# Particle Filters

CSCI 545 Introduction to Robotics
Instructor: Stefanos Nikolaidis

# Test for a Mobile Robot Localization

Algorithm : MCL
Sensor : USAT(Ultra Sonic)

Chosun Univ. Instrument & Control
Autonomous Robot Control systoem Lab
Date : 2009. 03

# Motivation

- What if the state distribution is not Gaussian?

- Performance of most conventional filters drops when the dynamics are non-linear

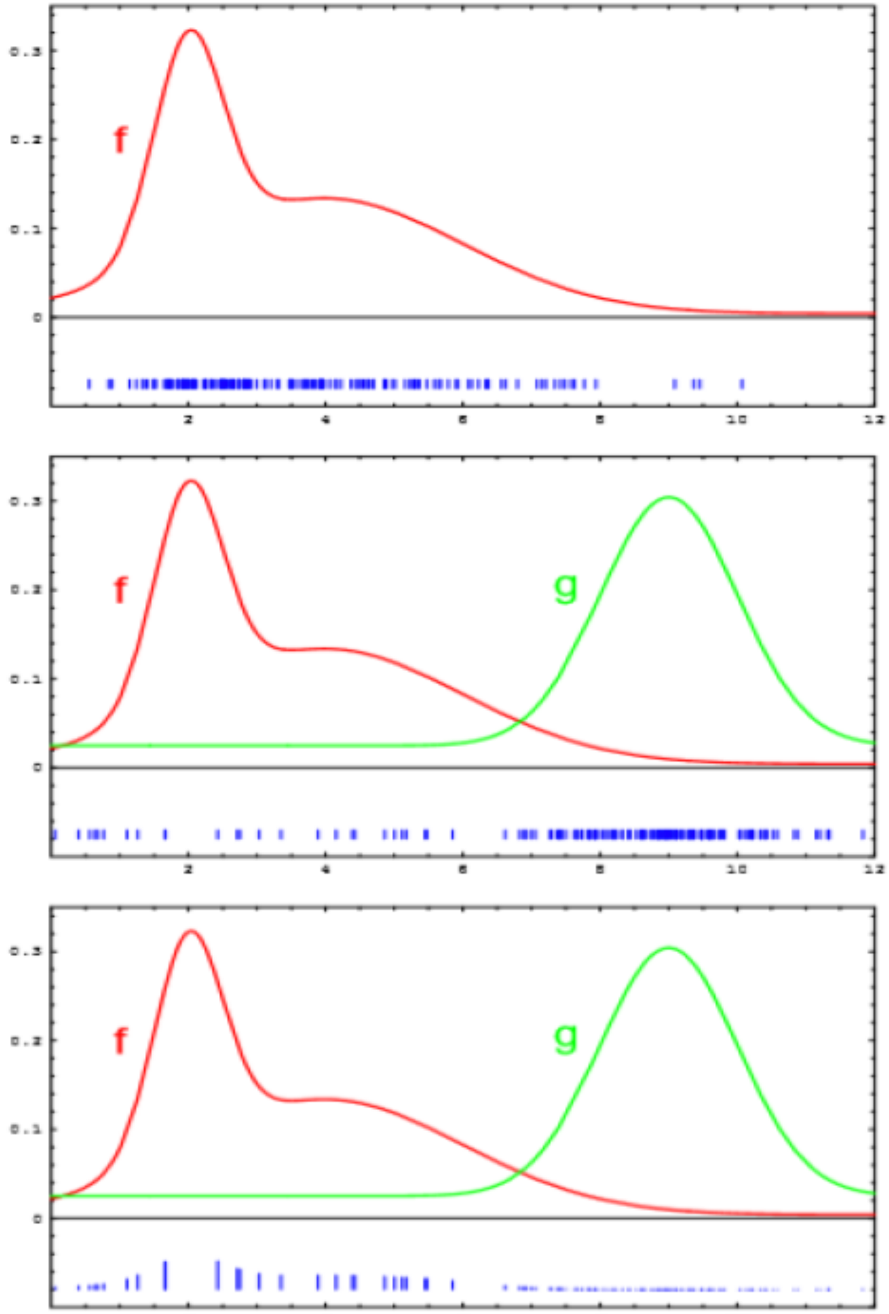
- Key Idea: Particle Filters represent the distribution with a number of samples

# Importance Sampling

- Assume that we have a distribution $f(x)$ that is difficult to draw samples from

- We can specify a distribution $g(x)$, draw samples from $g$ and multiply them with $\frac{f(x)}{g(x)}$

# Example



# Algorithm

**Algorithm Particle_filter**($\mathcal{X}_{t-1}, u_t, z_t$)**:**

$\bar{\mathcal{X}}_t = \mathcal{X}_t = \emptyset$

for $m = 1$ to $M$ do

  sample $x_t^{[m]} \sim p(x_t \mid u_t, x_{t-1}^{[m]})$

  $w_t^{[m]} = p(z_t \mid x_t^{[m]})$

  $\bar{\mathcal{X}}_t = \bar{\mathcal{X}}_t + \langle x_t^{[m]}, w_t^{[m]} \rangle$

endfor

for $m = 1$ to $M$ do

  draw $i$ with probability $\propto w_t^{[i]}$

  add $x_t^{[i]}$ to $\mathcal{X}_t$

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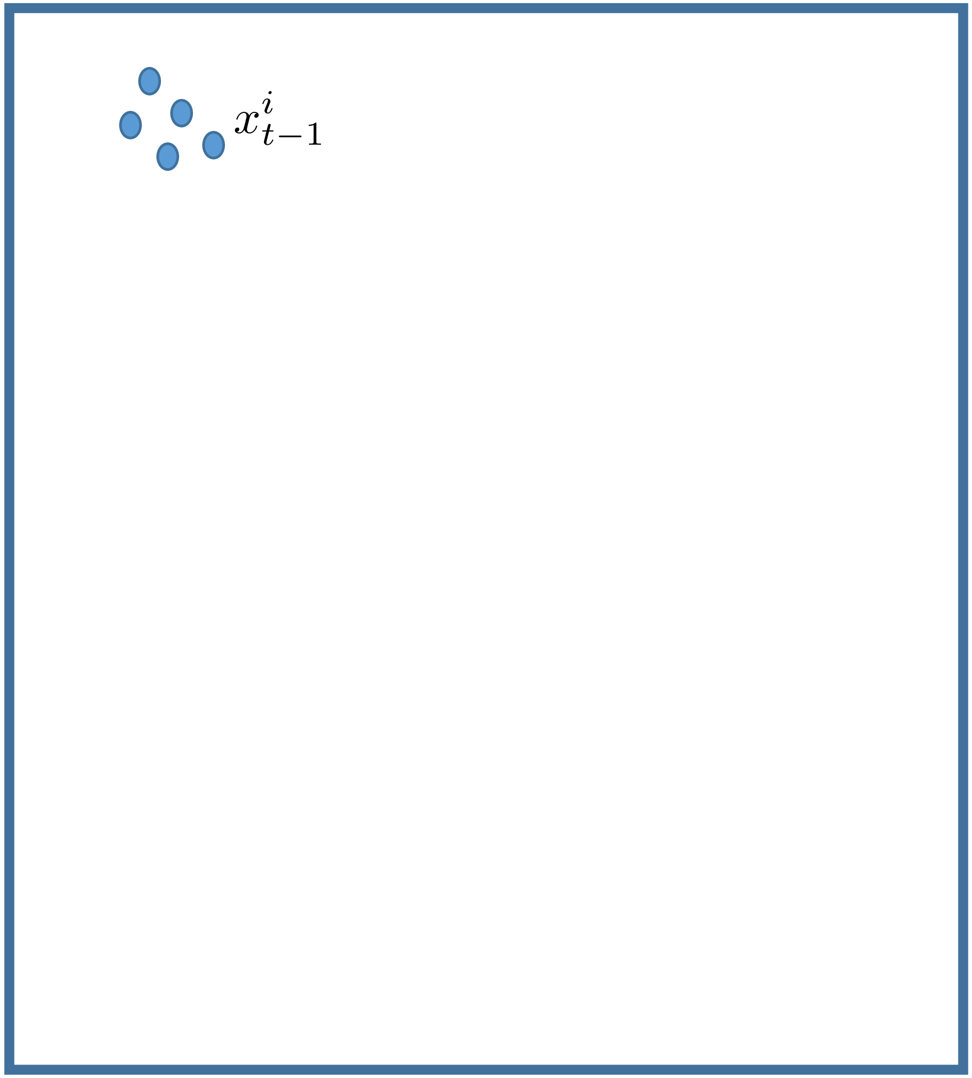
# Initial Distribution

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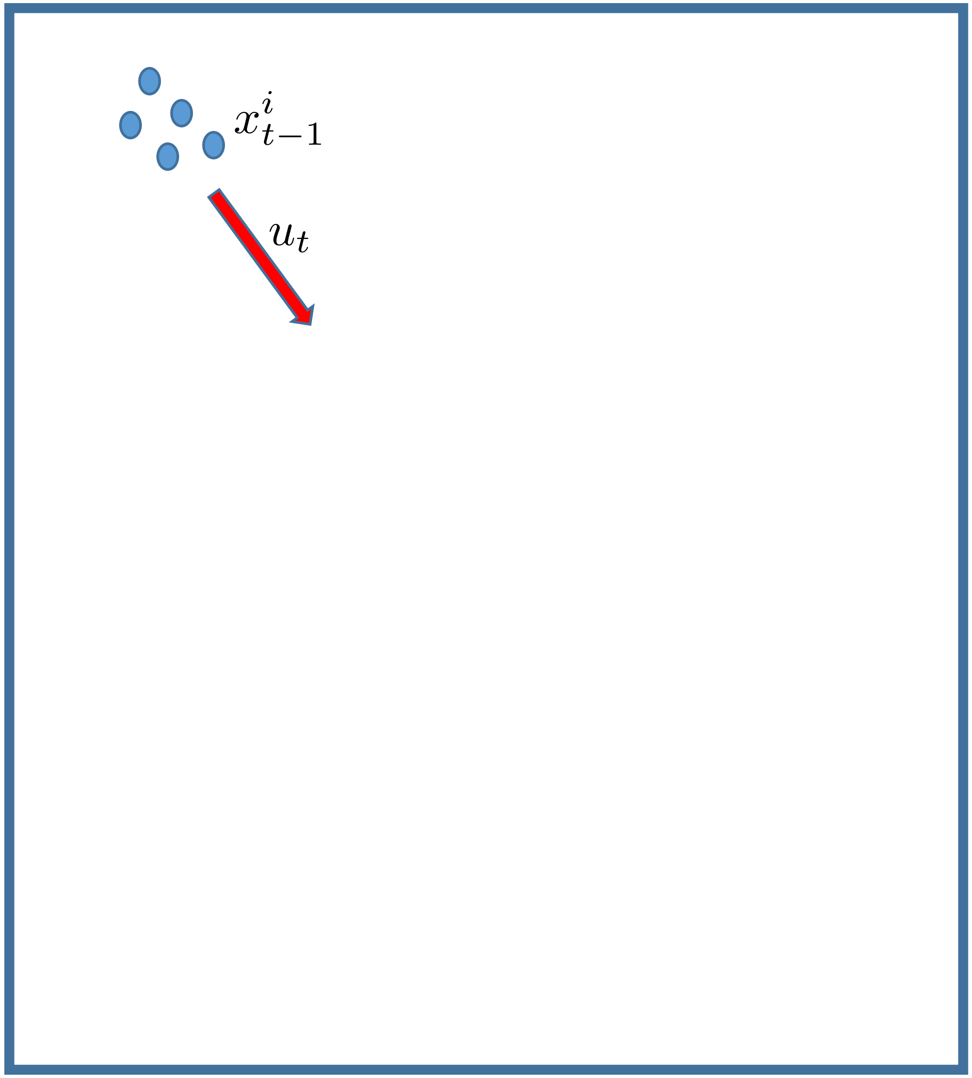
# Motion Model

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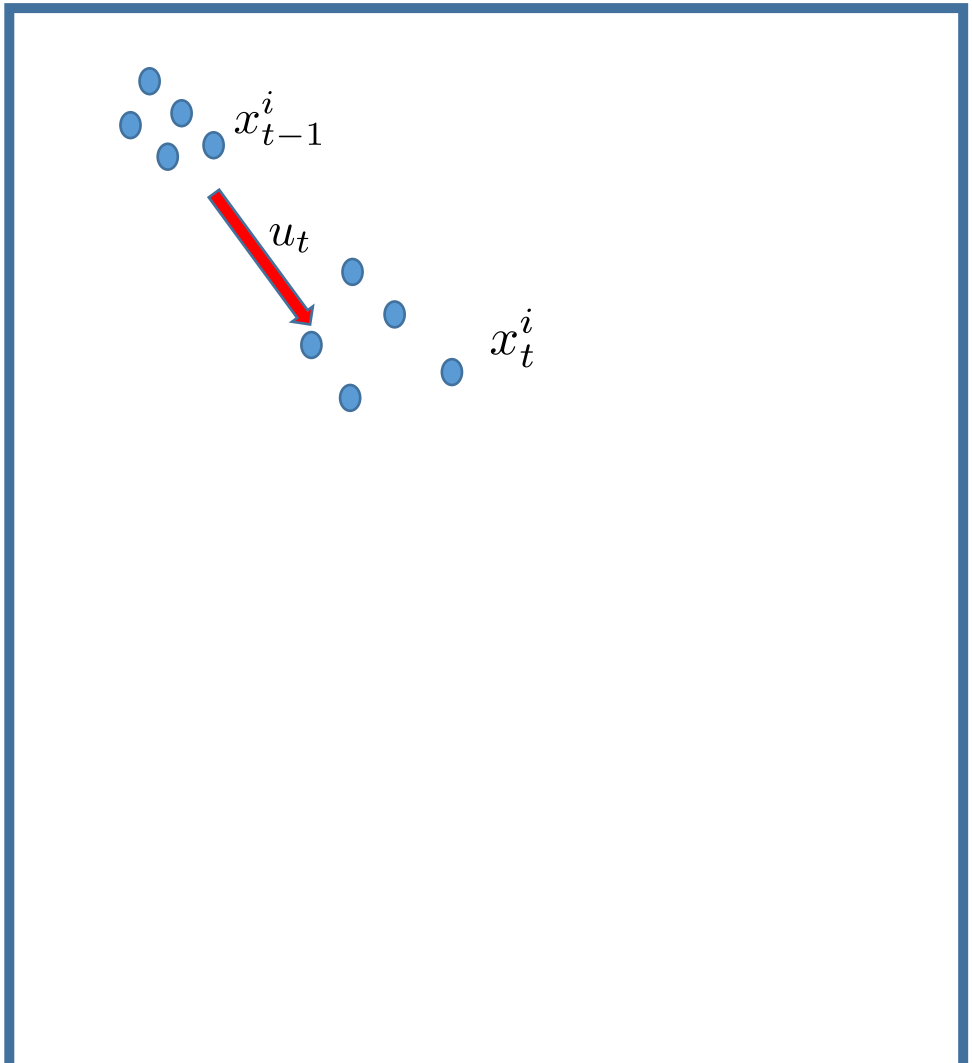


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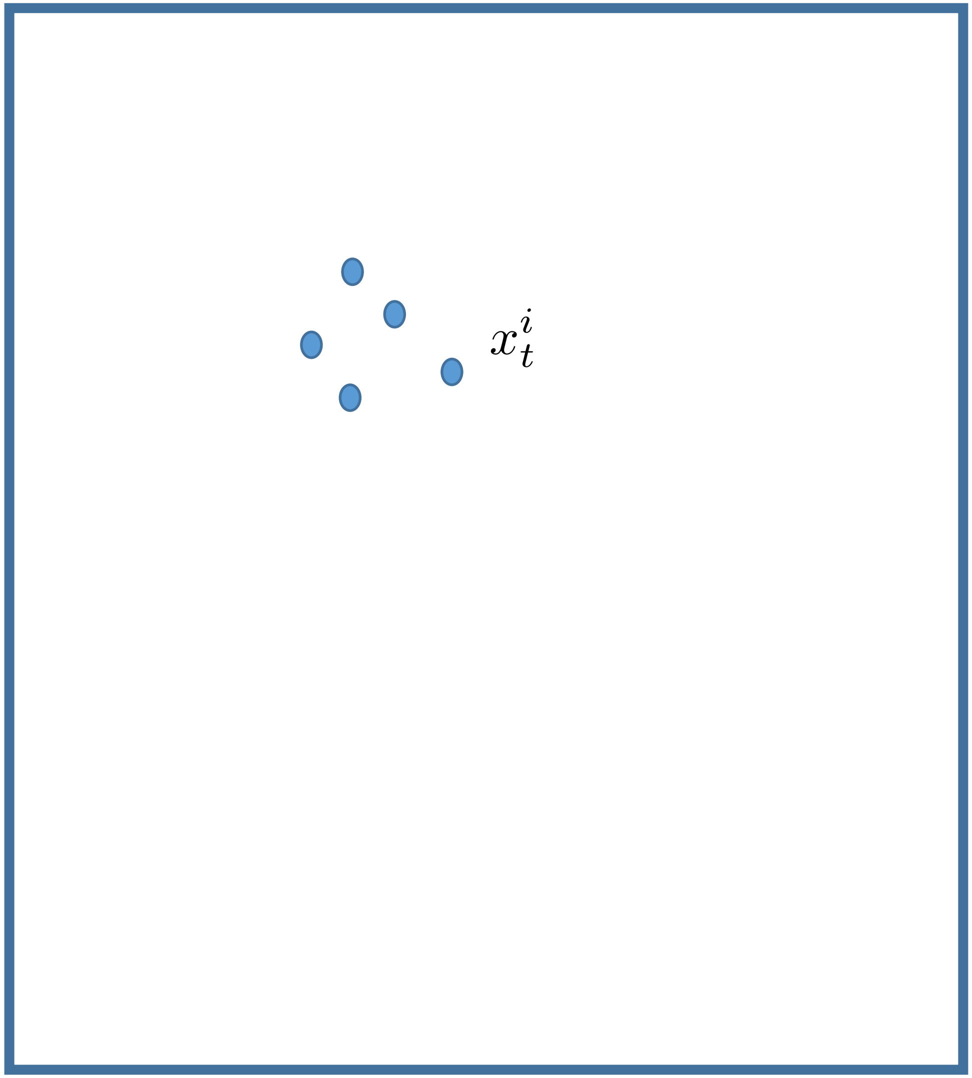


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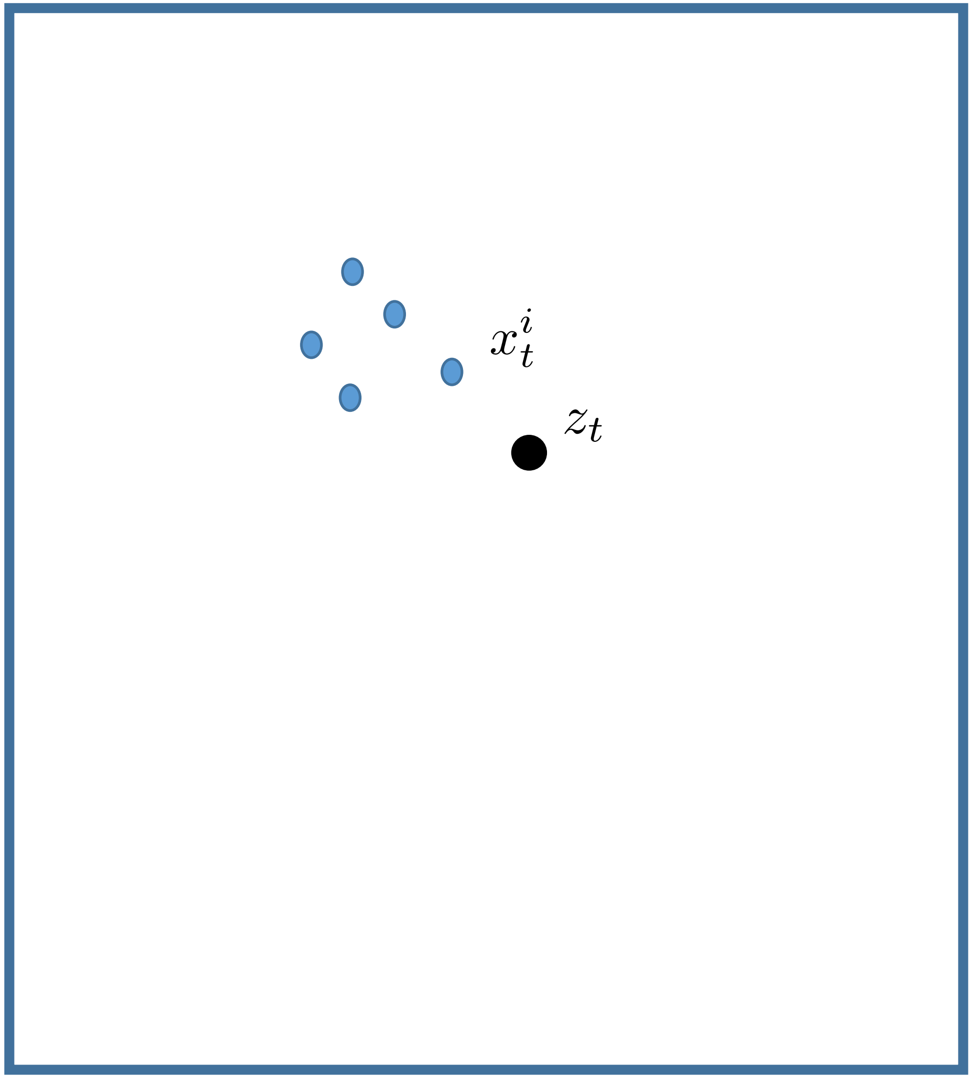
# Observation

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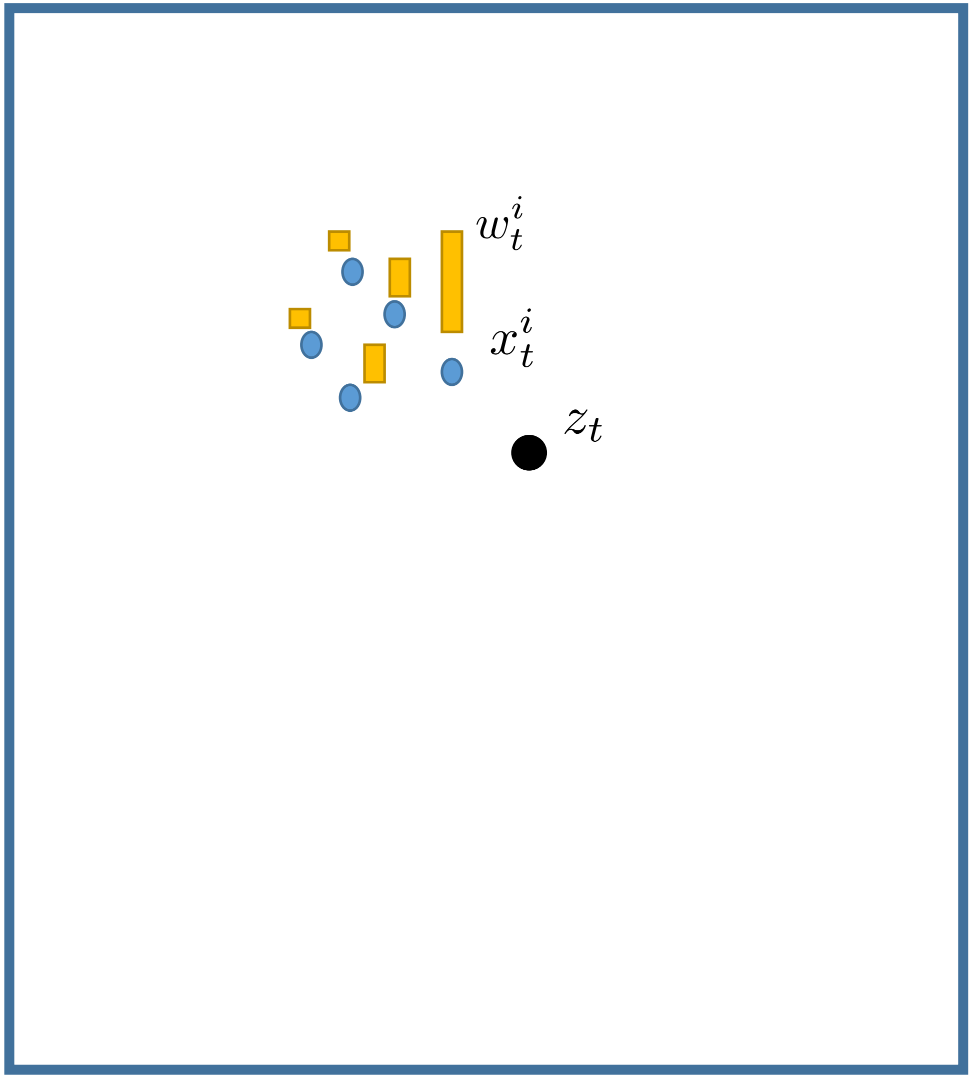
# Weight Assignment

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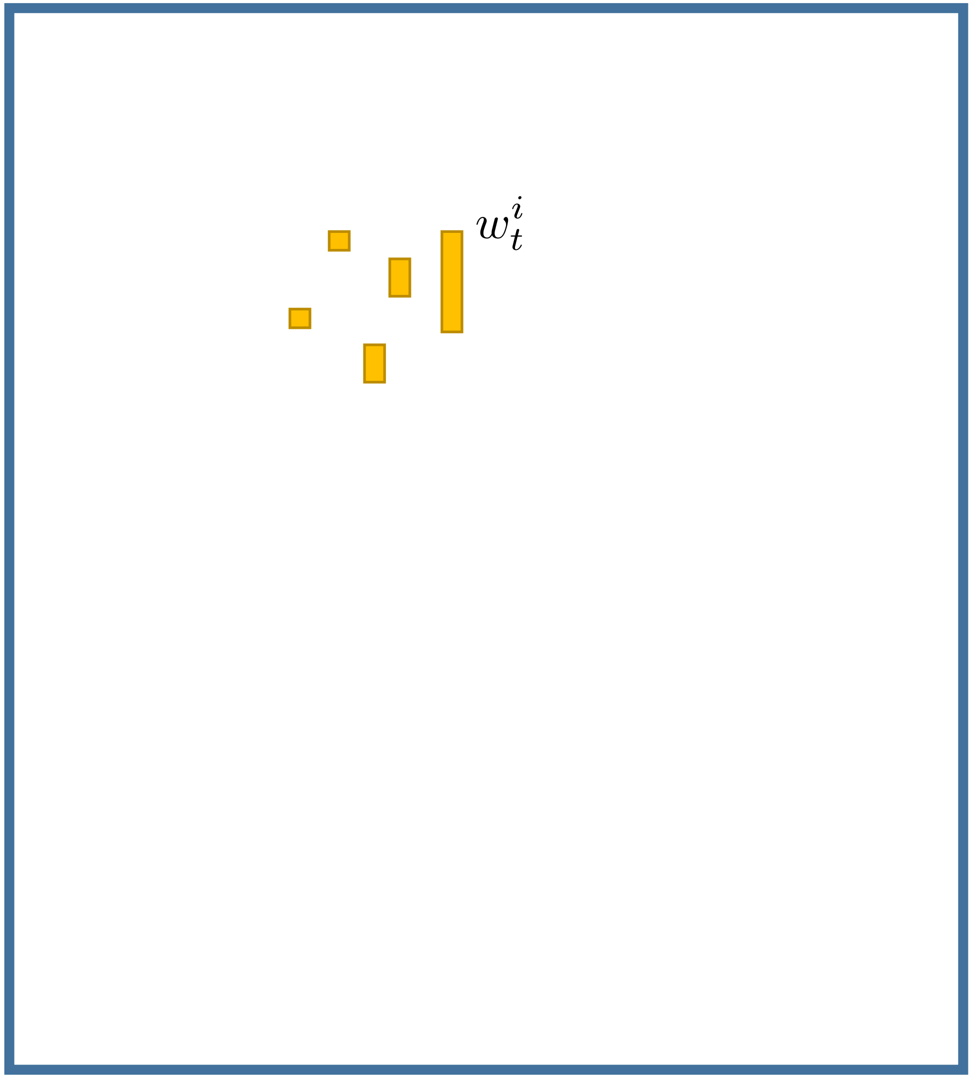

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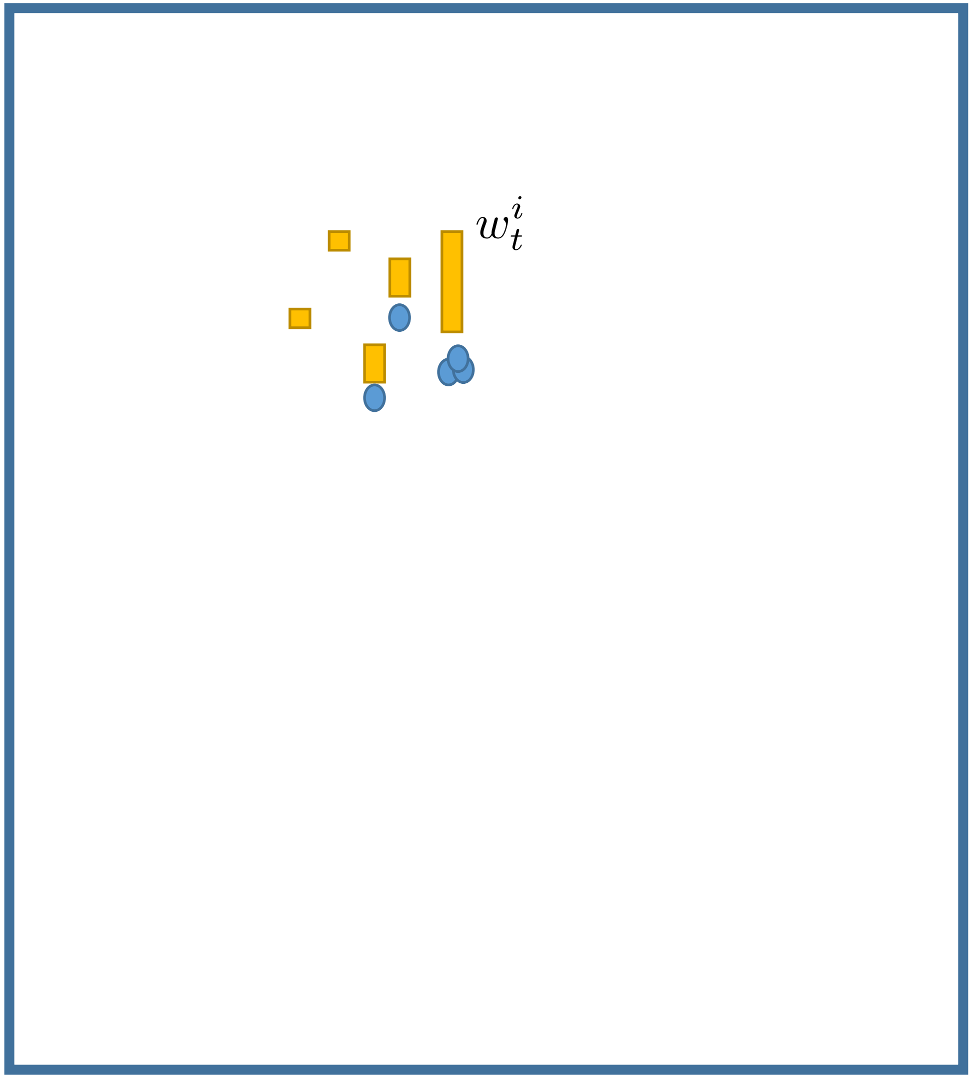


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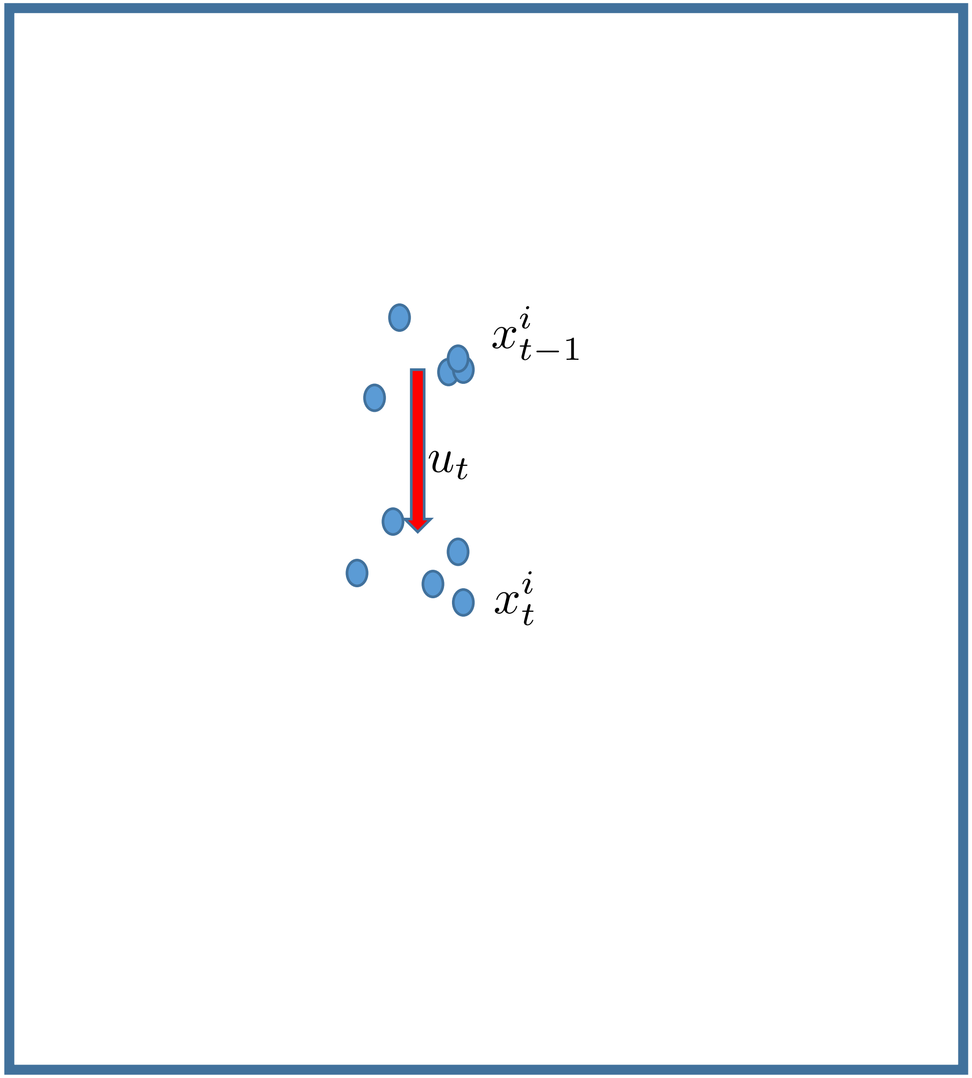


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$$b(x_{0:t}) = P(x_{0:t}|z_{1:t}, u_{1:t})$$

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b(x_{0:t}) &= P(x_{0:t} | z_{1:t}, u_{1:t}) \\
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$$b(x_{0:t}) = \eta P(z_t|x_t)P(x_t|x_{t-1}, u_t)b(x_{0:t-1})$$

$$w_t^{[m]} = \frac{target\ distribution}{proposal\ distribution}$$

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&= \eta P(z_t|x_t)
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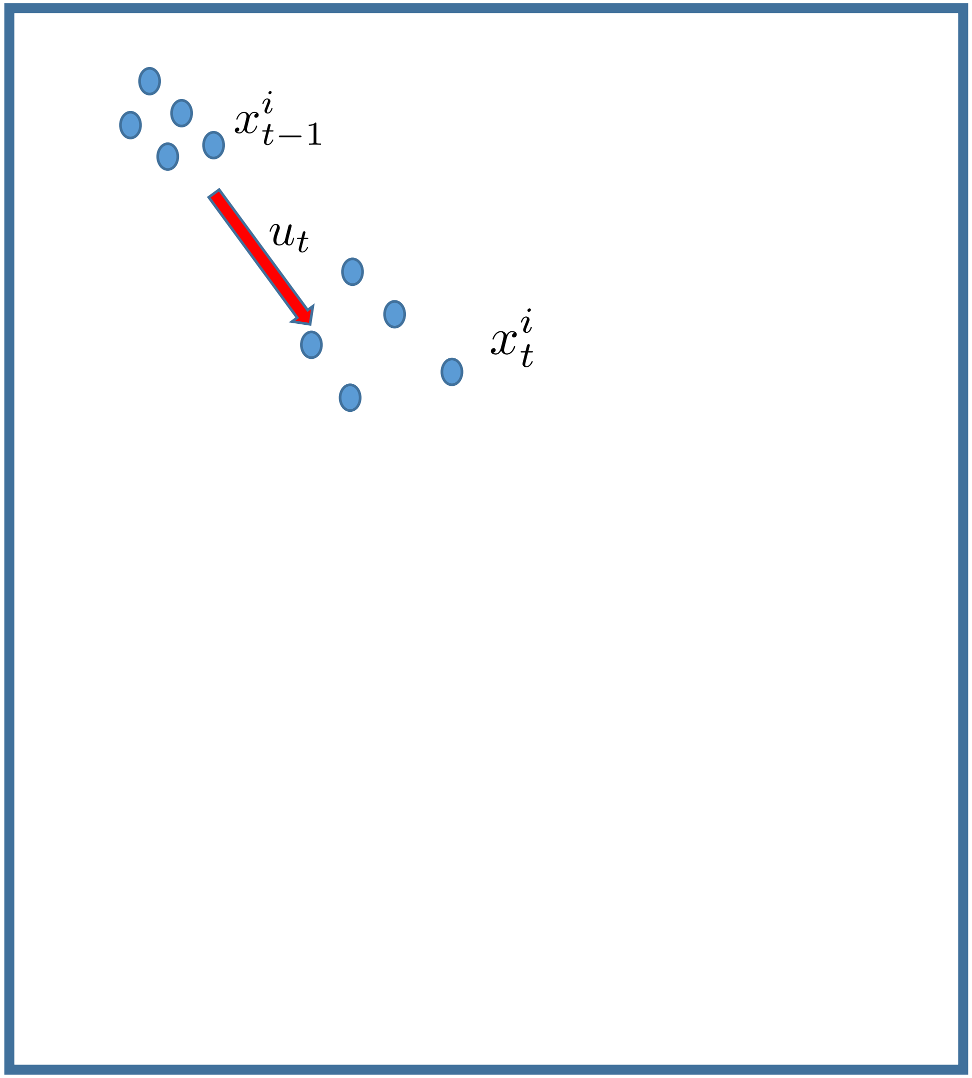

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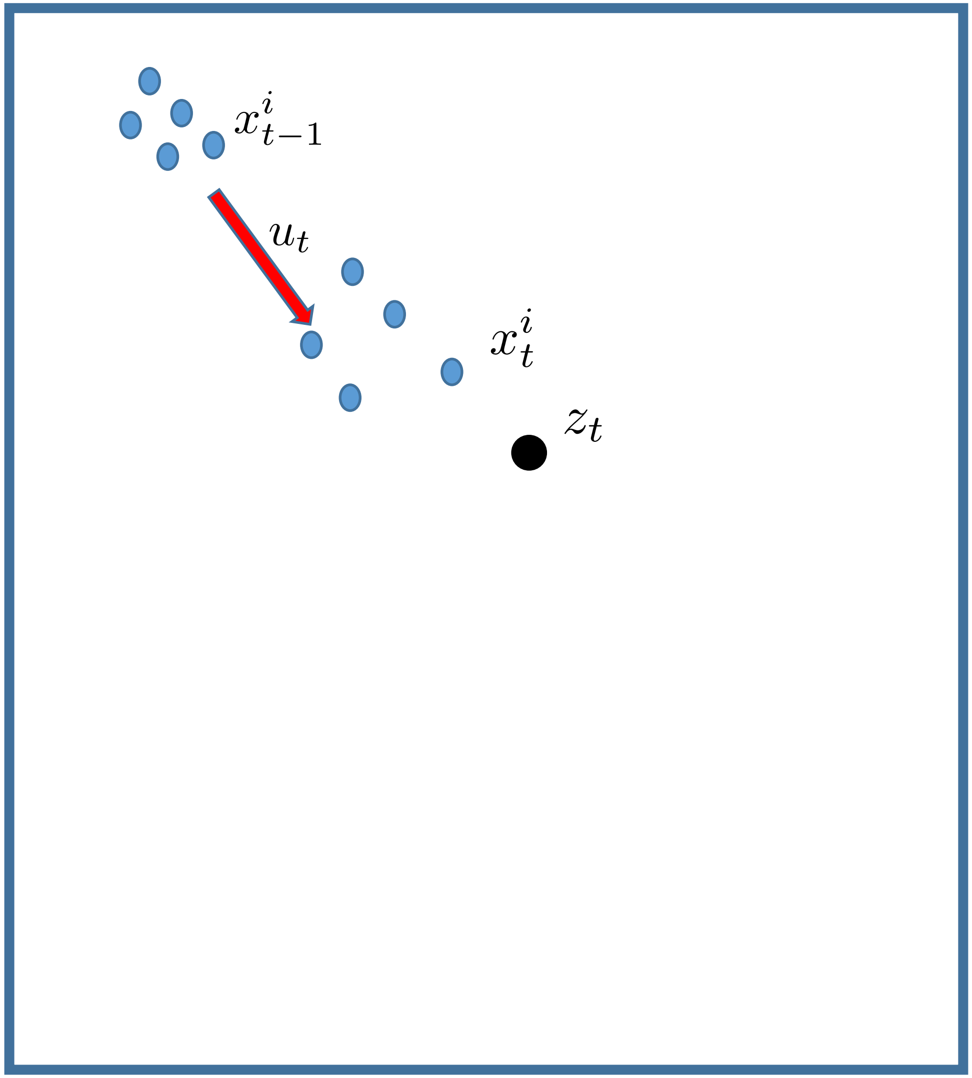

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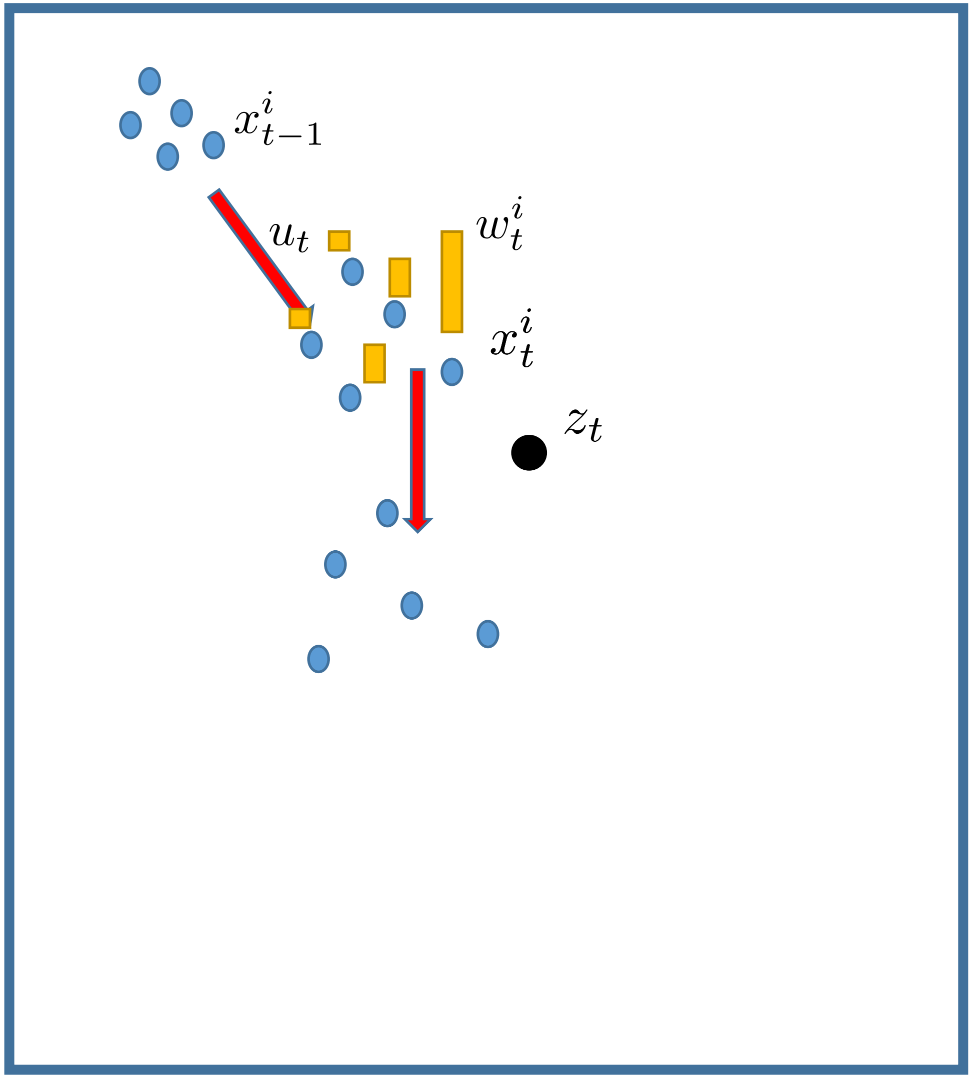
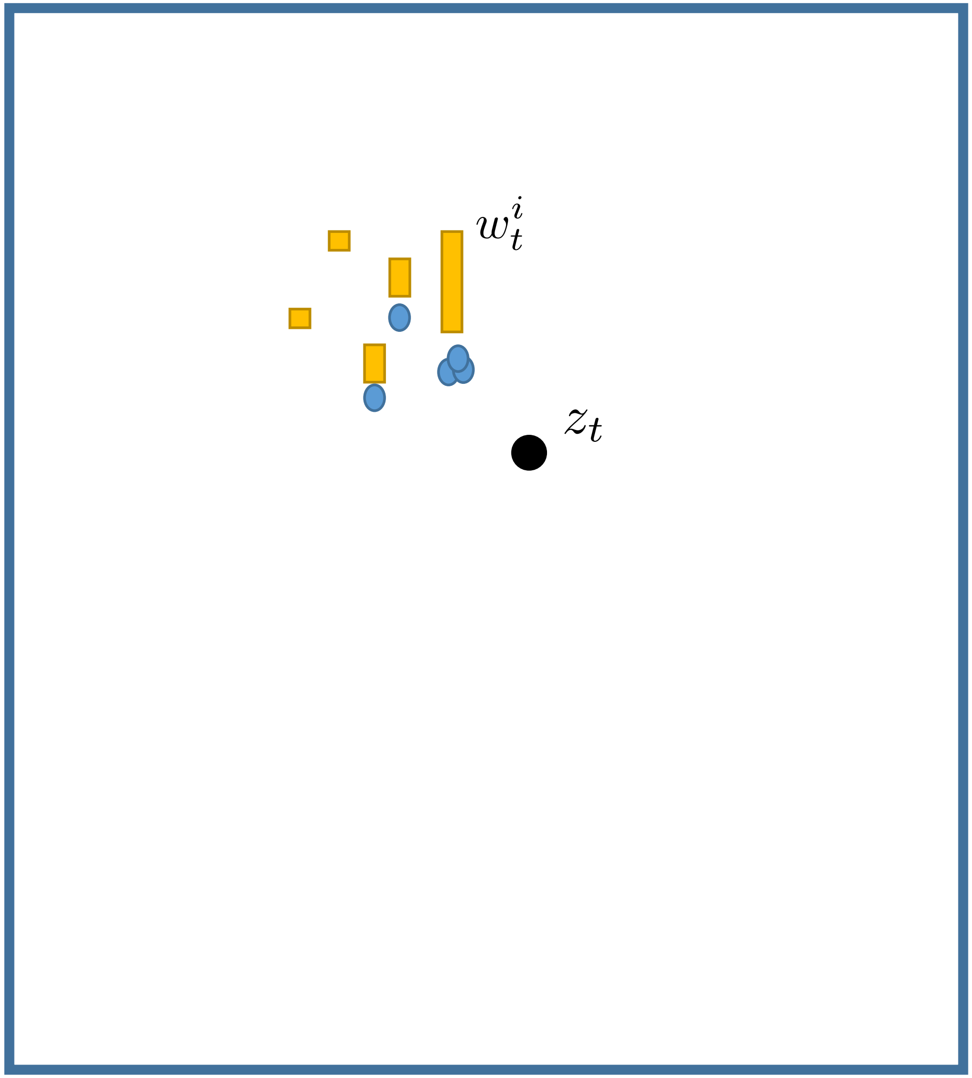


# Issues With Particle Filter



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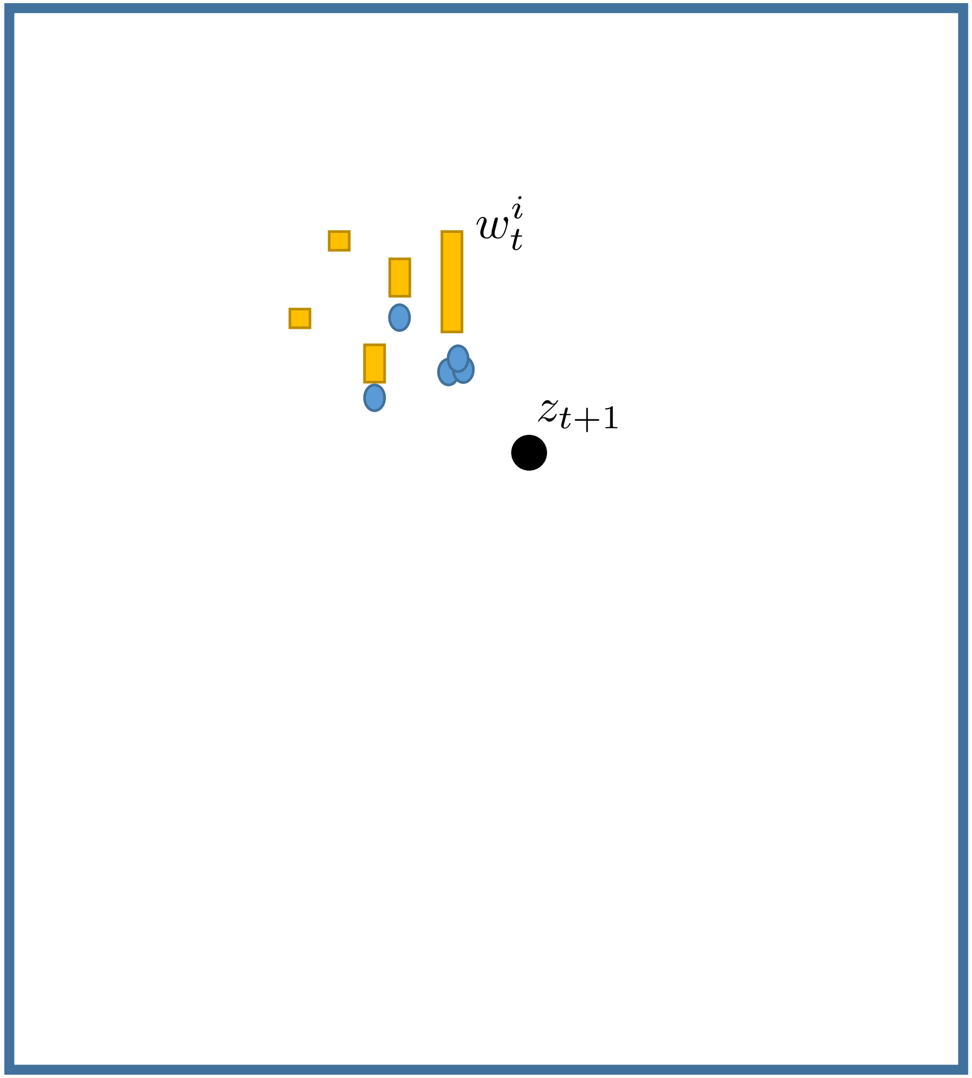

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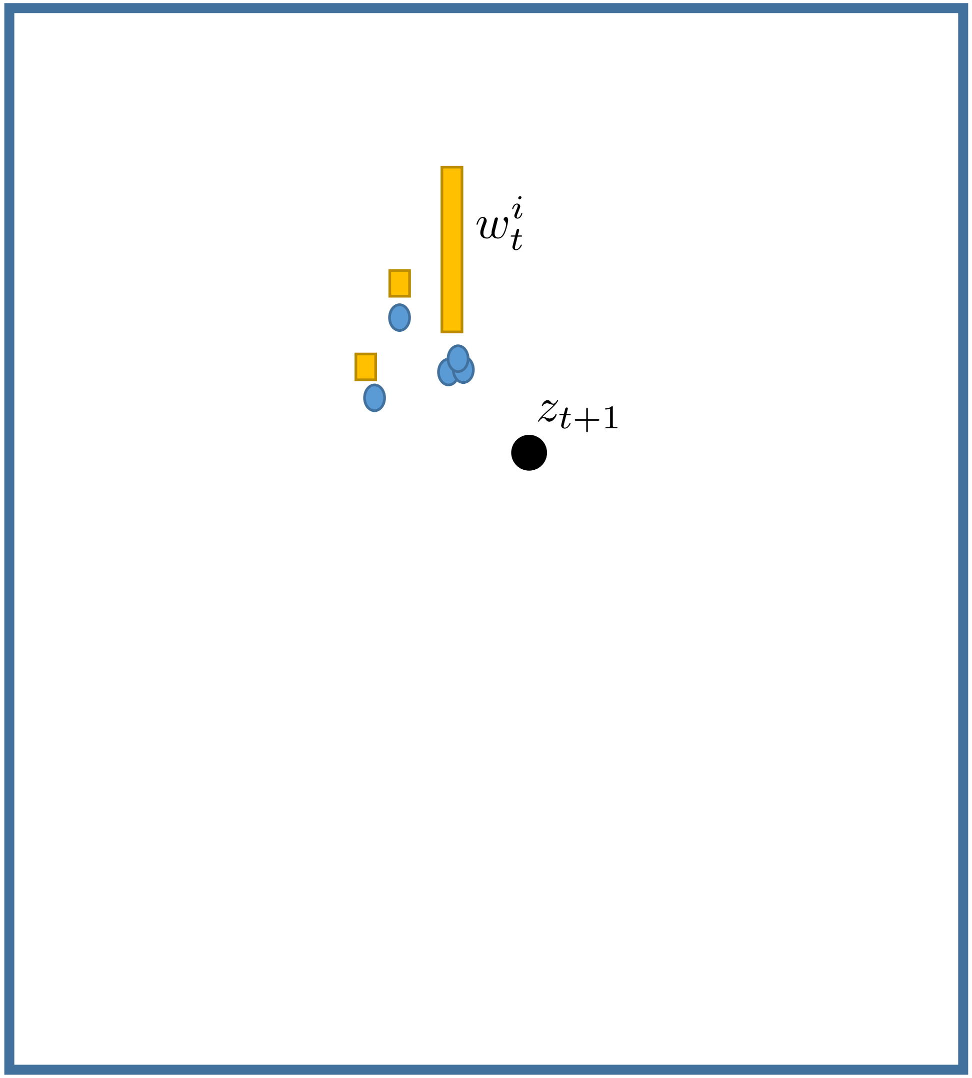

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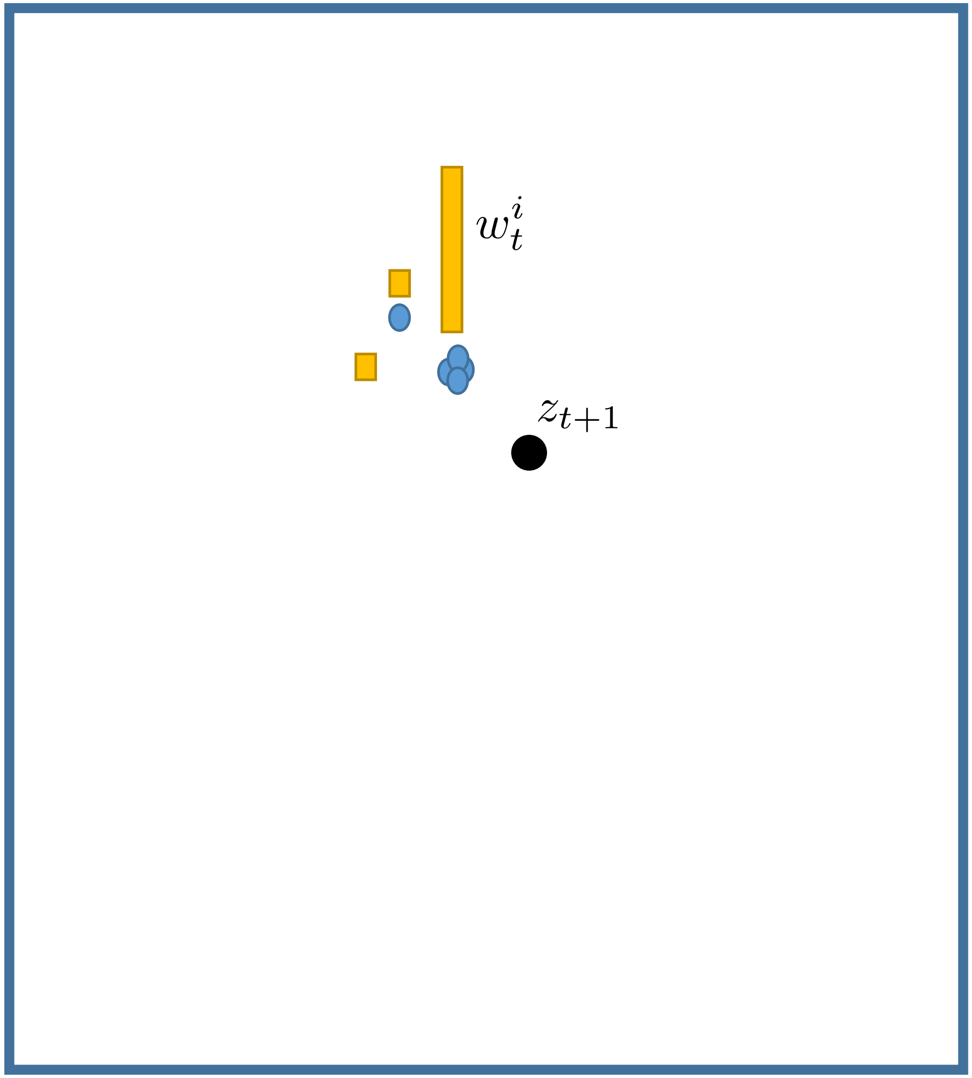

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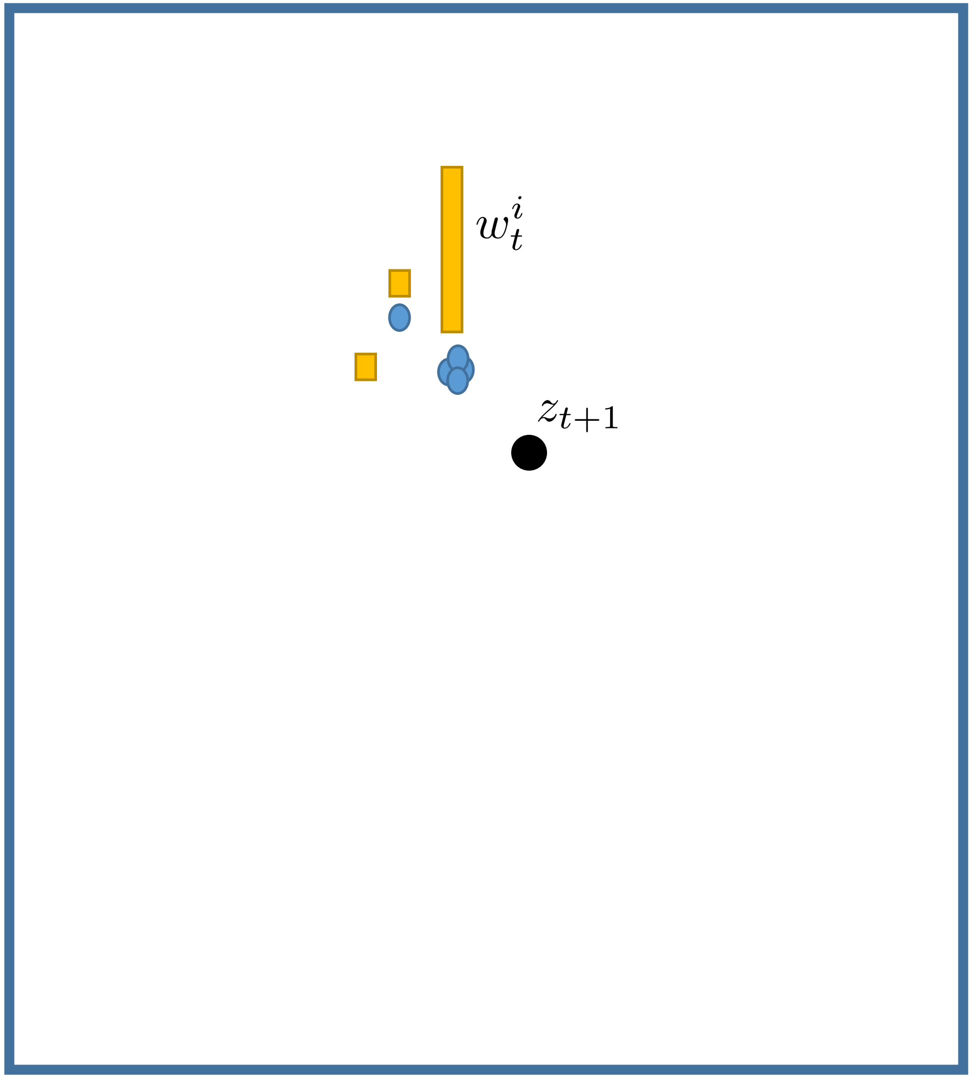


How can we address the sample impoverishment problem?