USC Viterbi School of Engineering

Motion and Sensor Models

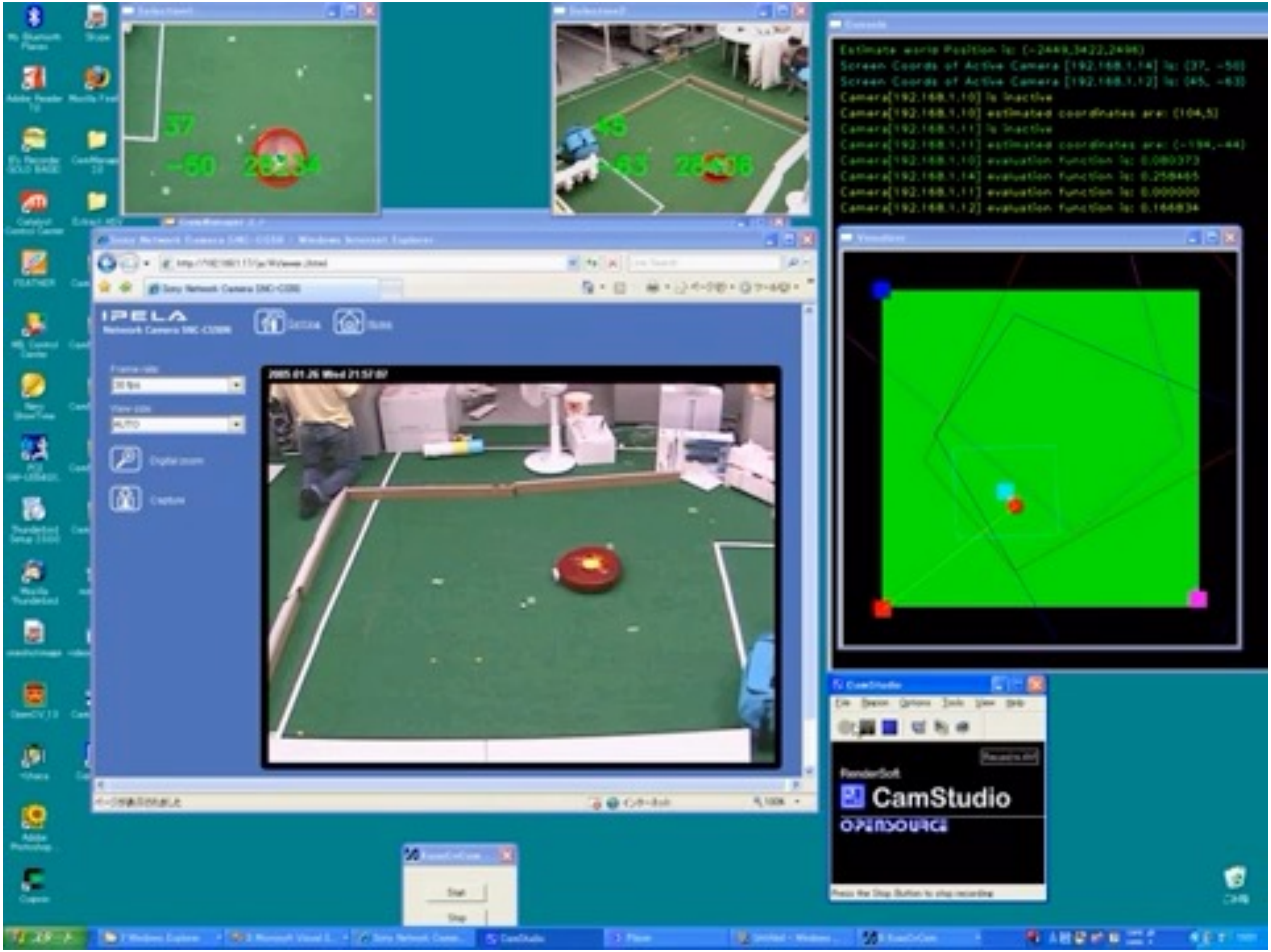
CSCI 545 Introduction to Robotics
Instructor: Stefanos Nikolaidis

Motion Model

- What is the transition function?

$$P(x_t | x_{t-1}, u_t)$$

2D Mobile Robot



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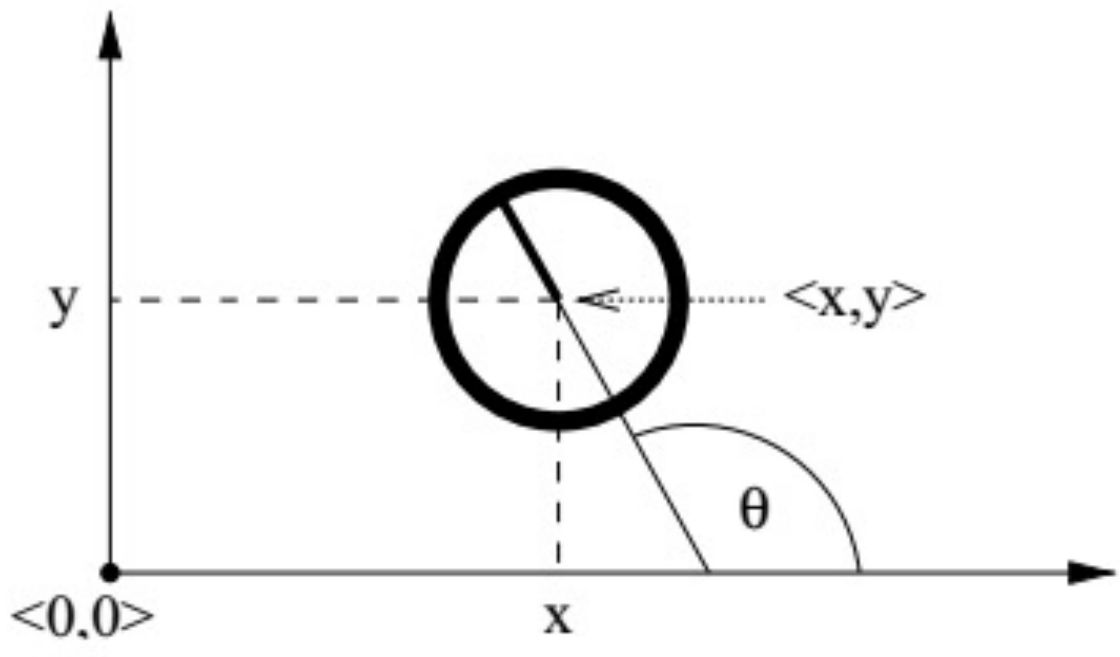
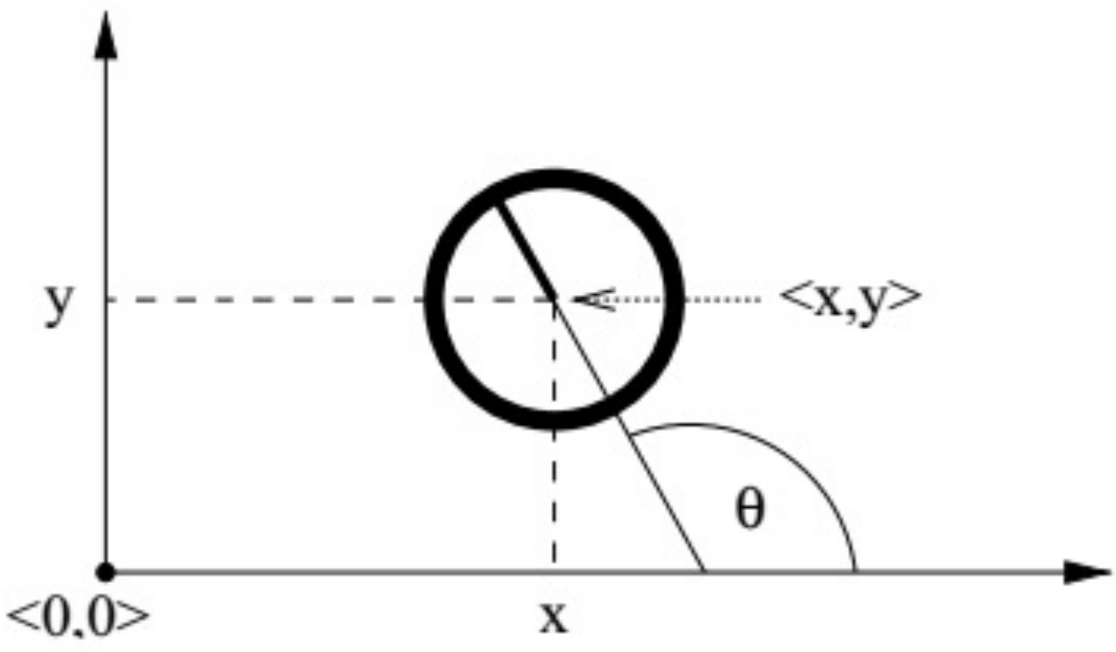


Robot Configuration

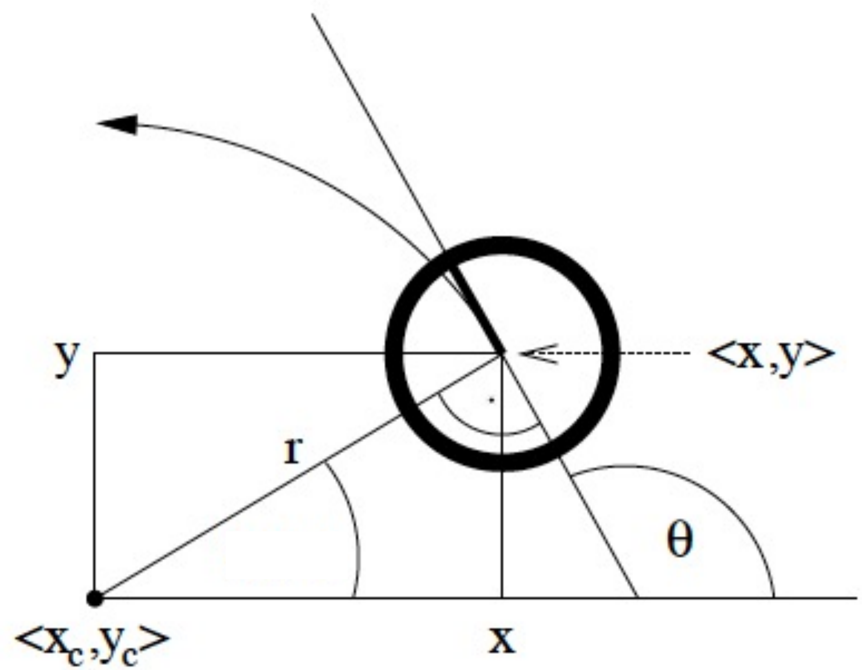
$$(x, y, \theta)$$



Velocity Motion Model

- We can assume that we provide two inputs, a translational and a rotational velocity

$$u_t = \begin{bmatrix} v_t \\ \omega_t \end{bmatrix}$$



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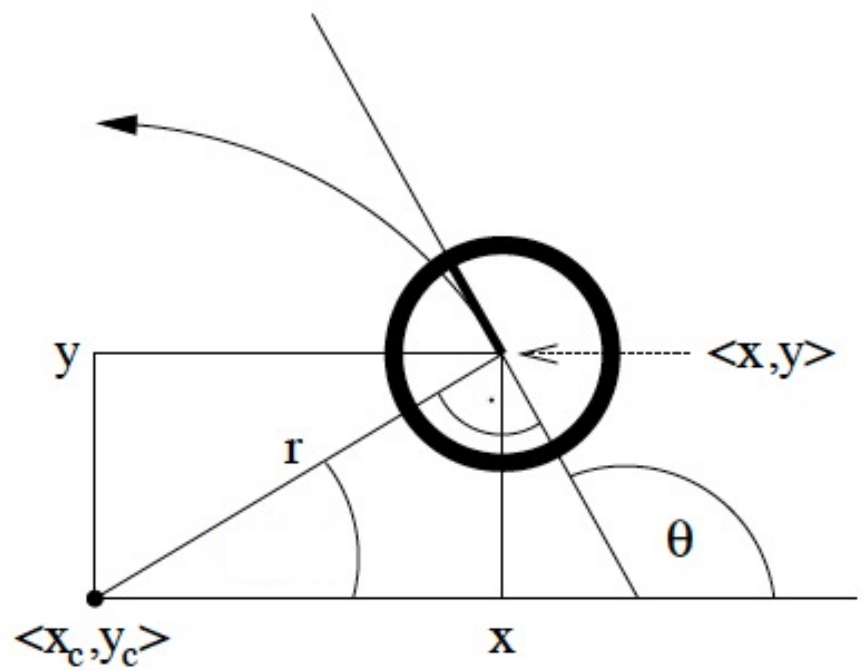

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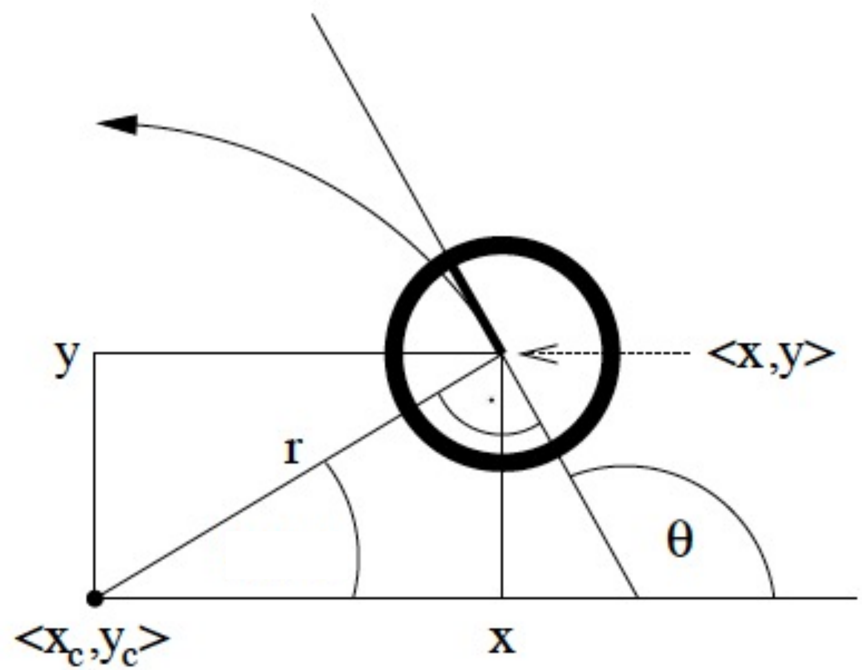
$$(v_t = \omega_t \cdot r_t)$$



Velocity Motion Model

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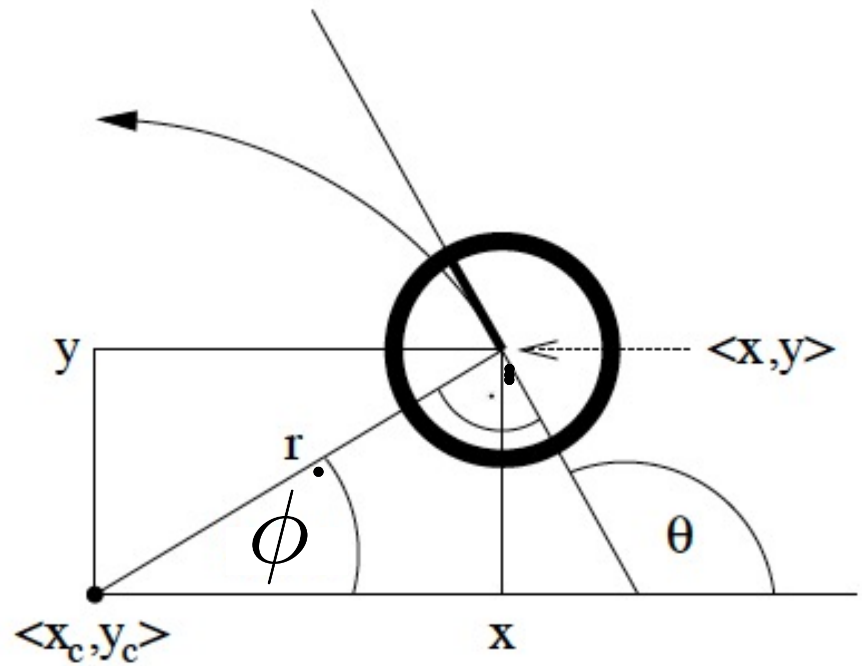
$$u_t = \begin{bmatrix} v_t \\ \omega_t \end{bmatrix}$$

$$\pi - \theta + \frac{\pi}{2} + \phi = \pi$$

$$\phi = \theta - \frac{\pi}{2}$$



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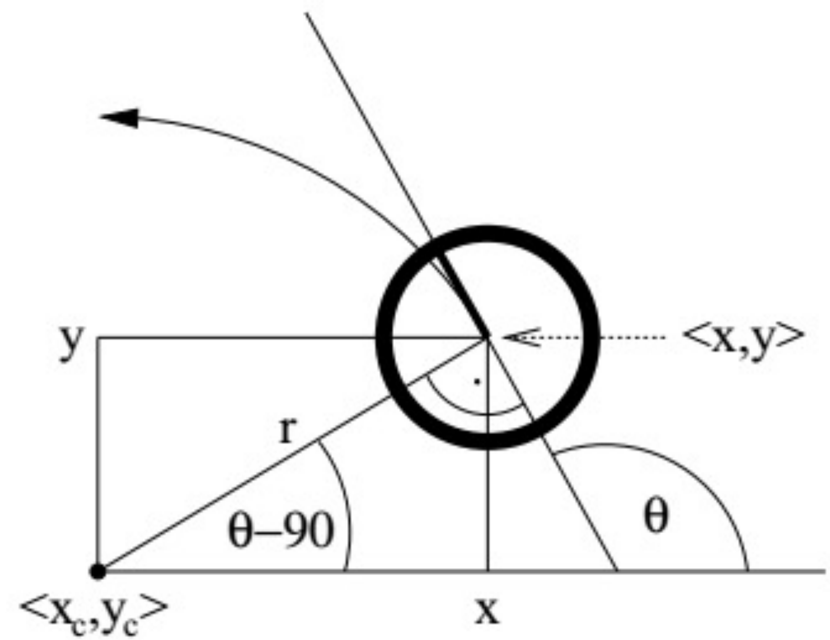

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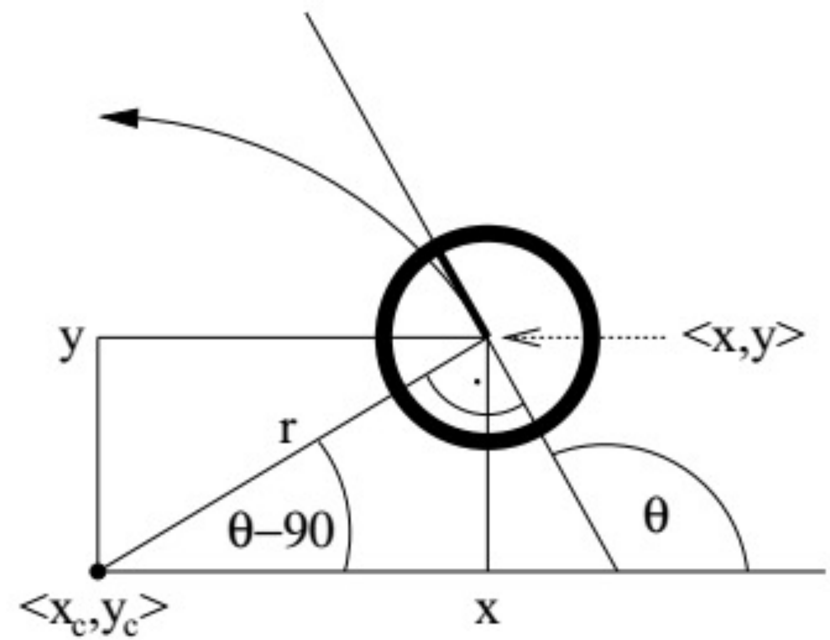
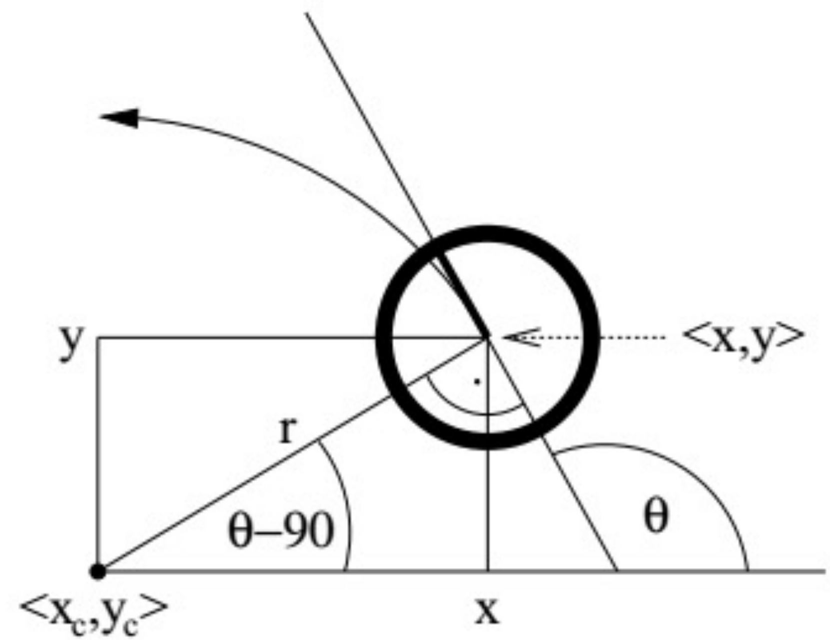


Velocity Motion Model

$$r_t = \frac{v_t}{\omega_t}$$

$$x_t = x_c + r_t cos(\theta_t - \pi/2)$$



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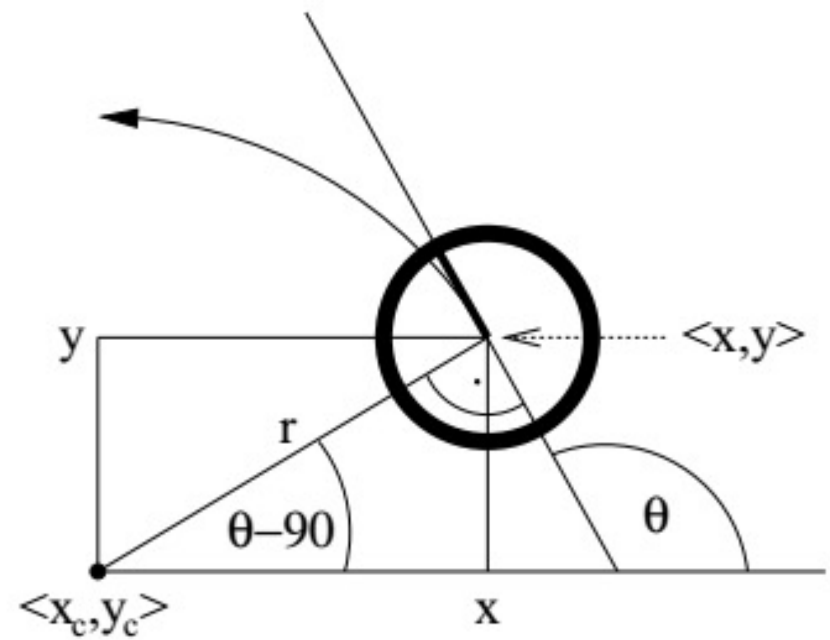

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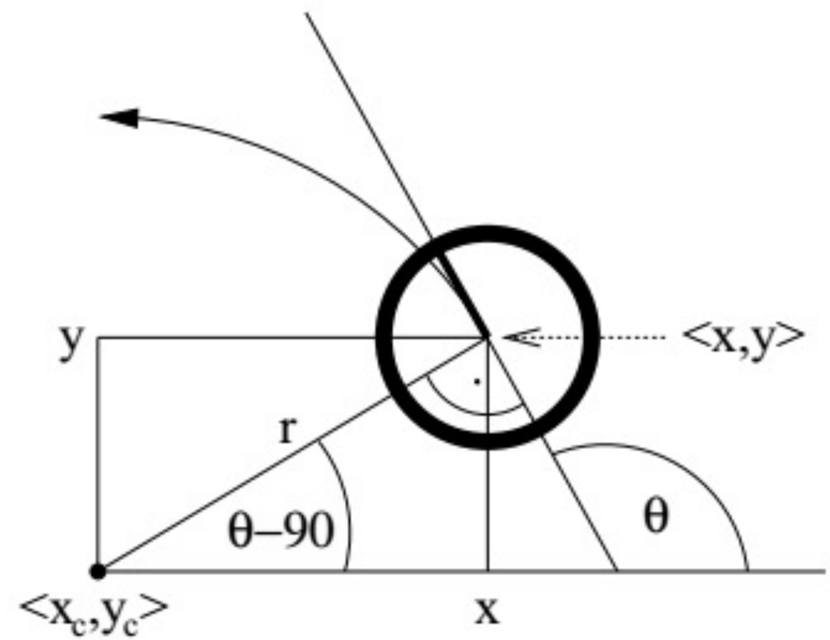

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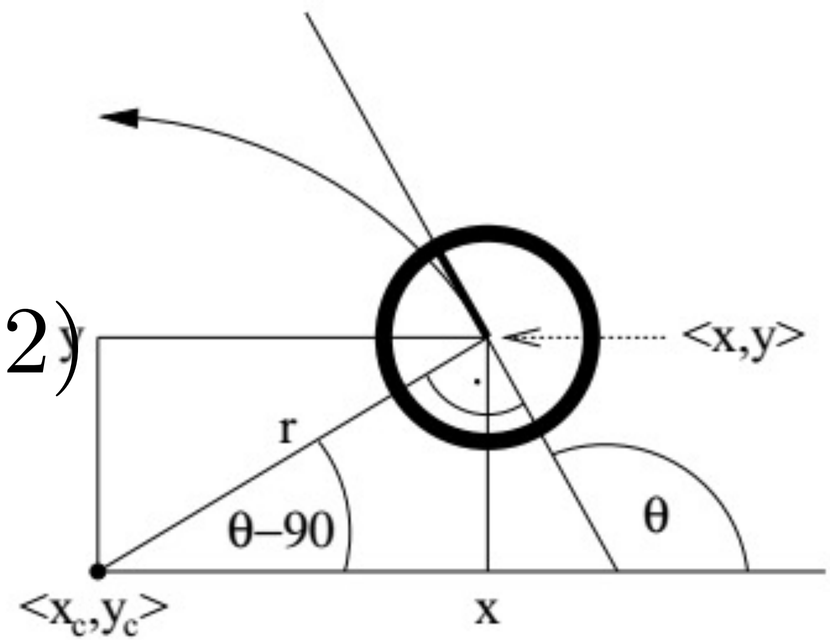

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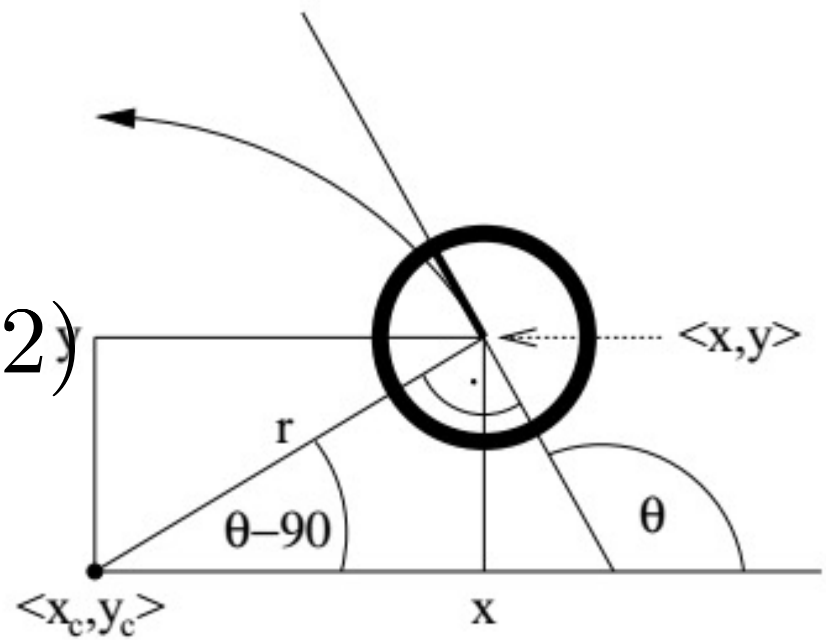

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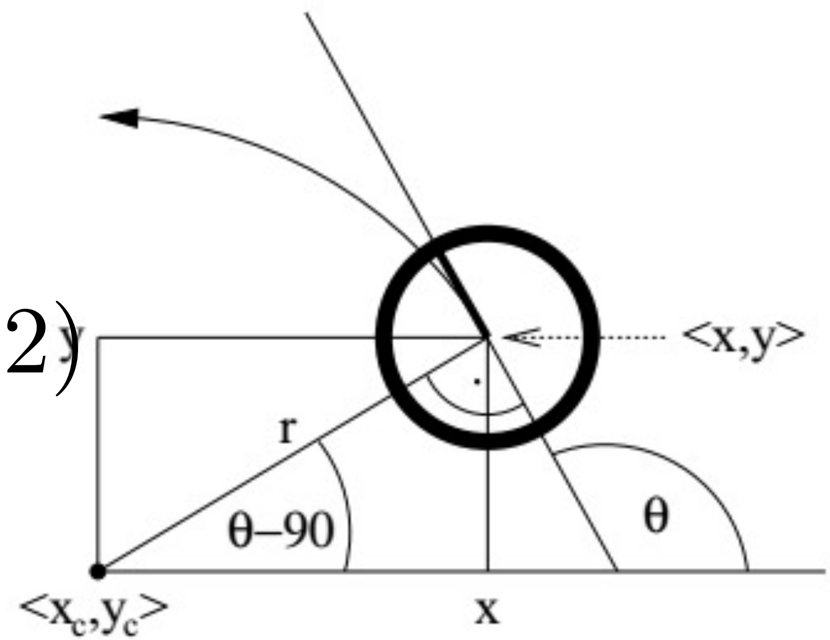

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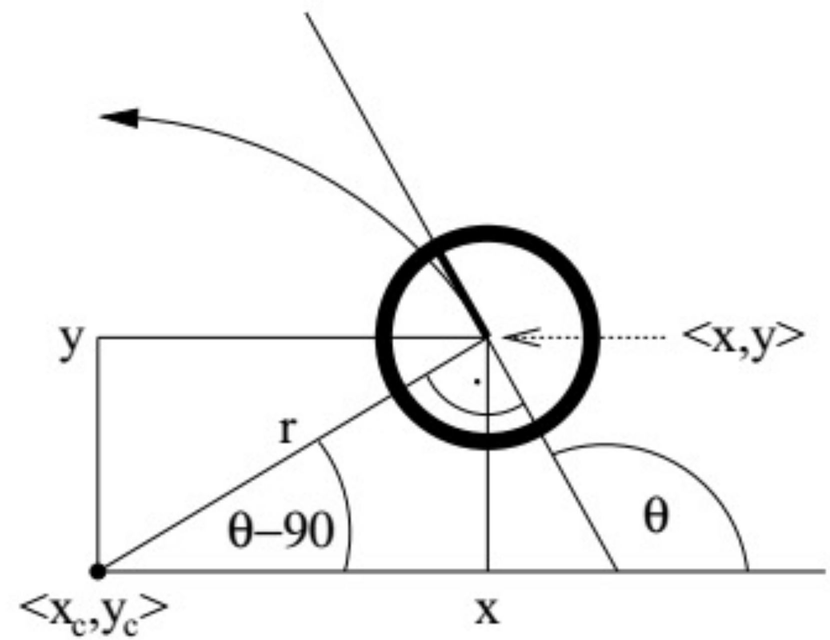

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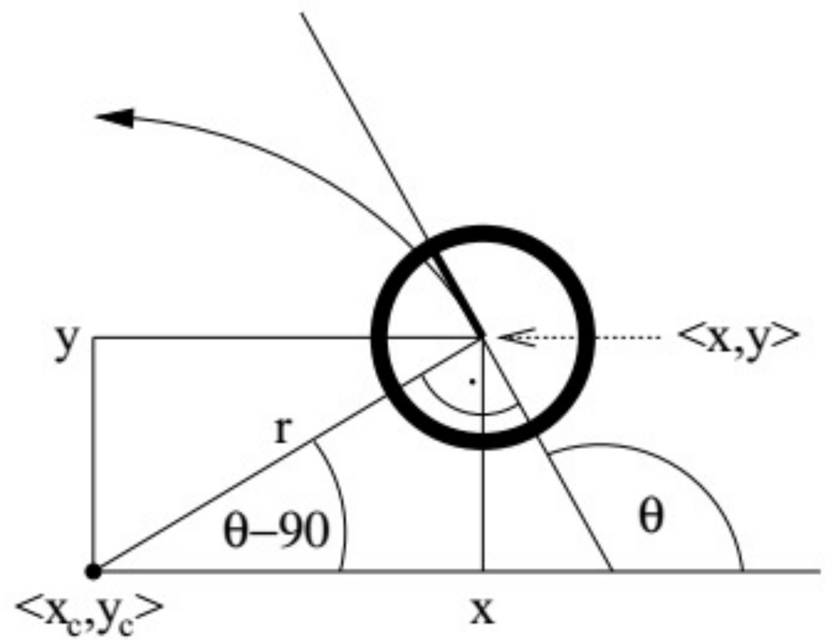
$$\theta_t = \theta_{t-1} + \omega_t * \Delta_t$$



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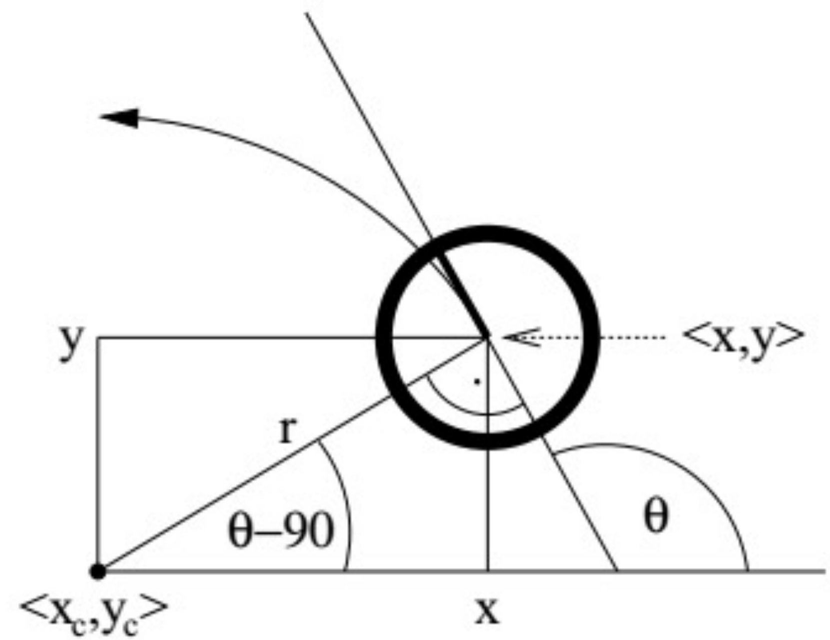

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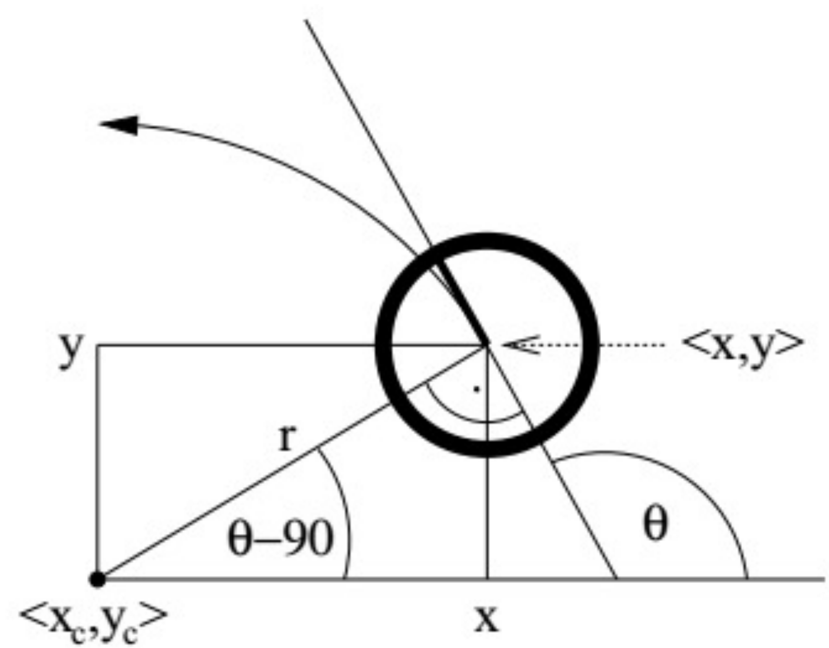

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$$x_t = x_c + r_t sin(\theta_{t-1} + \omega_t * \Delta_t)$$

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$$\theta_t = \theta_{t-1} + \omega_t * \Delta_t$$

$$x_t = x_{t-1} - r_t sin(\theta_{t-1}) + r_t sin(\theta_{t-1} + \omega_t * \Delta_t$$

$$y_t = y_{t-1} + r_t cos(\theta_{t-1}) - r_t cos(\theta_{t-1} + \omega_t * \Delta_t$$

Velocity Motion Model

$$\begin{pmatrix} \hat{v}_t \\ \hat{\omega}_t \end{pmatrix} = \begin{pmatrix} v_t \\ \omega_t \end{pmatrix} + \begin{pmatrix} \epsilon_{a_1|v_t|+a_2|\omega_t|} \\ \epsilon_{a_3|v_t|+a_4|\omega_t|} \end{pmatrix}$$

$$\theta_t = \theta_{t-1} + \hat{\omega}_t * \Delta_t$$

$$x_t = x_{t-1} - \frac{\hat{v}_t}{\hat{\omega}_t} sin(\theta_{t-1}) + \frac{\hat{v}_t}{\hat{\omega}_t} sin(\theta_{t-1} + \hat{\omega}_t * \Delta_t)$$

$$y_t = y_{t-1} + \frac{\hat{v}_t}{\hat{\omega}_t} cos(\theta_{t-1}) - \frac{\hat{v}_t}{\hat{\omega}_t} cos(\theta_{t-1} + \hat{\omega}_t * \Delta_t)$$

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$$\theta_t = \theta_{t-1} + \hat{\omega}_t * \Delta_t$$

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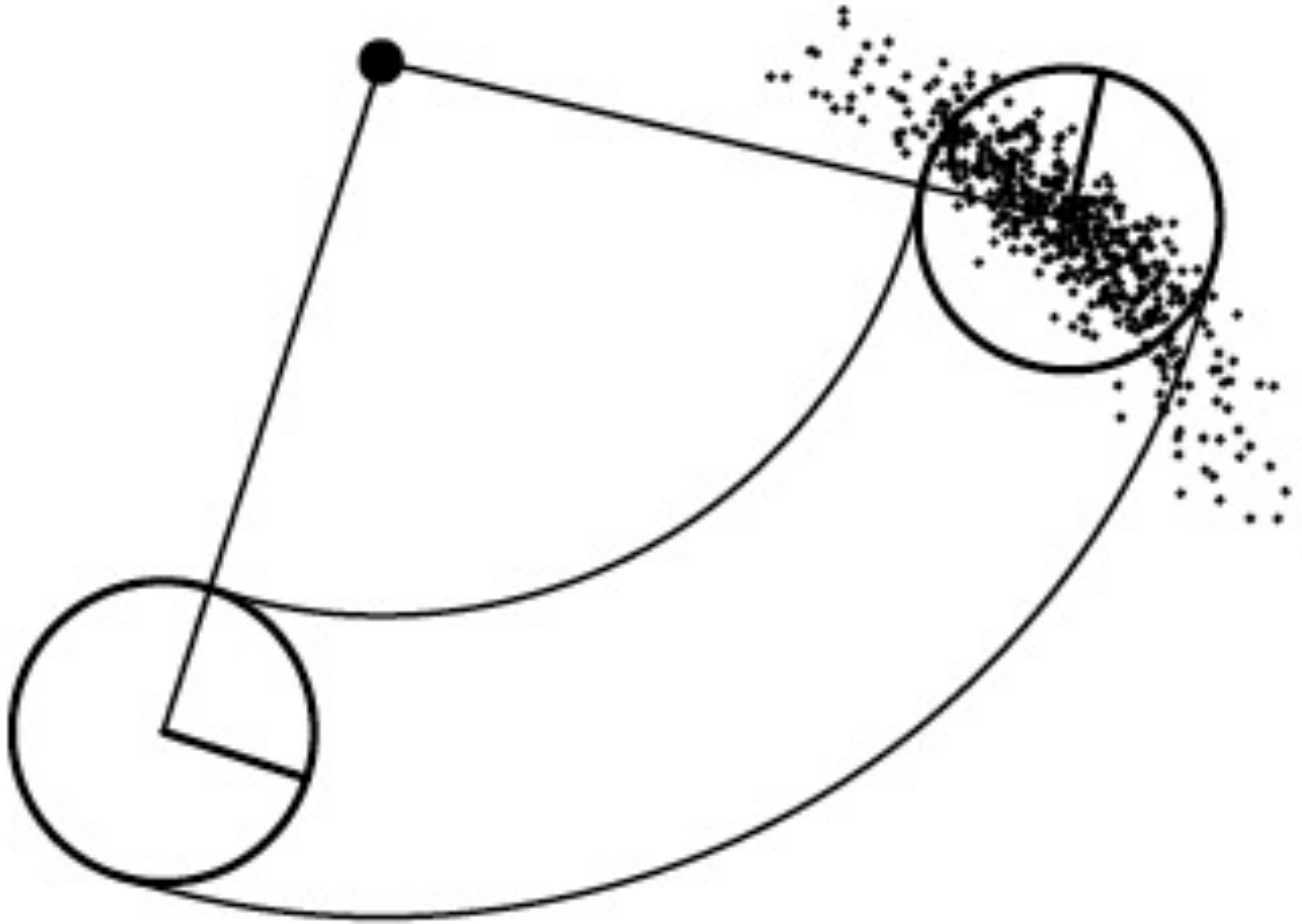
Velocity Motion Model

Algorithm sample_motion_model_velocity(u_t, x_{t-1})**:**

$\hat{v} = v + \textbf{sample}(\alpha_1|v| + \alpha_2|\omega|)$
$\hat{\omega} = \omega + \textbf{sample}(\alpha_3|v| + \alpha_4|\omega|)$
$\hat{\gamma} = \textbf{sample}(\alpha_5|v| + \alpha_6|\omega|)$
$x' = x - \frac{\hat{v}}{\hat{\omega}} \sin\theta + \frac{\hat{v}}{\hat{\omega}}\ sin(\theta + \hat{\omega}\Delta t)$
$y' = y + \frac{\hat{v}}{\hat{\omega}} \cos\theta - \frac{\hat{v}}{\hat{\omega}}\ cos(\theta + \hat{\omega}\Delta t)$
$\theta' = \theta + \hat{\omega}\Delta t + \hat{\gamma}\Delta t$
return $x_t = (x', y', \theta')^T$

Velocity Motion Model

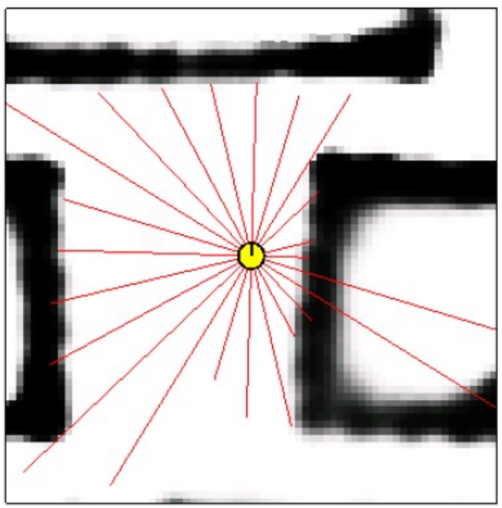


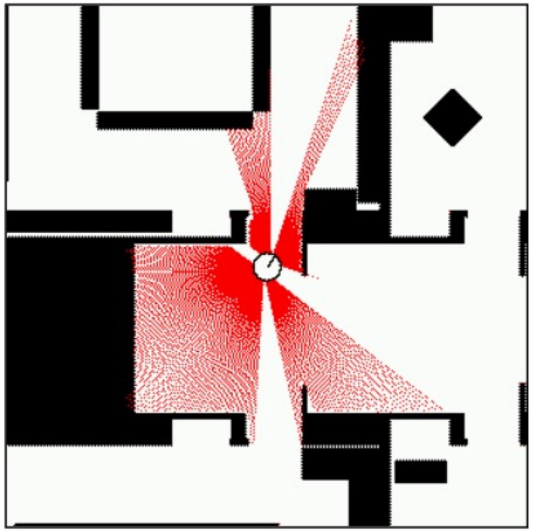
Sensor Model

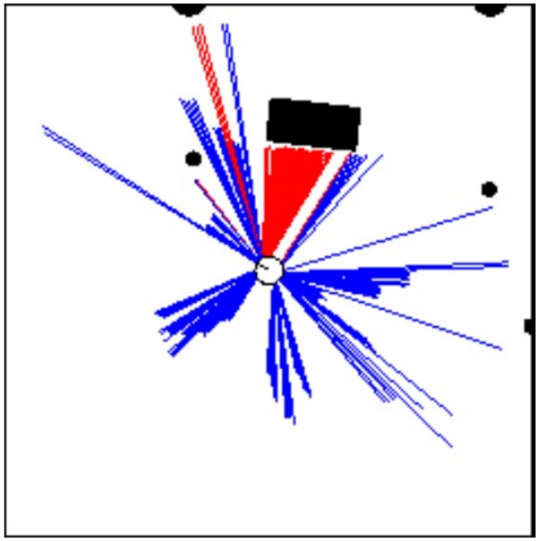
- What is the observation function?

$$P(z_t|x_t)$$

LiDAR Measurements







Measurements

$$z_t = \{z_t^1, z_t^2, ..., z_t^k\}$$

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$$p(z_t | x_t)$$

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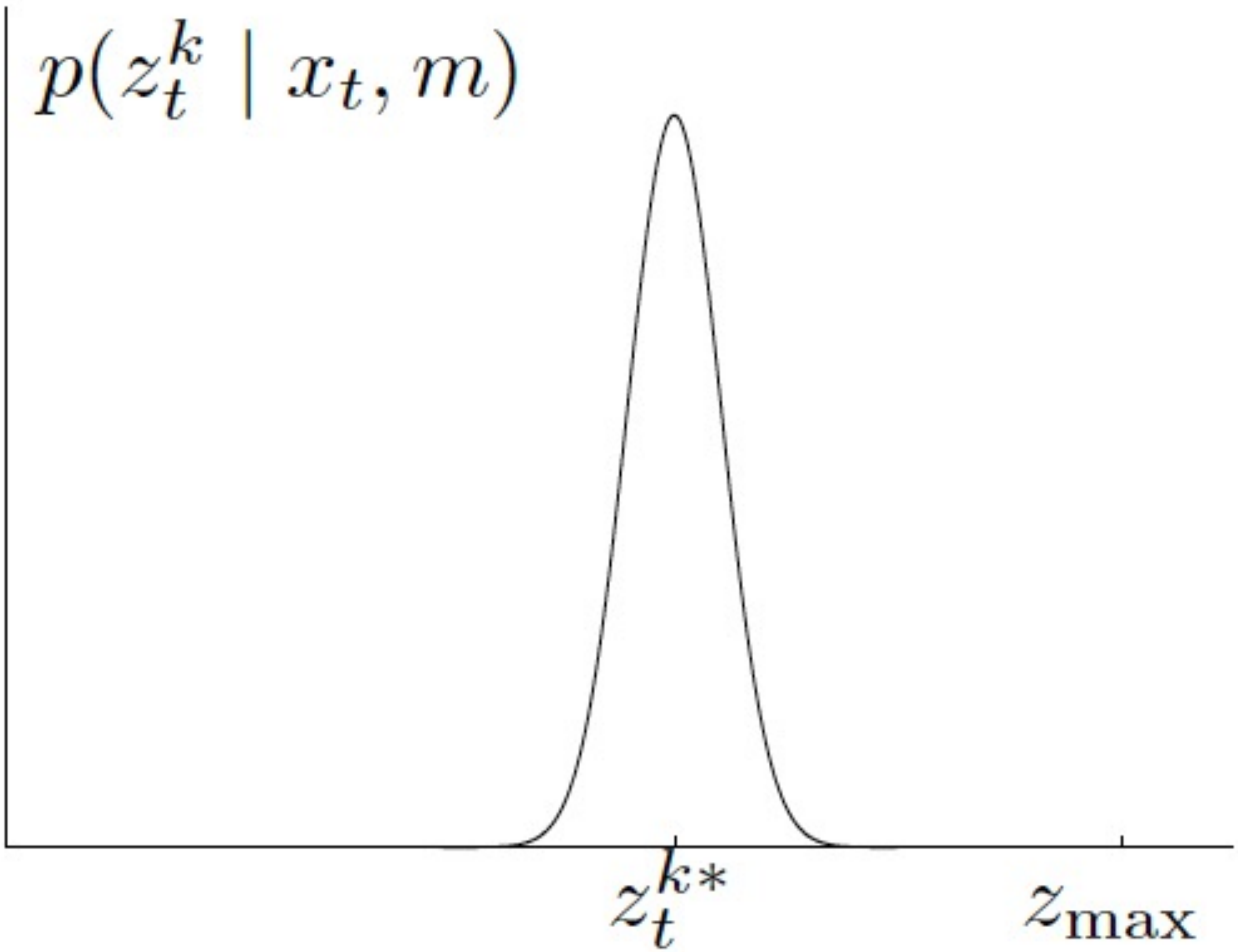
$$p(z_t | x_t, m)$$

Measurements

$$z_t = \{z_t^1, z_t^2, ..., z_t^k\}$$

$$p(z_t|x_t, m) = p(z_t^1|x_t, m)p(z_t^1|x_t, m)...p(z_t^k|x_t, m).$$

Sources of Error: Local Noise



Other Sources of Error:

- Unexpected Objects

- No Detection

- Random Errors

Feature-Based Sensor Models

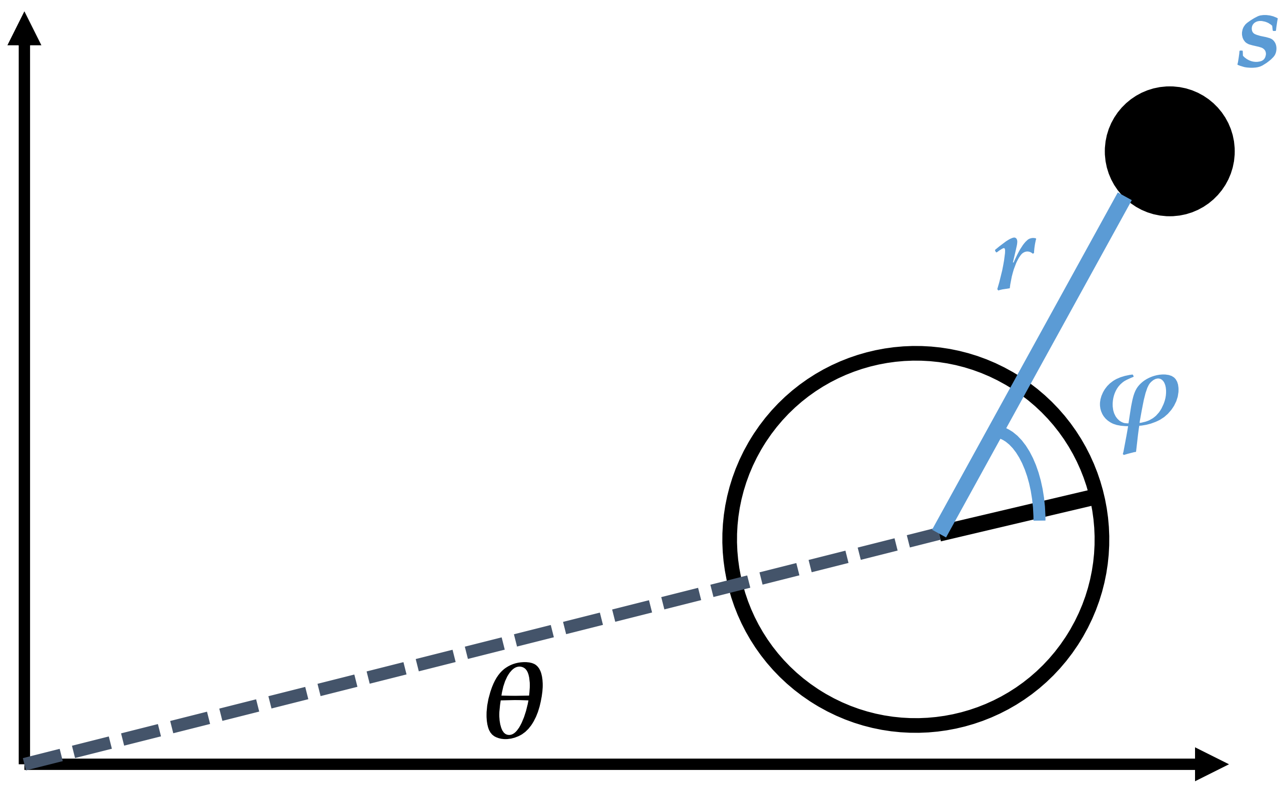
Fully Autonomous Robot Navigation based on a Prior Map without Artificial Landmarks

Feature-Based Sensor Models

$$p(z_t|x_t, m) = p(z_t^1|x_t, m)p(z_t^1|x_t, m)...p(z_t^k|x_t, m).$$

$$p(z_t|x_t, m) = \Pi_{i=1}^k p(r_t^i, \phi_t^i, s_t^i|x_t, m)$$

Feature-Based Sensor Models



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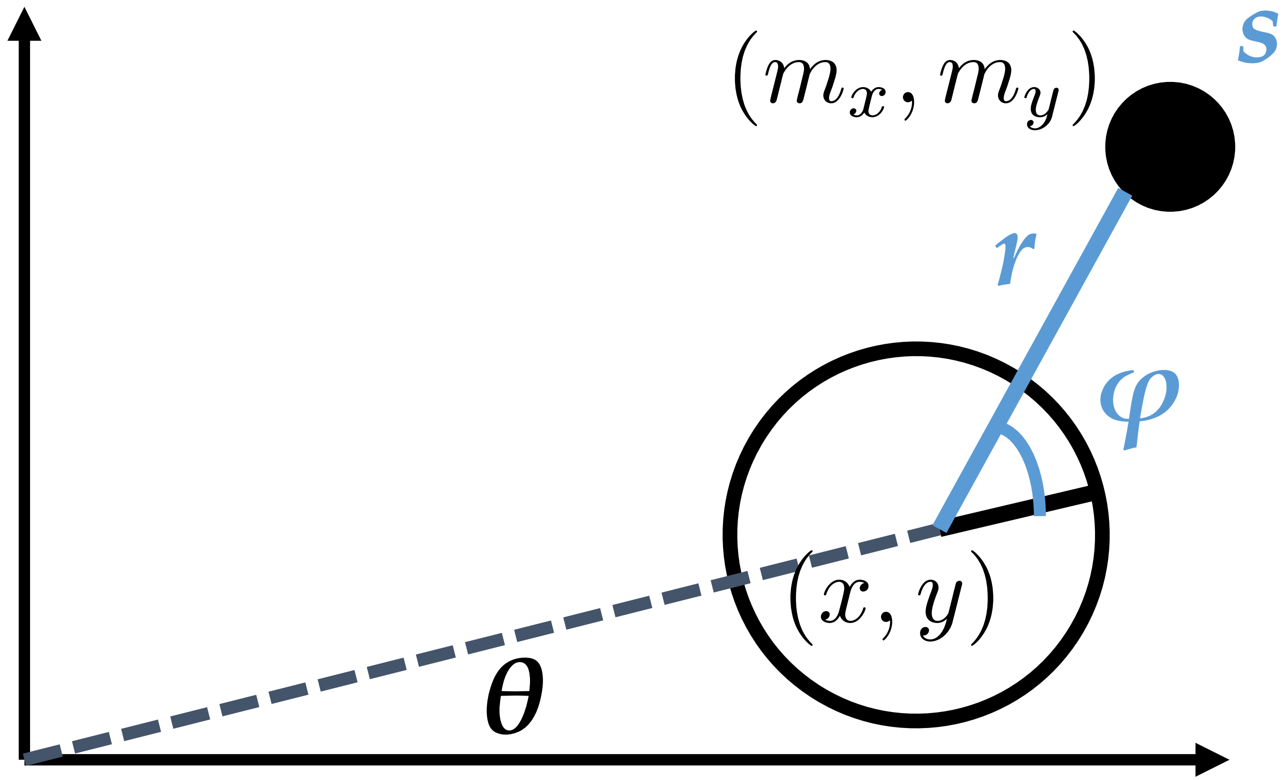

Feature-Based Sensor Models

$$\begin{pmatrix} r_t^i \\ \phi_t^i \\ s_t^i \end{pmatrix} = \begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \text{atan2}(m_{j,y} - y, m_{j,x} - x) - \theta \\ s_j \end{pmatrix} + \begin{pmatrix} \varepsilon_{\sigma_r^2} \\ \varepsilon_{\sigma_\phi^2} \\ \varepsilon_{\sigma_s^2} \end{pmatrix}$$