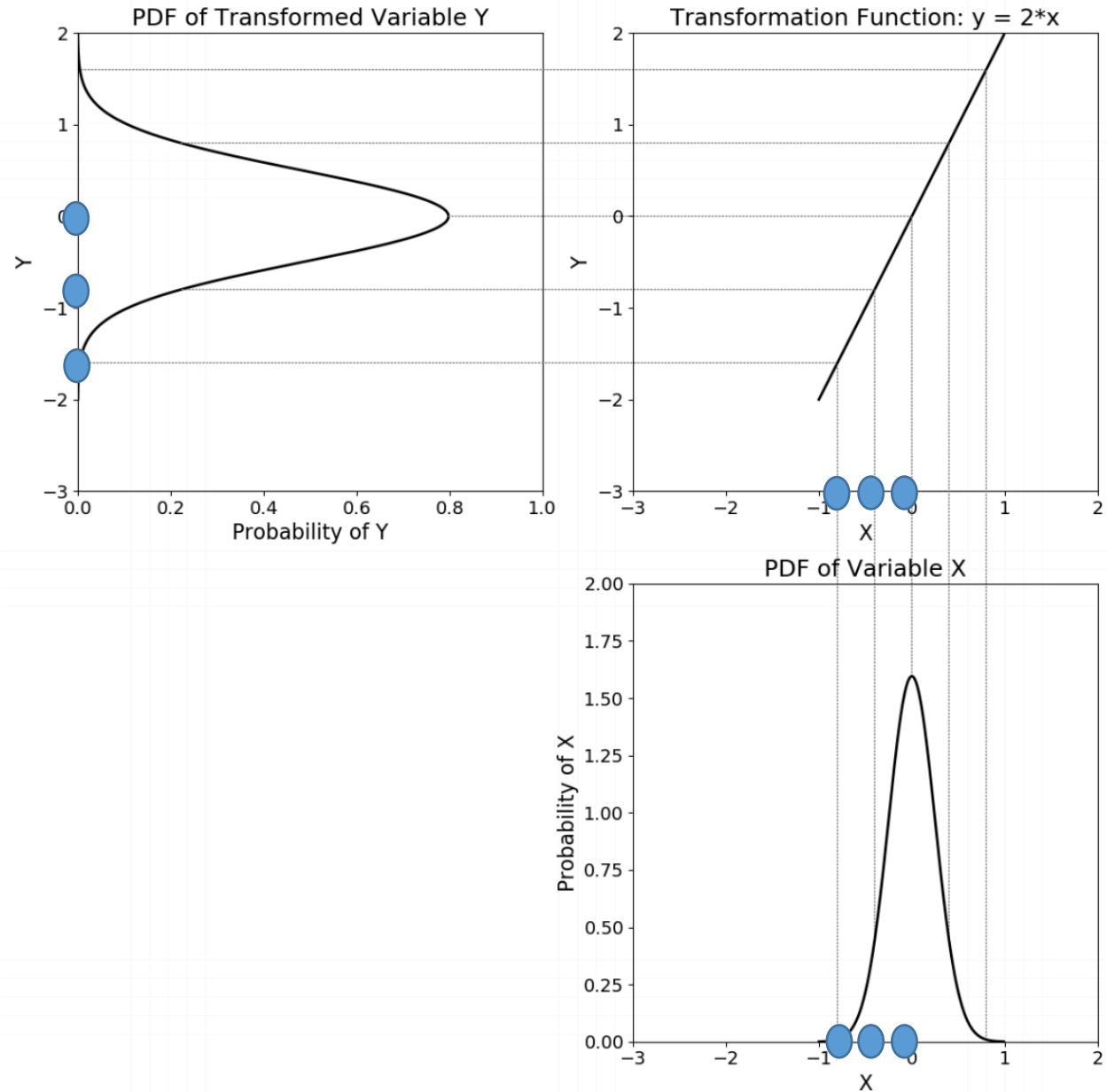
$$x_t = g(u_t, x_{t-1}) + \epsilon$$

$$z_t = h(x_t) + \delta$$

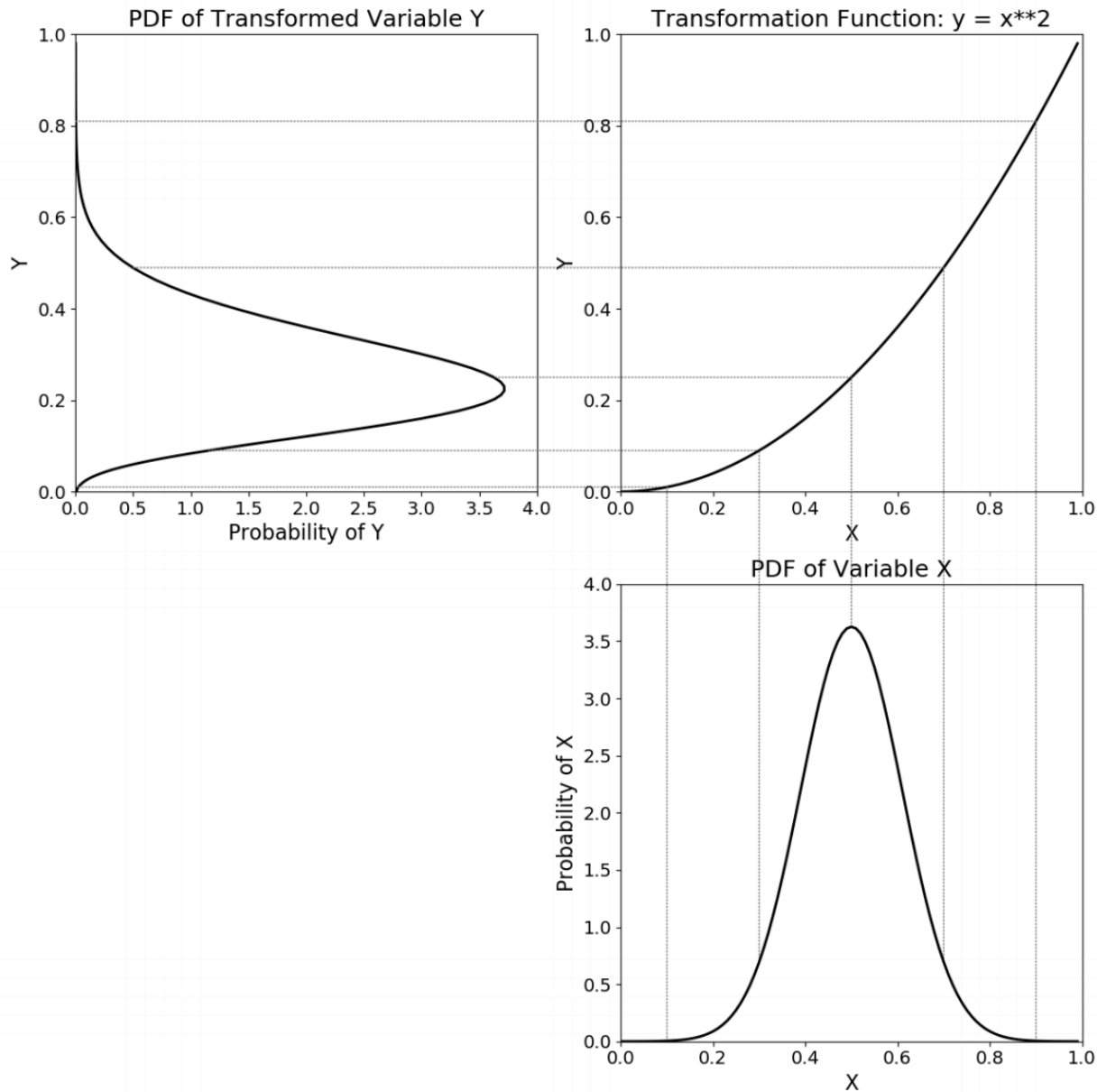
Linearity Assumption



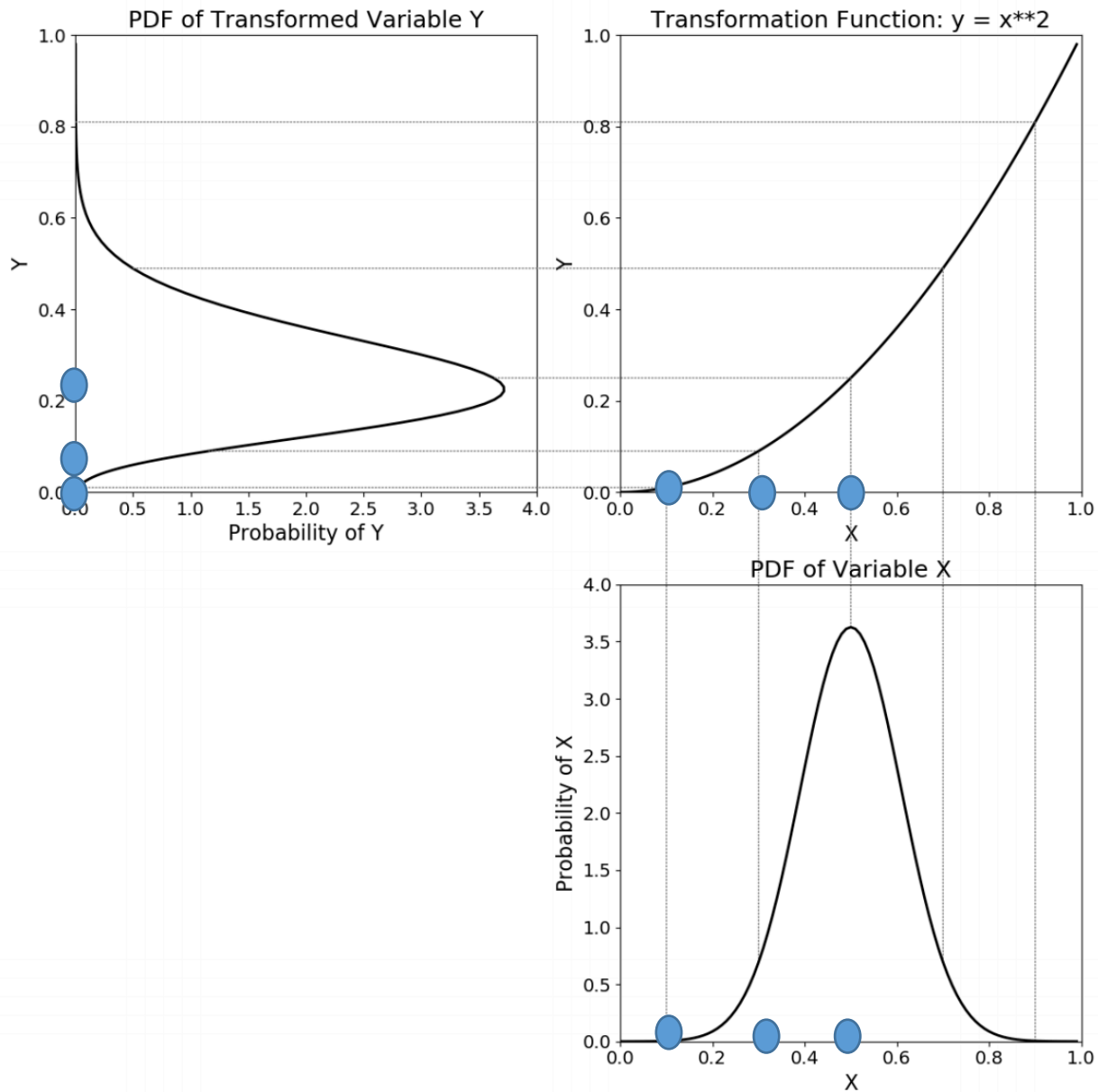
Linearity Assumption



Linearity Assumption



Linearity Assumption

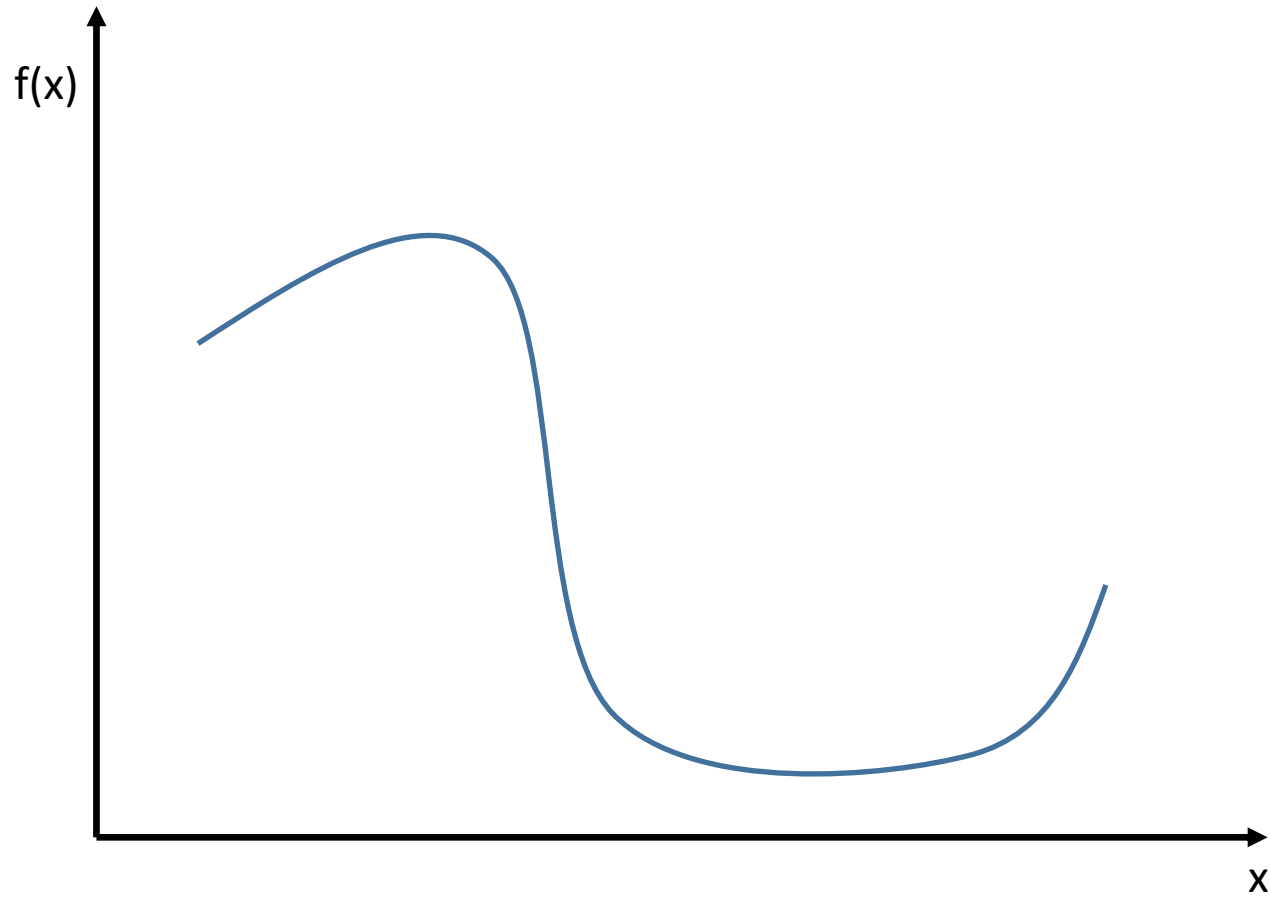


Linearization

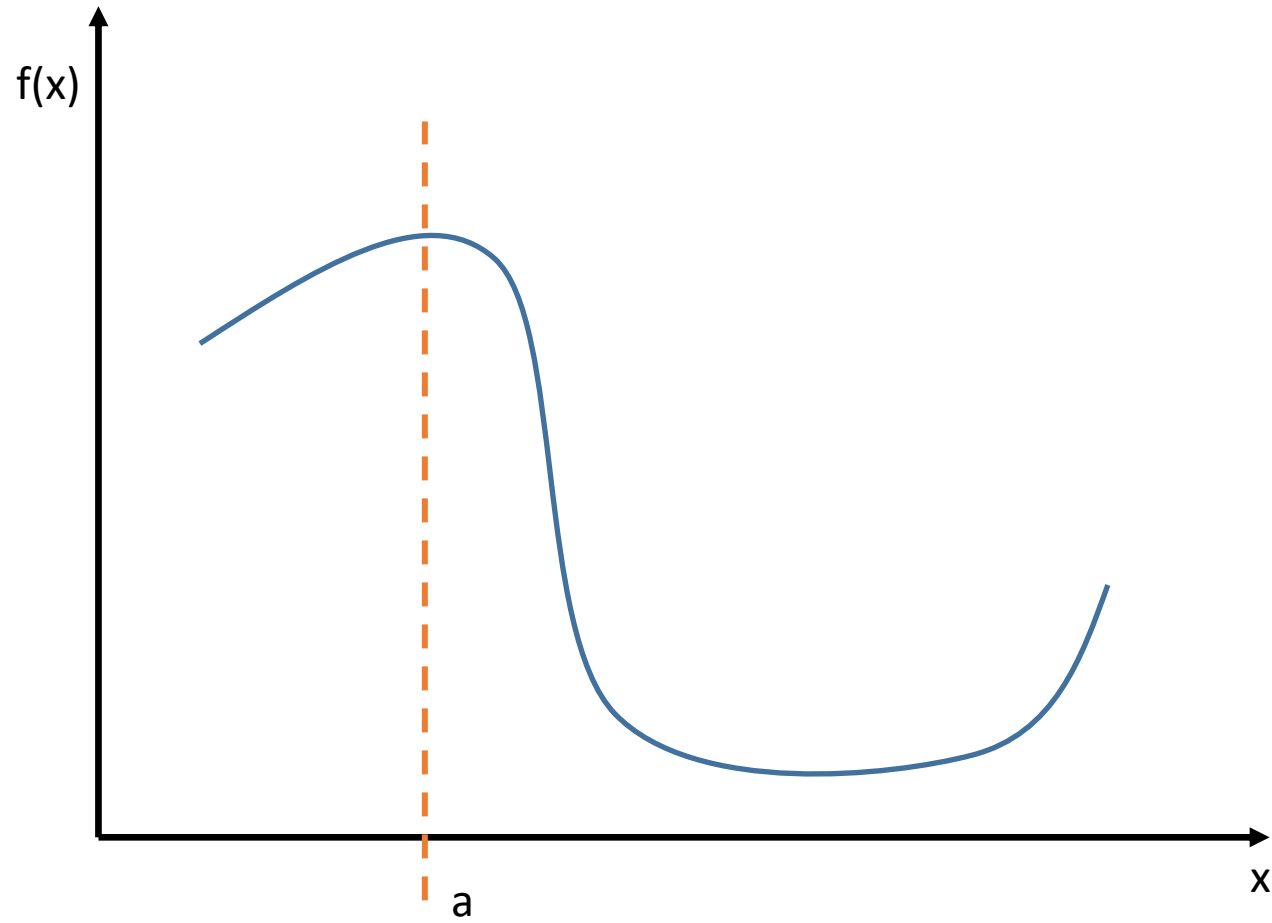
We construct a linear approximation using function g 's value and slope.

$$x_t = g(u_t, x_{t-1}) + \epsilon$$

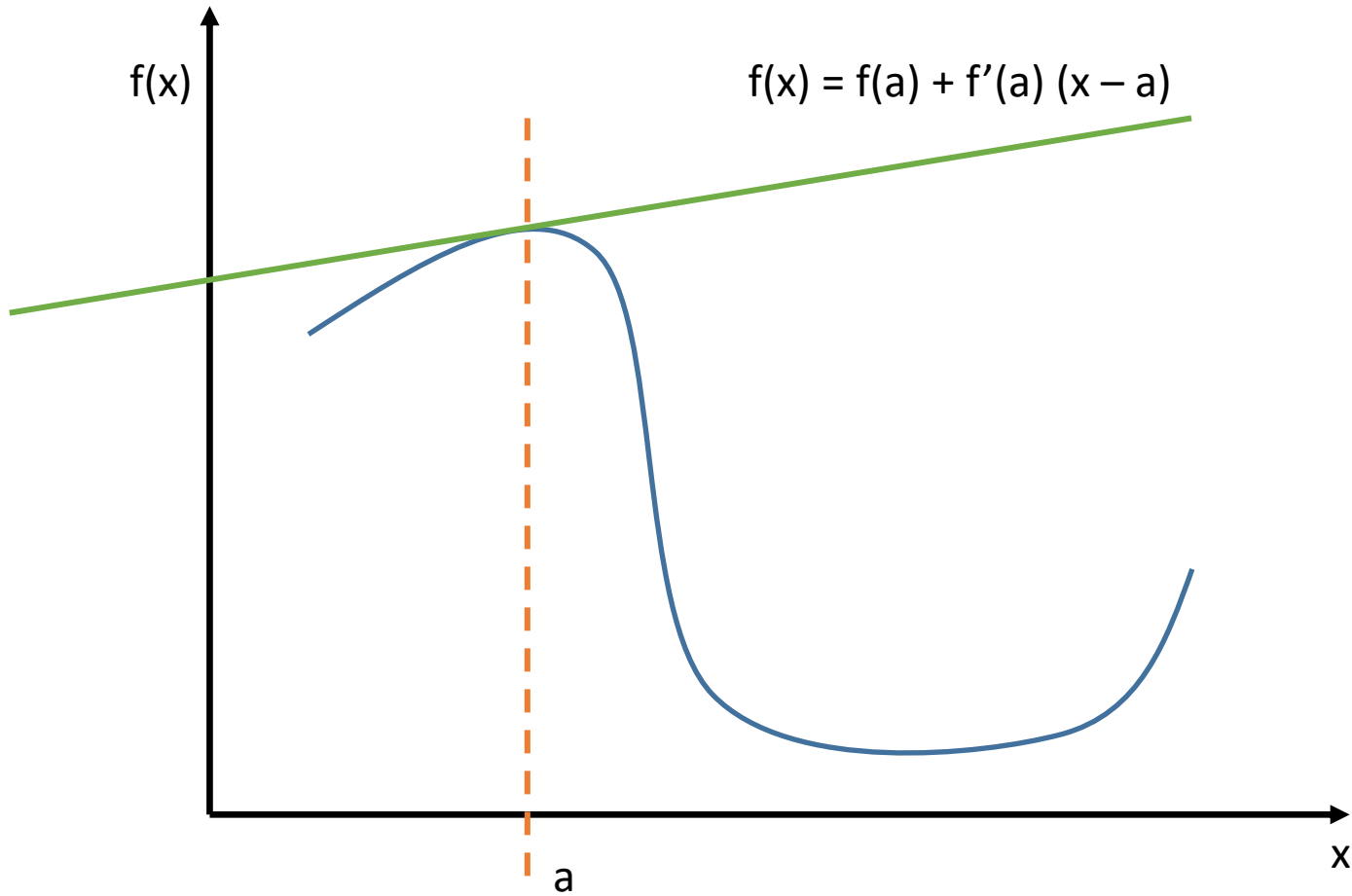
First-order Taylor Expansion



First-order Taylor Expansion



First-order Taylor Expansion



Linearization

We construct a linear approximation using function g 's value and slope.

$$x_t = g(u_t, x_{t-1}) + \epsilon$$

$$\text{Slope: } g'(u_t, x_{t-1}) = \frac{\partial g(u_t, x_{t-1})}{\partial x_{t-1}}$$

Linearization

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But what about the value?

Linearization

We construct a linear approximation using function g 's value and slope.

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$$\text{Slope: } g'(u_t, x_{t-1}) = \frac{\partial g(u_t, x_{t-1})}{\partial x_{t-1}}$$

But what about the value?

$$\begin{aligned} g(u_t, x_{t-1}) &\approx g(u_t, \mu_{t-1}) + g'(u_t, \mu_{t-1}) * (x_{t-1} - \mu_{t-1}) \\ &\approx g(u_t, \mu_{t-1}) + G_t(u_t, \mu_{t-1}) * (x_{t-1} - \mu_{t-1}) \end{aligned}$$

Linearization

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$$p(x_t | u_t, x_{t-1}) =$$

Linearization

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$$\begin{aligned} g(u_t, x_{t-1}) &\approx g(u_t, \mu_{t-1}) + g'(u_t, \mu_{t-1}) * (x_{t-1} - \mu_{t-1}) \\ &\approx g(u_t, \mu_{t-1}) + G_t(u_t, \mu_{t-1}) * (x_{t-1} - \mu_{t-1}) \end{aligned}$$

$$p(x_t | u_t, x_{t-1}) = \det(2\pi R)^{-\frac{1}{2}} \exp\left\{-\frac{1}{2} (x_t - g(u_t, \mu_{t-1}) - G_t(u_t, \mu_{t-1})(x_{t-1} - \mu_{t-1}))^T R^{-1} (x_t - g(u_t, \mu_{t-1}) - G_t(u_t, \mu_{t-1})(x_{t-1} - \mu_{t-1}))\right\}$$

Measurements

$$z_t = h(x_t) + \delta$$

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We construct a linear approximation using function h 's value and slope.

$$h(x_t) \approx h(\bar{\mu}_t) + H(\mu_t)(x_t - \bar{\mu}_t)$$

Kalman Filter

1: **Algorithm Kalman_filter**($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$):

2: $\bar{\mu}_t = A_t \mu_{t-1} + B_t u_t$

3: $\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$

4: $K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$

5: $\mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$

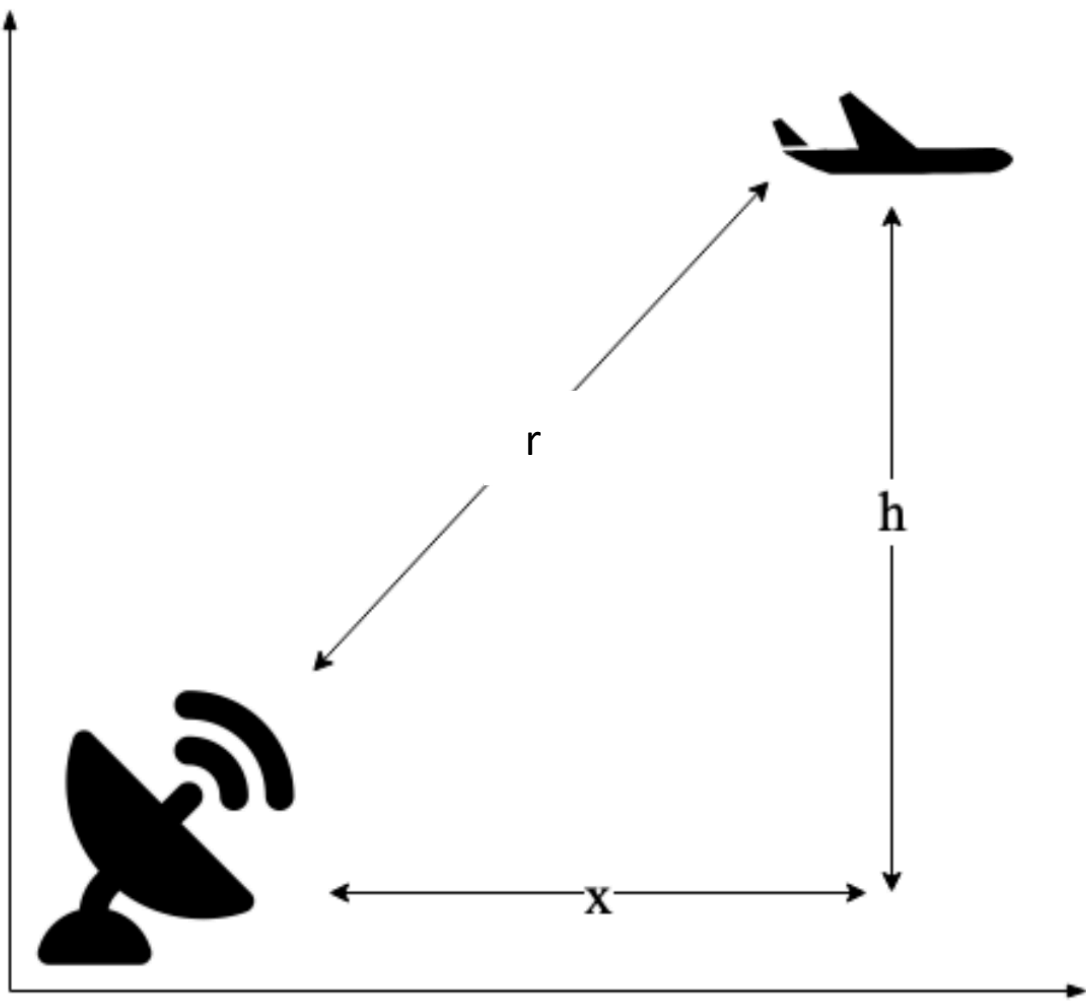
6: $\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$

7: return μ_t, Σ_t

Extended Kalman Filter

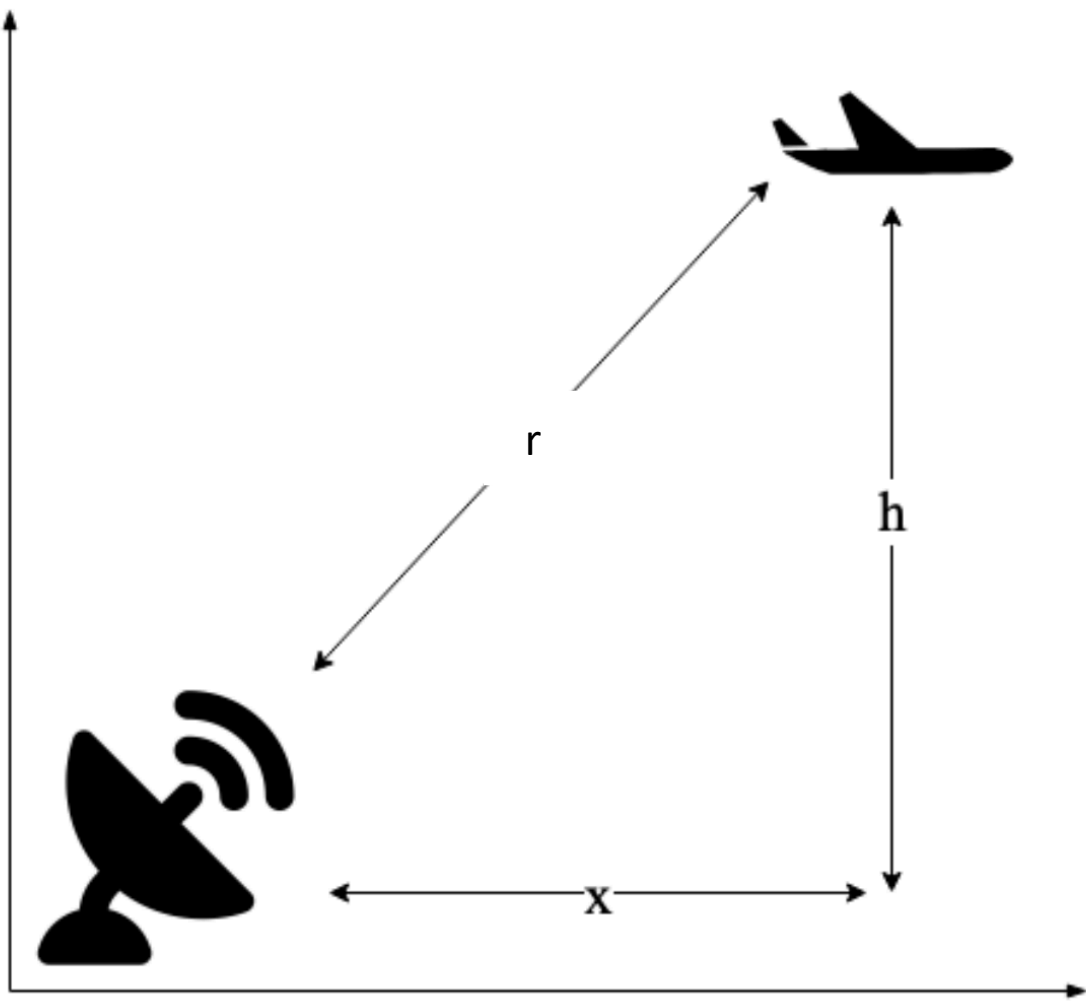
```
1:   Algorithm Extended_Kalman_filter( $\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$ ):  
2:      $\bar{\mu}_t = g(u_t, \mu_{t-1})$   
3:      $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$   
4:      $K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1}$   
5:      $\mu_t = \bar{\mu}_t + K_t(z_t - h(\bar{\mu}_t))$   
6:      $\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$   
7:     return  $\mu_t, \Sigma_t$ 
```

Example



$$\dot{x} = v$$

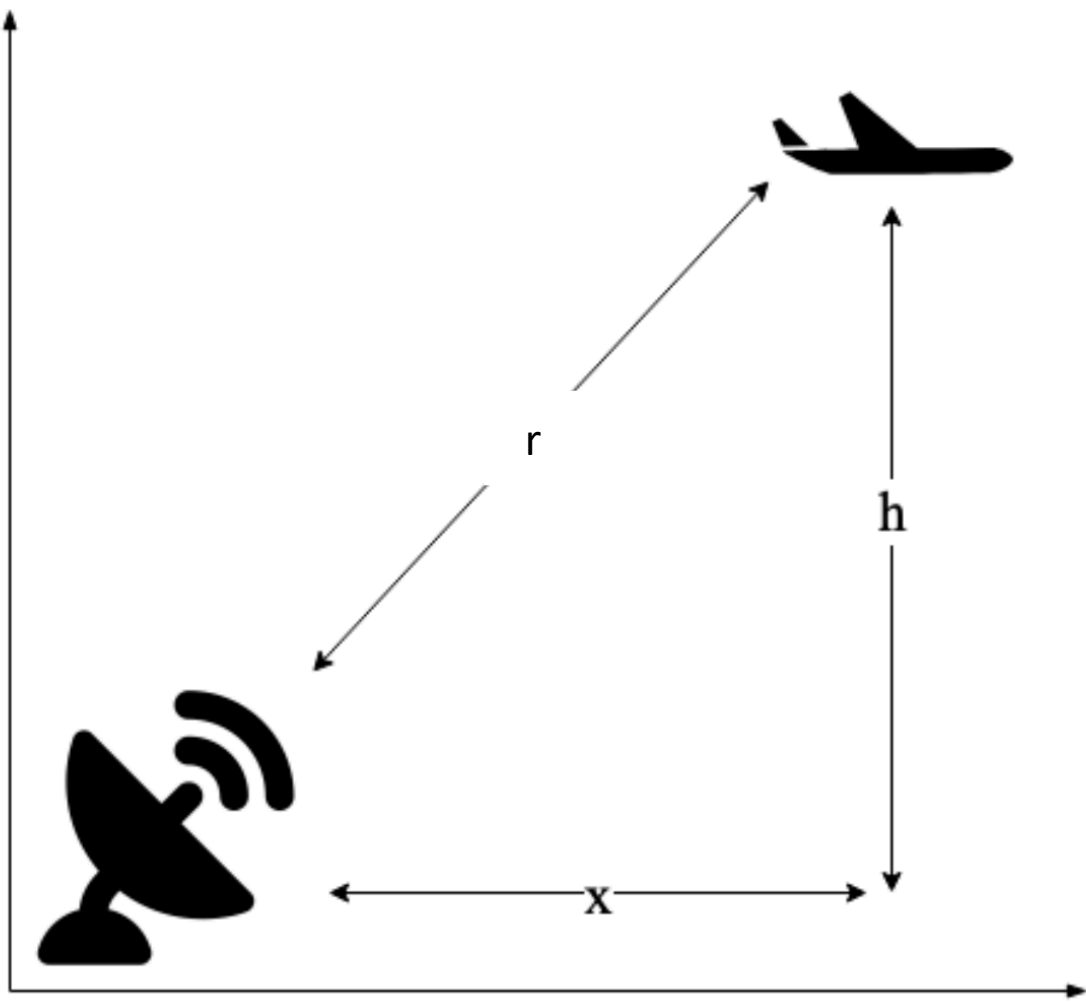
Example



$$\dot{x} = v$$

$$\dot{v} = 0$$

Example

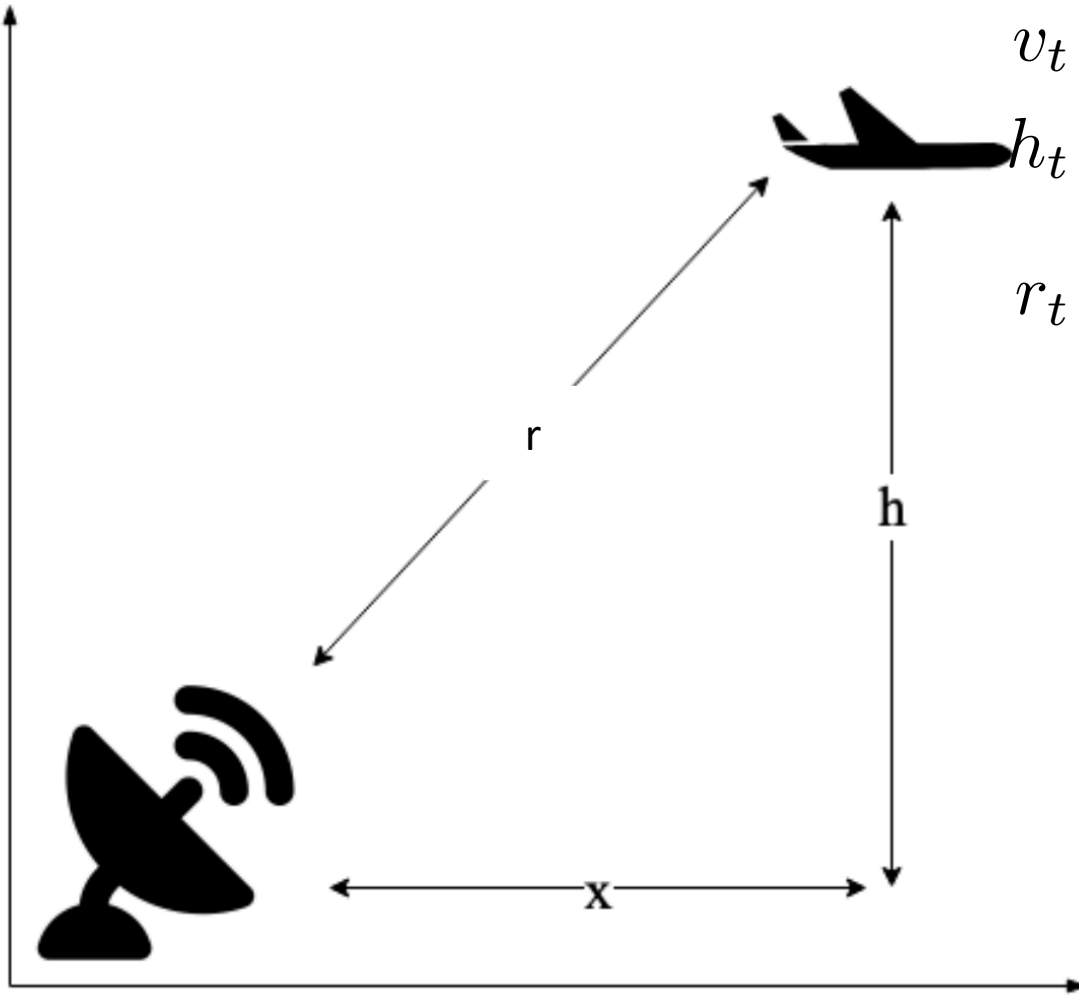


$$\dot{x} = v$$

$$\dot{v} = 0$$

$$\dot{h} = 0$$

Example



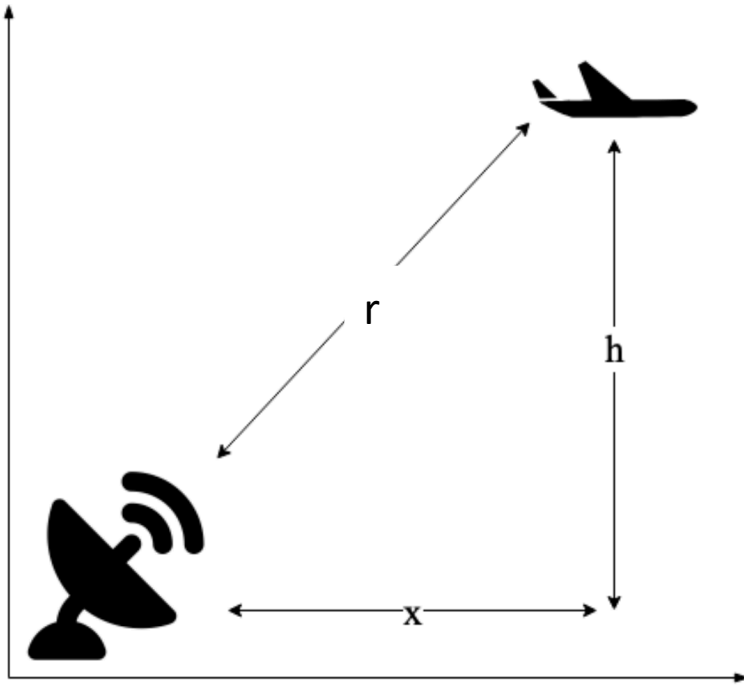
$$x_t = x_{t-1} + v_{t-1} * \Delta_t$$

$$v_t = v_{t-1}$$

$$h_t = h_{t-1}$$

$$r_t = \sqrt{x_t^2 + h_t^2}$$

Example



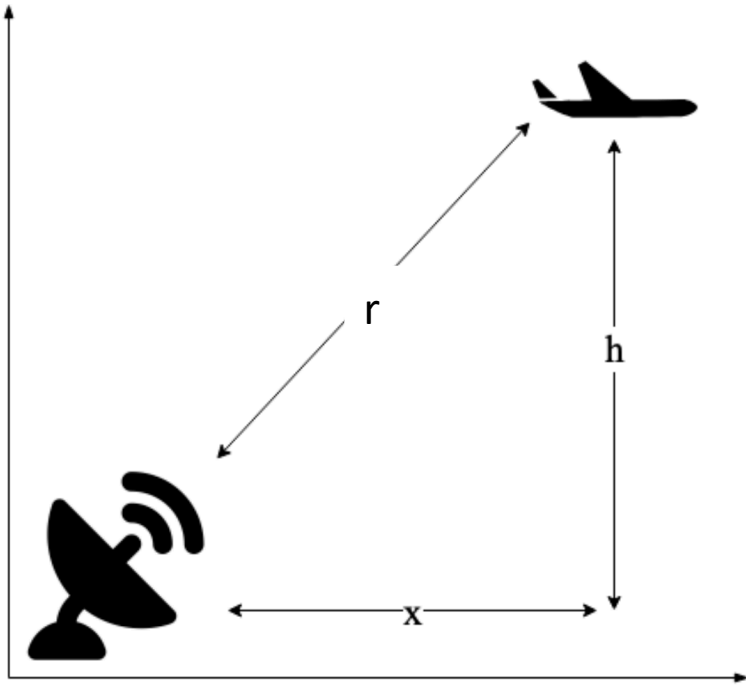
$$x_t = x_{t-1} + v_{t-1} * \Delta_t$$

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Example



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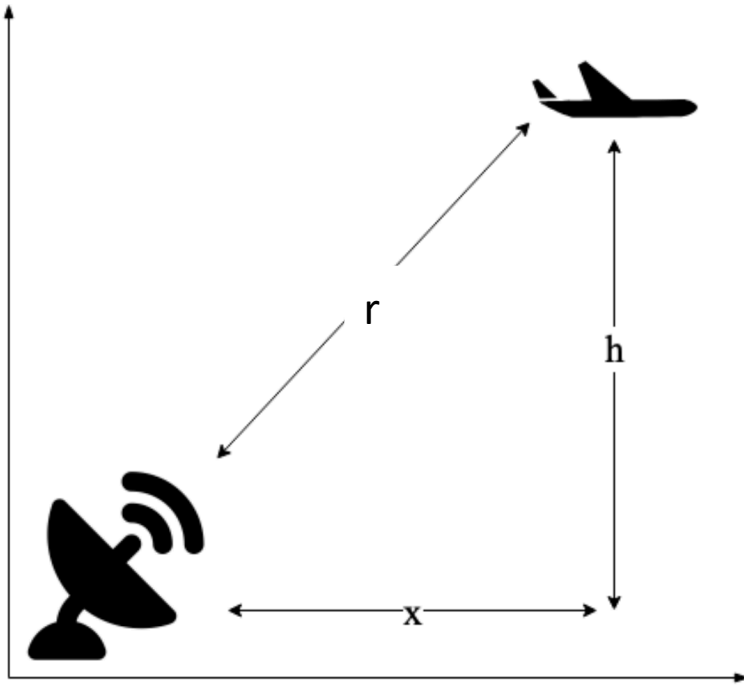
$$r_t = \sqrt{x_t^2 + h_t^2}$$

$$x_{1,t} = x_t$$

$$x_{2,t} = v_t$$

$$x_{3,t} = h_t$$

Example



$$x_t = x_{t-1} + v_{t-1} * \Delta_t$$

$$v_t = v_{t-1}$$

$$h_t = h_{t-1}$$

$$r_t = \sqrt{x_t^2 + h_t^2}$$

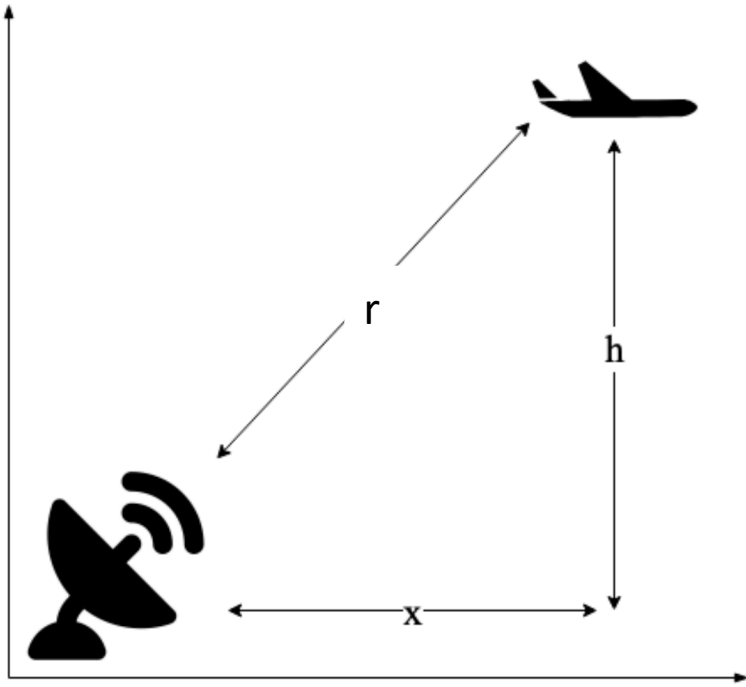
$$x_{1,t} = x_t$$

$$x_{2,t} = v_t$$

$$x_{3,t} = h_t$$

$$\begin{bmatrix} x_{1,t} \\ x_{2,t} \\ x_{3,t} \end{bmatrix} = \begin{bmatrix} \phantom{x_{1,t-1}} \\ \phantom{x_{2,t-1}} \\ \phantom{x_{3,t-1}} \end{bmatrix} \cdot \begin{bmatrix} x_{1,t-1} \\ x_{2,t-1} \\ x_{3,t-1} \end{bmatrix}$$

Example



$$x_t = x_{t-1} + v_{t-1} * \Delta_t$$

$$v_t = v_{t-1}$$

$$h_t = h_{t-1}$$

$$r_t = \sqrt{x_t^2 + h_t^2}$$

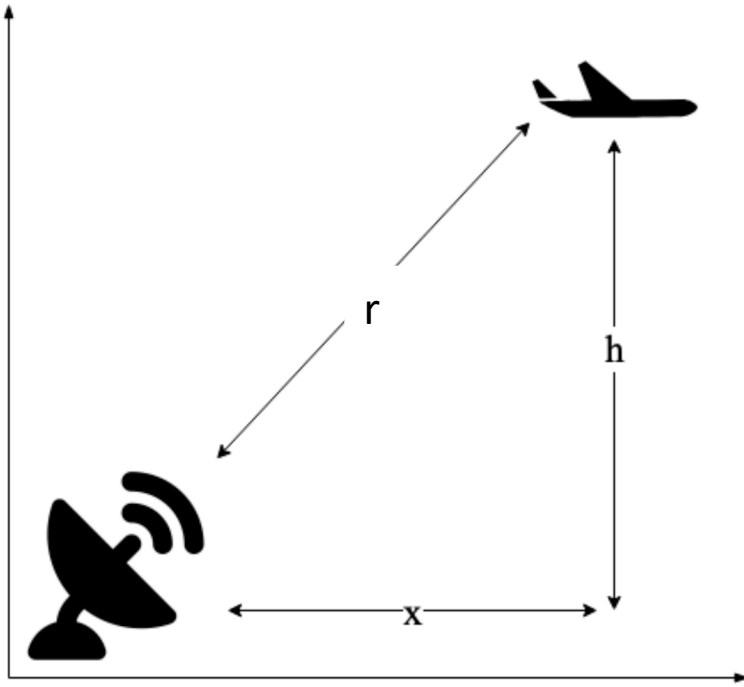
$$x_{1,t} = x_t$$

$$x_{2,t} = v_t$$

$$x_{3,t} = h_t$$

$$\begin{bmatrix} x_{1,t} \\ x_{2,t} \\ x_{3,t} \end{bmatrix} = \begin{bmatrix} 1 & \Delta_t & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_{1,t-1} \\ x_{2,t-1} \\ x_{3,t-1} \end{bmatrix}$$

Example



$$x_t = x_{t-1} + v_{t-1} * \Delta_t$$

$$v_t = v_{t-1}$$

$$h_t = h_{t-1}$$

$$r_t = \sqrt{x_t^2 + h_t^2}$$

$$x_{1,t} = x_t$$

$$x_{2,t} = v_t$$

$$x_{3,t} = h_t$$

$$\begin{bmatrix} x_{1,t} \\ x_{2,t} \\ x_{3,t} \end{bmatrix} = \begin{bmatrix} 1 & \Delta_t & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_{1,t-1} \\ x_{2,t-1} \\ x_{3,t-1} \end{bmatrix}$$

Are the dynamics linear?

Example

$$g(x_{t-1}, u_t) \approx g(u_t, \mu_{t-1}) + G_t(u_t, \mu_{t-1}) * (x_{t-1} - \mu_{t-1})$$

Example

$$\begin{aligned} g(x_{t-1}, u_t) &\approx g(u_t, \mu_{t-1}) + G_t(u_t, \mu_{t-1}) * (x_{t-1} - \mu_{t-1}) \\ &\approx \begin{bmatrix} \mu_{1,t-1} + \Delta_t \mu_{2,t-1} \\ \mu_{2,t-1} \\ \mu_{3,t-1} \end{bmatrix} + \end{aligned}$$

Example

$$\begin{aligned} g(x_{t-1}, u_t) &\approx g(u_t, \mu_{t-1}) + G_t(u_t, \mu_{t-1}) * (x_{t-1} - \mu_{t-1}) \\ &\approx \begin{bmatrix} \mu_{1,t-1} + \Delta_t \mu_{2,t-1} \\ \mu_{2,t-1} \\ \mu_{3,t-1} \end{bmatrix} + \begin{bmatrix} 1 & \Delta_t & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{1,t-1} - \mu_{1,t-1} \\ x_{2,t-1} - \mu_{2,t-1} \\ x_{3,t-1} - \mu_{3,t-1} \end{bmatrix} \end{aligned}$$

Example

$$\begin{aligned} g(x_{t-1}, u_t) &\approx g(u_t, \mu_{t-1}) + G_t(u_t, \mu_{t-1}) * (x_{t-1} - \mu_{t-1}) \\ &\approx \begin{bmatrix} \mu_{1,t-1} + \Delta_t \mu_{2,t-1} \\ \mu_{2,t-1} \\ \mu_{3,t-1} \end{bmatrix} + \begin{bmatrix} 1 & \Delta_t & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{1,t-1} - \mu_{1,t-1} \\ x_{2,t-1} - \mu_{2,t-1} \\ x_{3,t-1} - \mu_{3,t-1} \end{bmatrix} \\ &\approx \begin{bmatrix} 1 & \Delta_t & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mu_{1,t-1} \\ \mu_{2,t-1} \\ \mu_{3,t-1} \end{bmatrix} + \begin{bmatrix} 1 & \Delta_t & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{1,t-1} - \mu_{1,t-1} \\ x_{2,t-1} - \mu_{2,t-1} \\ x_{3,t-1} - \mu_{3,t-1} \end{bmatrix} \end{aligned}$$

Extended Kalman Filter

```
1:   Algorithm Extended_Kalman_filter( $\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$ ):  
2:      $\bar{\mu}_t = g(u_t, \mu_{t-1})$   
3:      $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$   
4:      $K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1}$   
5:      $\mu_t = \bar{\mu}_t + K_t(z_t - h(\bar{\mu}_t))$   
6:      $\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$   
7:     return  $\mu_t, \Sigma_t$ 
```

Kalman Filter

1: **Algorithm Kalman_filter**($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$):

2: $\bar{\mu}_t = A_t \mu_{t-1} + B_t u_t$

3: $\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$

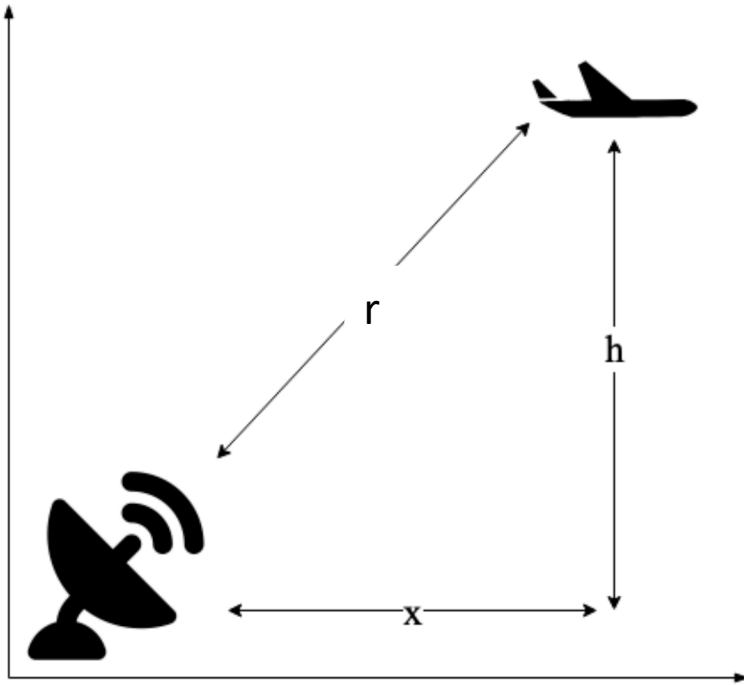
4: $K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$

5: $\mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$

6: $\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$

7: return μ_t, Σ_t

Example



$$x_t = x_{t-1} + v_{t-1} * \Delta_t$$

$$v_t = v_{t-1}$$

$$h_t = h_{t-1}$$

$$r_t = \sqrt{x_t^2 + h_t^2}$$

$$x_{1,t} = x_t$$

$$x_{2,t} = v_t$$

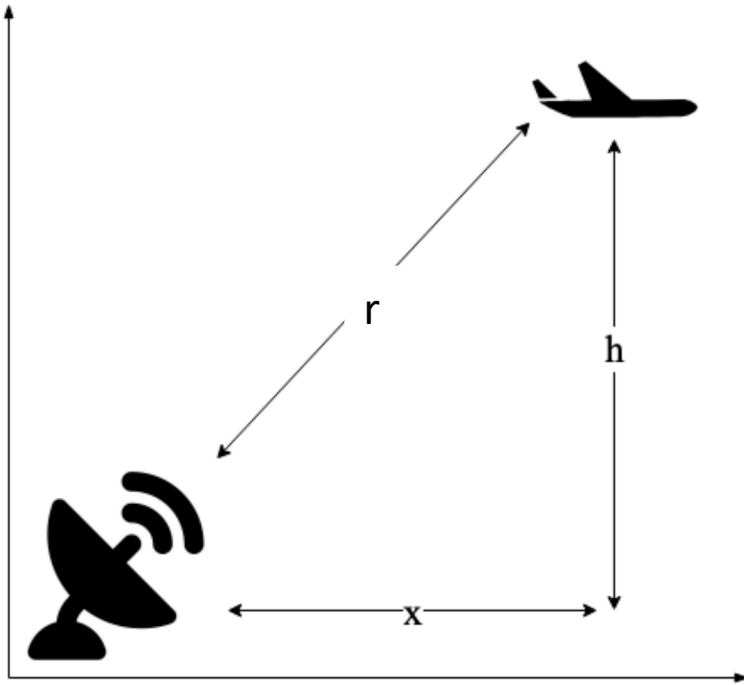
$$x_{3,t} = h_t$$

$$z_t = h(x_t)$$

$$= \sqrt{x_{1,t}^2 + x_{3,t}^2} + \delta$$

Are the measurements linear?

Example



$$x_{t+1} = x_t + v_t * \Delta_t$$

$$v_{t+1} = v_t$$

$$h_{t+1} = h_t$$

$$r_t = \sqrt{x_t^2 + h_t^2}$$

$$x_{1,t} = x_t$$

$$x_{2,t} = v_t$$

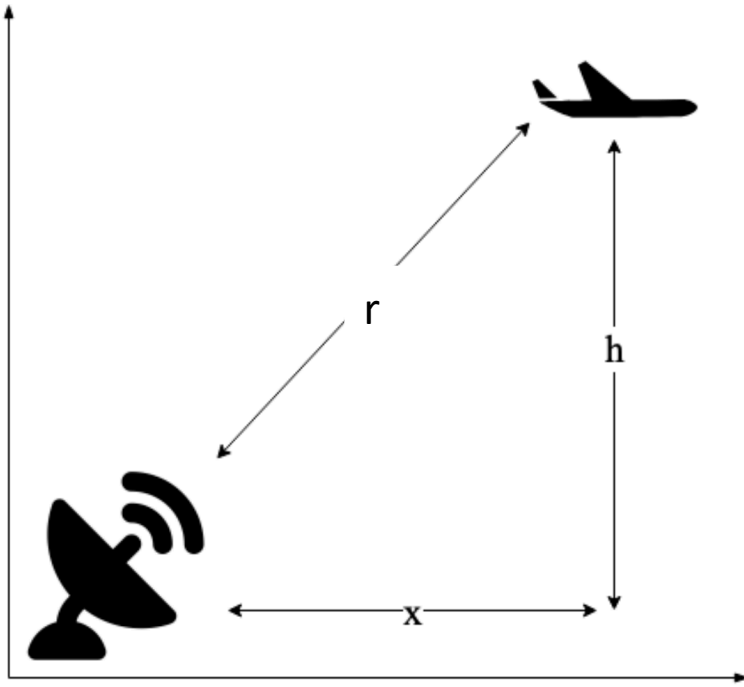
$$x_{3,t} = h_t$$

$$z_t = h(x_t) + \delta$$

$$= \sqrt{x_{1,t}^2 + x_{3,t}^2} + \delta$$

We will linearize at the mean!

Example



$$x_{t+1} = x_t + v_t * \Delta_t$$

$$v_{t+1} = v_t$$

$$h_{t+1} = h_t$$

$$r_t = \sqrt{x_t^2 + h_t^2}$$

$$x_{1,t} = x_t$$

$$x_{2,t} = v_t$$

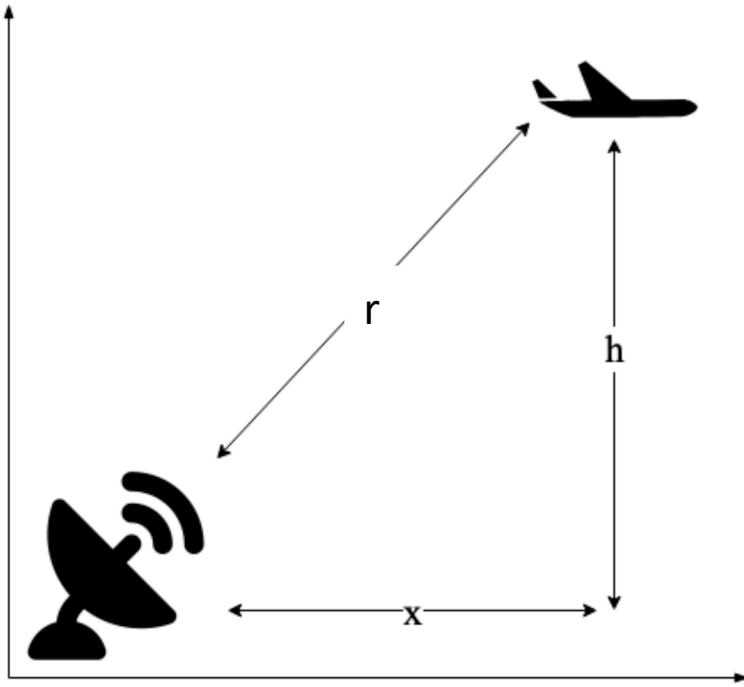
$$x_{3,t} = h_t$$

$$z_t = h(x_t) + \delta$$

$$= \sqrt{x_{1,t}^2 + x_{3,t}^2} + \delta$$

$$h(x_t) = h(\mu_t) + H_t(\mu_t)(z_t - \mu_t)$$

Example



$$x_{t+1} = x_t + v_t * \Delta_t$$

$$v_{t+1} = v_t$$

$$h_{t+1} = h_t$$

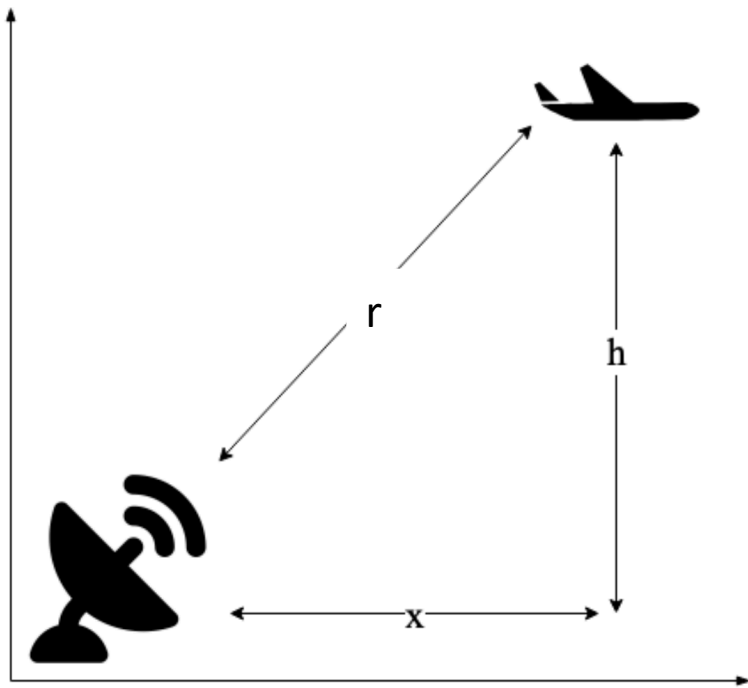
$$r_t = \sqrt{x_t^2 + h_t^2}$$

$$\begin{aligned} z_t &= h(x_t) + \delta \\ &= \sqrt{x_{1,t}^2 + x_{3,t}^2} + \delta \end{aligned}$$

$$h(x_t) = h(\mu_t) + H_t(\mu_t)(x_t - \mu_t)$$

$$H_t = \begin{bmatrix} \frac{\partial h}{\partial x_{t,1}} & \frac{\partial h}{\partial x_{t,2}} & \frac{\partial h}{\partial x_{t,3}} \end{bmatrix}$$

Example



$$x_{t+1} = x_t + v_t * \Delta_t$$

$$v_{t+1} = v_t$$

$$h_{t+1} = h_t$$

$$r_t = \sqrt{x_t^2 + h_t^2}$$

$$z_t = h(x_t) + \delta$$

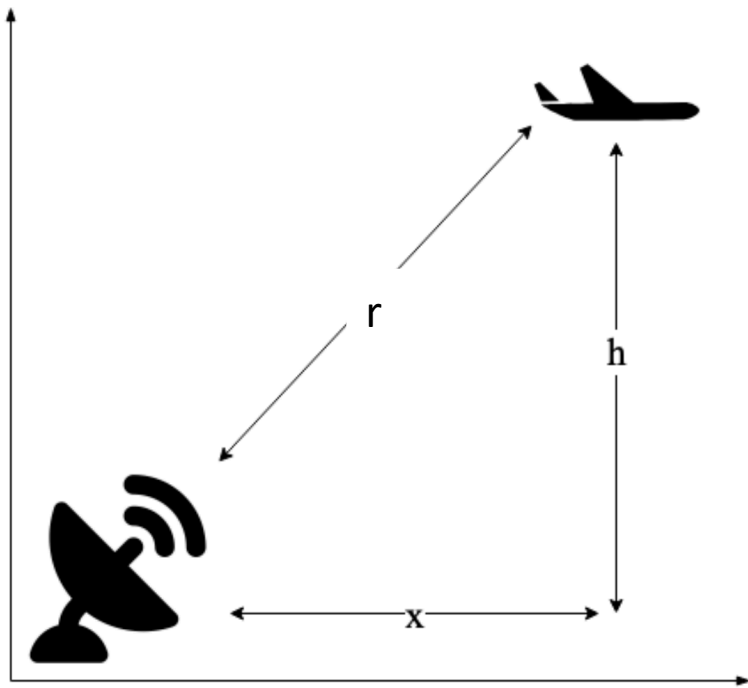
$$= \sqrt{x_{1,t}^2 + x_{3,t}^2} + \delta$$

$$h(x_t) = h(\mu_t) + H_t(\mu_t)(x_t - \mu_t)$$

$$H_t = \begin{bmatrix} \frac{\partial h}{\partial x_{t,1}} & \frac{\partial h}{\partial x_{t,2}} & \frac{\partial h}{\partial x_{t,3}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2x_{t,1}}{2\sqrt{x_{t,1}^2 + x_{t,3}^2}} & 0 & \frac{2x_{t,3}}{2\sqrt{x_{t,1}^2 + x_{t,3}^2}} \end{bmatrix}$$

Example



$$x_{t+1} = x_t + v_t * \Delta_t$$

$$v_{t+1} = v_t$$

$$h_{t+1} = h_t$$

$$r_t = \sqrt{x_t^2 + h_t^2}$$

$$z_t = h(x_t) + \delta$$

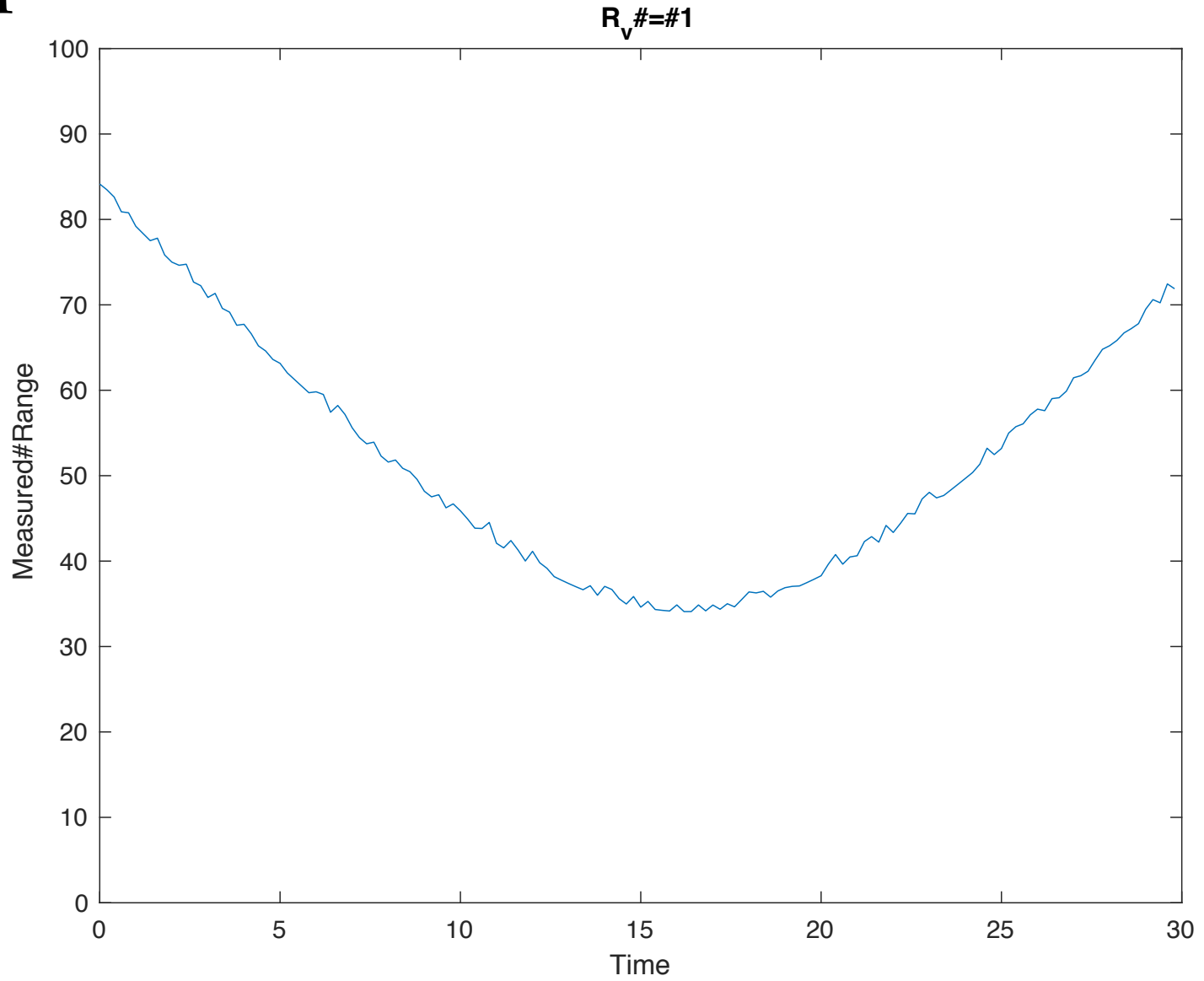
$$= \sqrt{x_{1,t}^2 + x_{3,t}^2} + \delta$$

$$h(x_t) = h(\mu_t) + H_t(\mu_t)(x_t - \mu_t)$$

$$H_t = \begin{bmatrix} \frac{\partial h}{\partial x_{t,1}} & \frac{\partial h}{\partial x_{t,2}} & \frac{\partial h}{\partial x_{t,3}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2x_{t,1}}{2\sqrt{x_{t,1}^2 + x_{t,3}^2}} & 0 & \frac{2x_{t,3}}{2\sqrt{x_{t,1}^2 + x_{t,3}^2}} \end{bmatrix}$$

Example



Measured range r

Example

