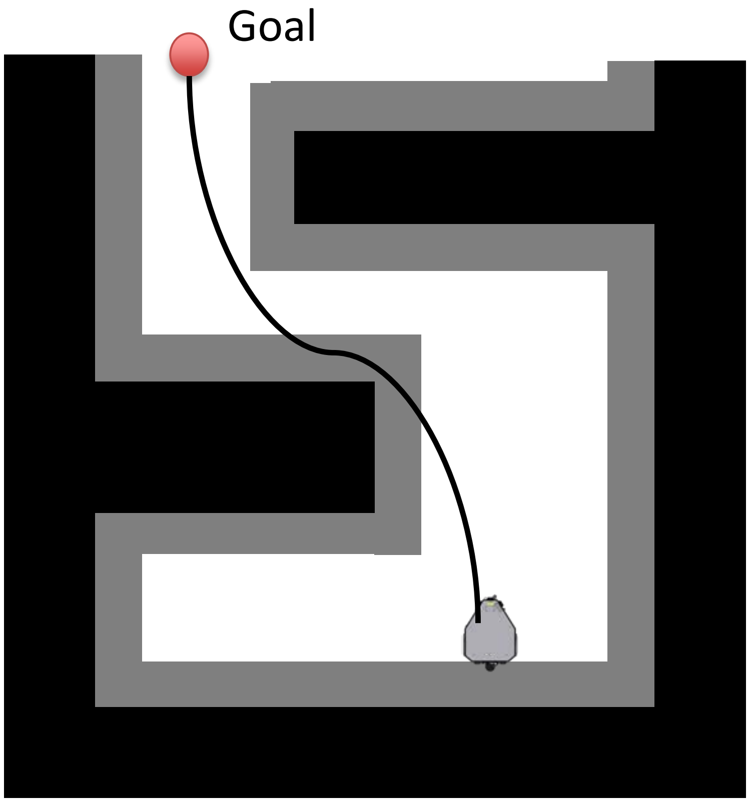
16-843 – Manipulation Algorithms

Configuration Spaces

Katharina Muelling

Problem Definition

Consider the problem of robot motion planning:



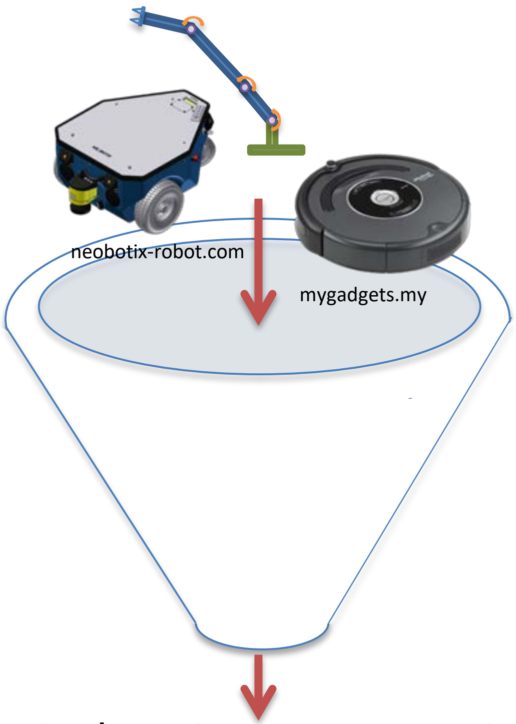
How to avoid collisions of parts of the robot with the environment if modeled as point?

Robot:

neobotix-robot.com

Outline

Can we create a space in which all robots can be treated as points?



Single point representation

Configuration Space

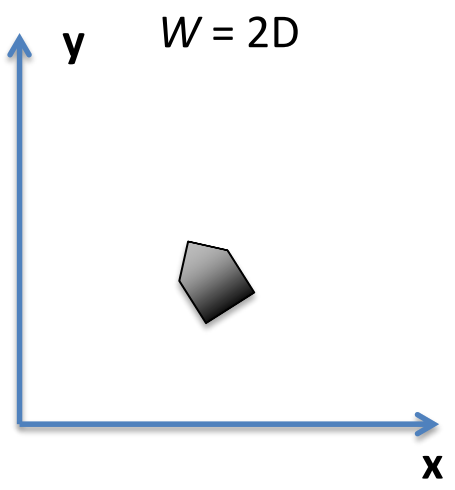
Algorithms

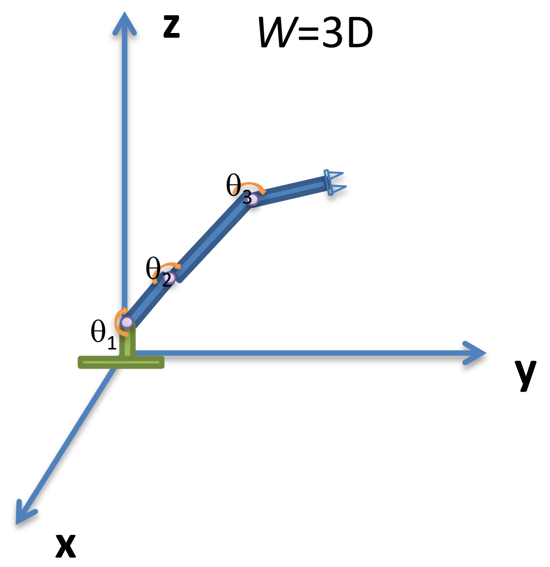
Configuration Space

Definitions

Work space: The world in which the robot lives and occupies space.

The set of reachable points by an end-effector.





3-parameter specification (x,y,θ)

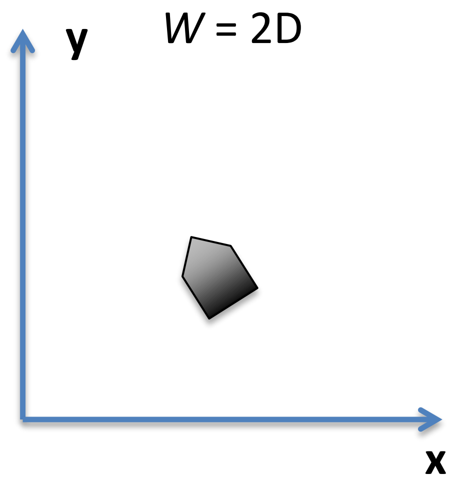
6-parameter specification $(x,y,z,\alpha,\beta,\gamma)$

Configuration Space

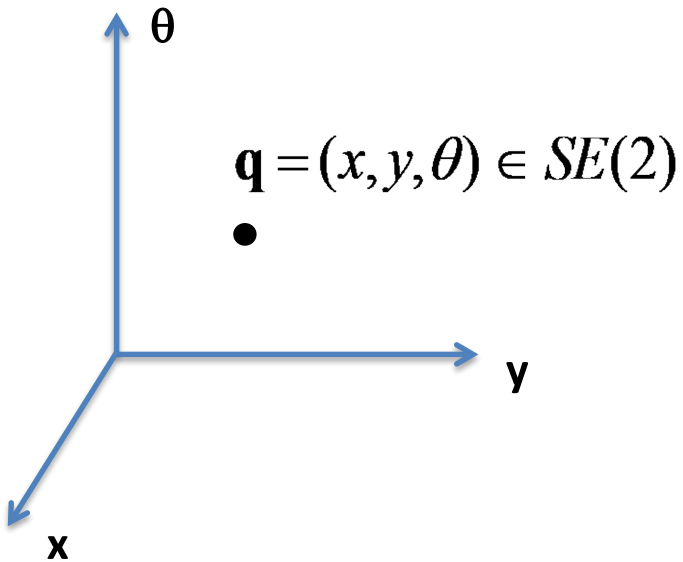
Definitions

Configuration:

The **configuration** of a robot is the sufficient and complete specification of the position of every point of the physical system



World space



Configuration Space

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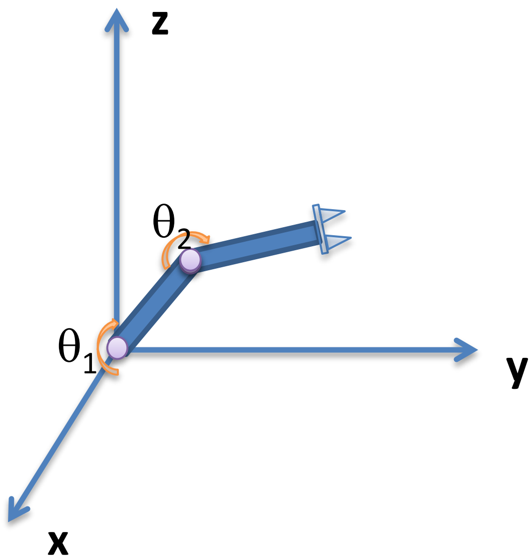
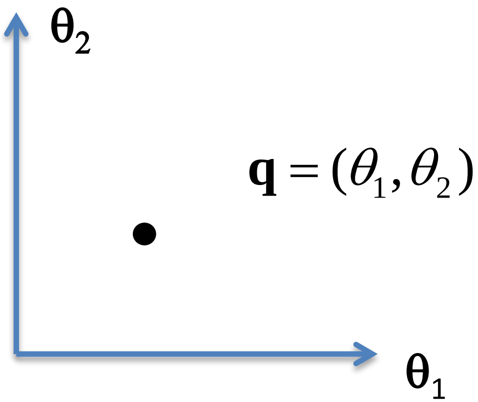


World space



Configuration Space

Configuration Space

Definitions

Configuration:

The **configuration** of a robot is the sufficient and complete specification of the position of every point of the physical system

The configuration **q** is usually expressed as a vector of length n, where n defines the degree of freedom of the robot

$$\mathbf{q} = [q_1, q_2, \ldots, q_n]$$

Configuration Space:

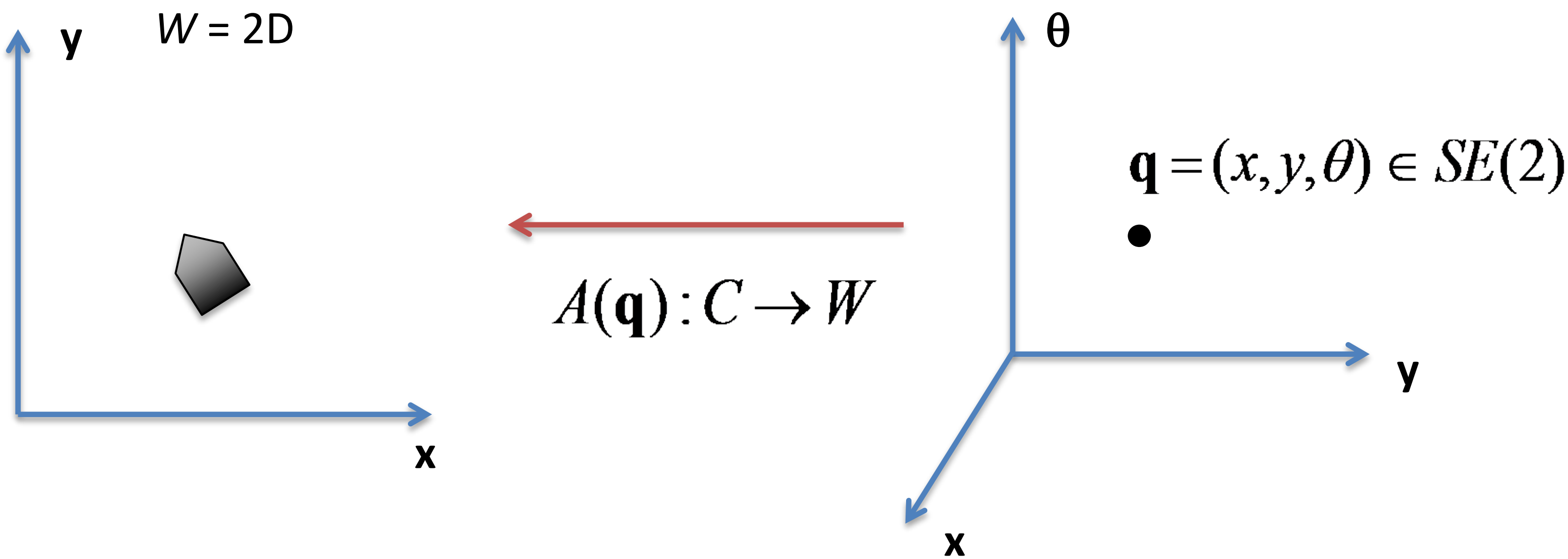
The set *C* of all possible configurations.

$$\mathbf{q} \in C$$

Read:
Spatial Planning: a Configuration Space
T. Lozano-Perez, 1980

Configuration Space

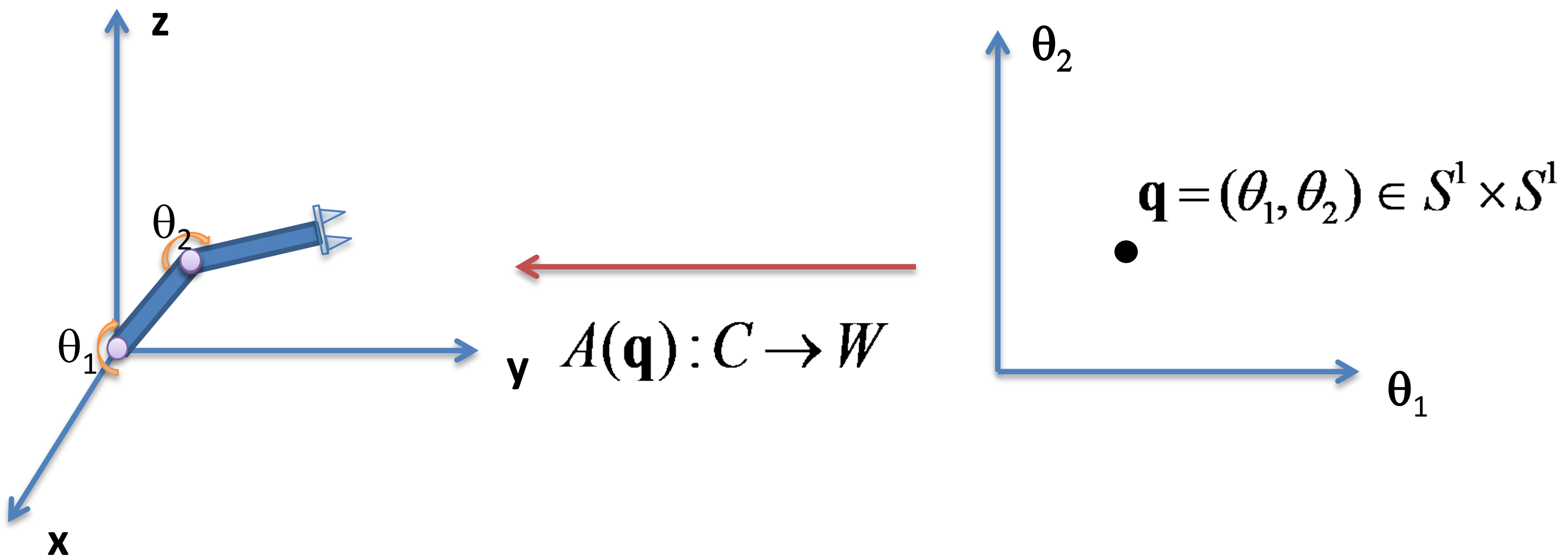
<u>Examples</u>



A(q) maps from point to the physical robot's location in $\mathbb{R}^2$

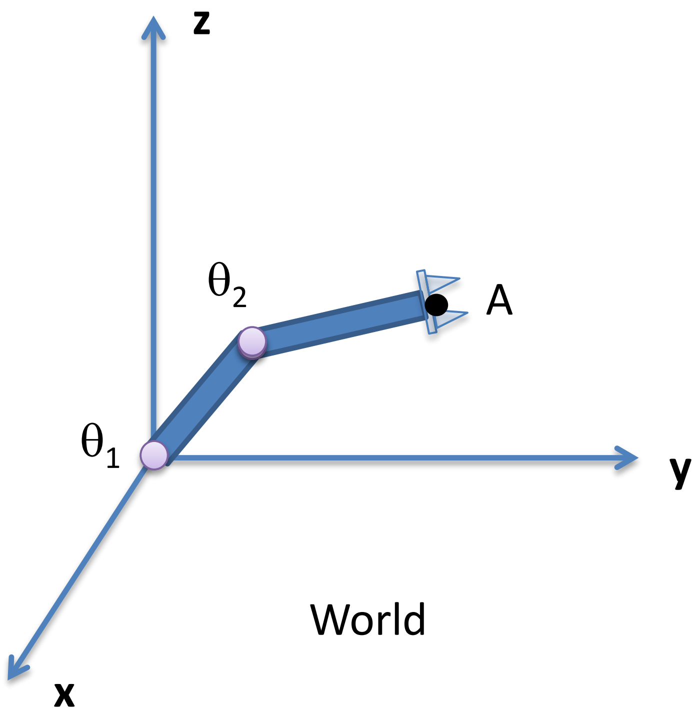
Configuration Space

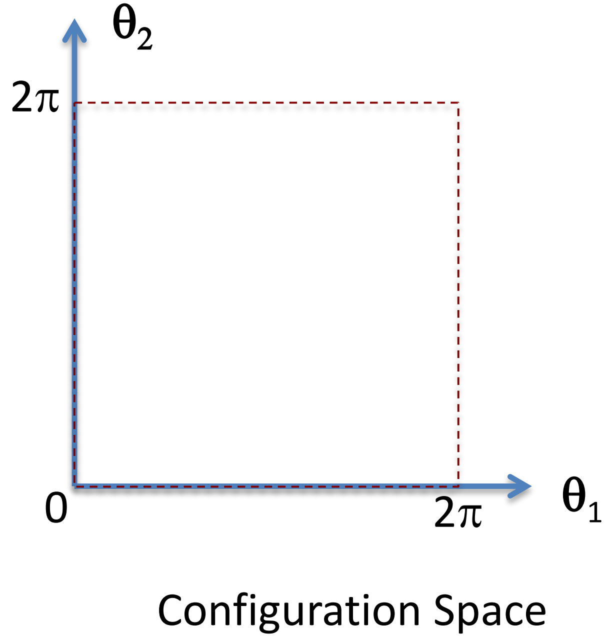
Examples



A(q) maps from point to the physical robot's location in $\mathbb{R}^3$

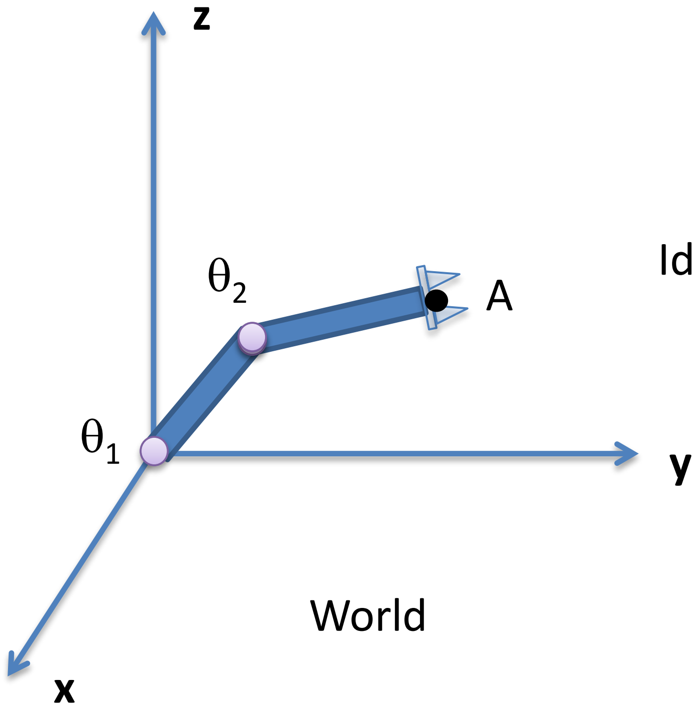
Configuration Space





Topology of this space is in not necessarily the one of a Cartesian space.

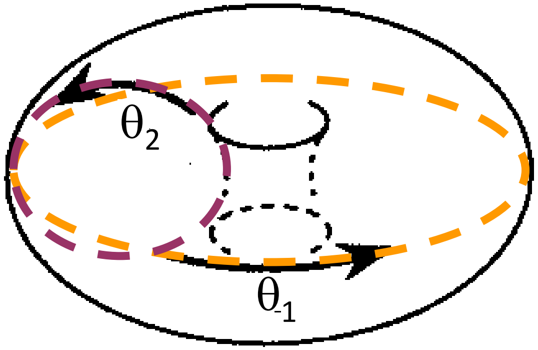
Configuration Space



World

Identifying 0 and 2π

$$C = S^1 \times S^1$$



Configuration Space

Topology of this space is in not necessarily the one of a Cartesian space.

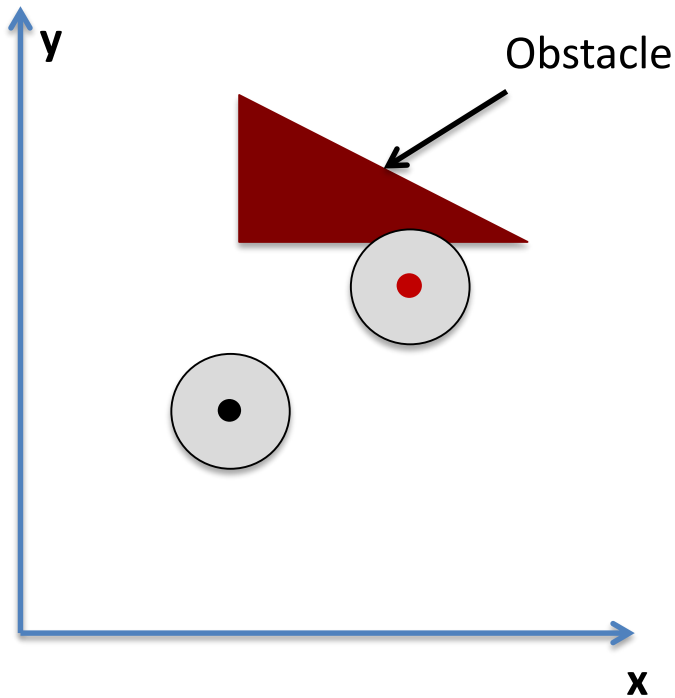
Motion Planning

Motion planning:
Instead of framing the motion planning problem as finding a path in the work space, we can define it as finding a path in the configuration space.

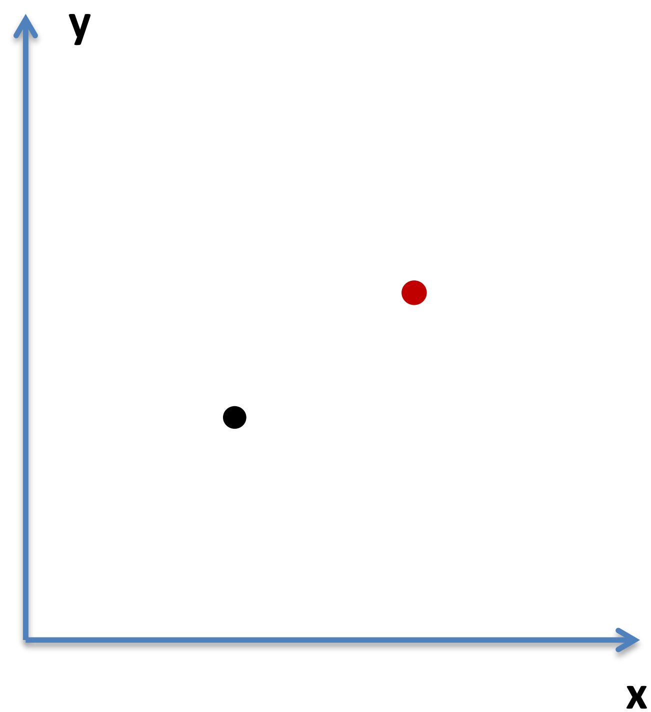
Problem:
1) A World
2) An obstacle region $O \subset W$
3) A robot A defined in *W:* $A \subset W$
4) Find a collision free path from $\mathbf{q}_I \in C$ to $\mathbf{q}_G \in C$

Configuration Space Obstacles

Where do we put the obstacle in configuration space?



World



Configuration Space

$$\mathbf{q} = (x, y) \in \Re^2$$

Configuration Space Obstacles

Obstacle representation in configuration space

- Obstacles make certain configurations in configuration space impossible
- Remove the configurations in configuration space that cause the robot to collide with obstacles or cause some specific links of the robot to collide

Obstacle Region:

$A \subset W$ $\quad$ $O \subset W$

$$C_{obs} = \{q \in C \mid A(q) \cap O \neq \varnothing\}$$

The world contains obstacle region *O* and rigid robot *A*

Free Space:

$$C_{free} = C \setminus C_{obs}$$

Note: C_{free} is open set. Sometimes need to consider closure of C_{free}.

Configuration Space Obstacles

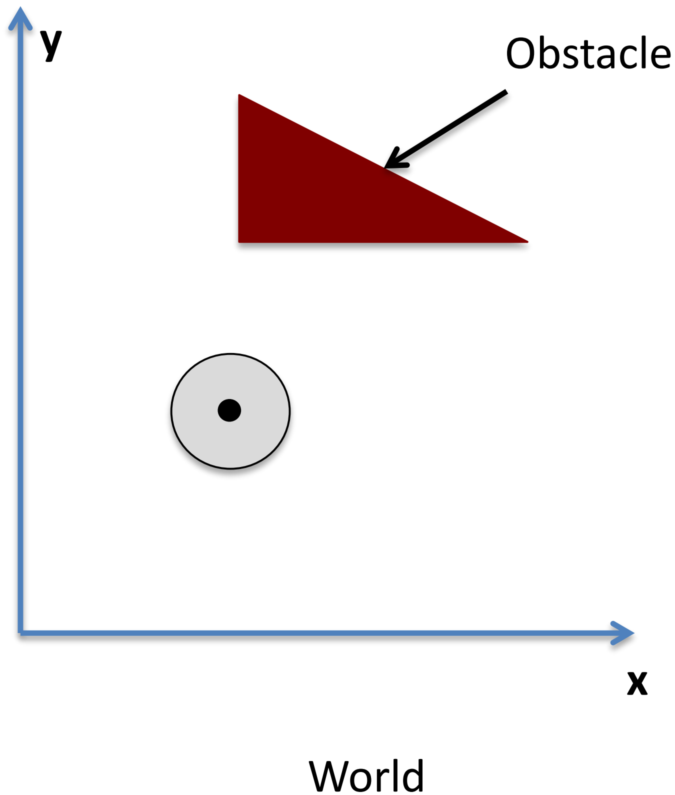
Motion planning:
Find a path in *C* such that no configuration along the path intersects with an obstacle

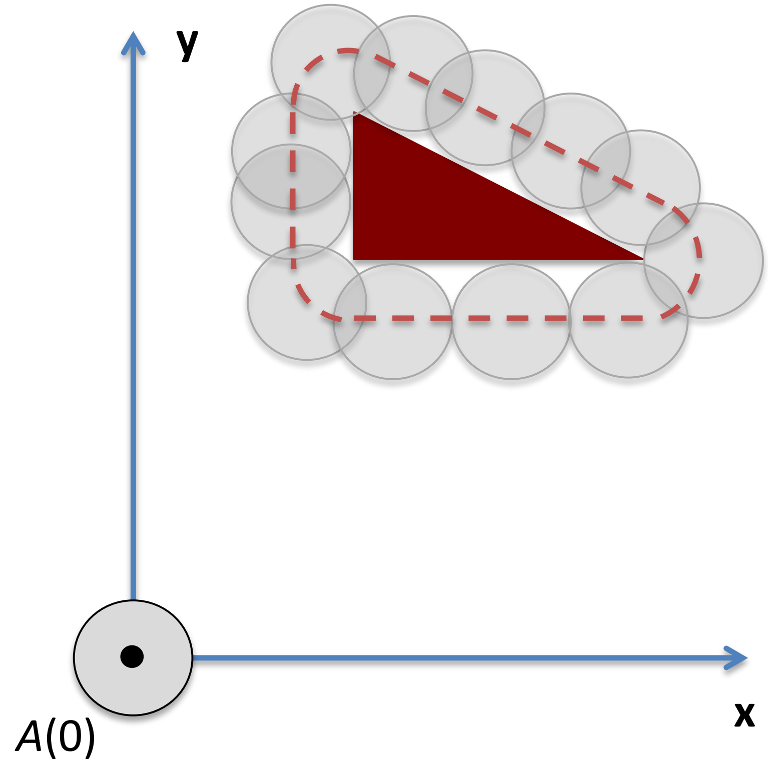
The Piano Mover's Problem:

1) A World $W = \Re^2$ or $W = \Re^3$
2) An obstacle region $O \subset W$
3) A robot A defined in *W:* $A \subset W$
4) $C_{free} = C \setminus C_{obs}$
5) A configuration $\mathbf{q}_I \in C_{free}$ as initial configuration and $\mathbf{q}_G \in C_{free}$ as goal configuration (query pair)
6) Compute continuous path $\tau : [0,1] \to C_{free}$ such that $\tau(0) = \mathbf{q}_I$ and $\tau(1) = \mathbf{q}_G$

Configuration Space Obstacles

Where do we put the obstacle in configuration space?





$$\mathbf{q} = (x, y) \in \Re^2$$

Configuration Space Obstacles

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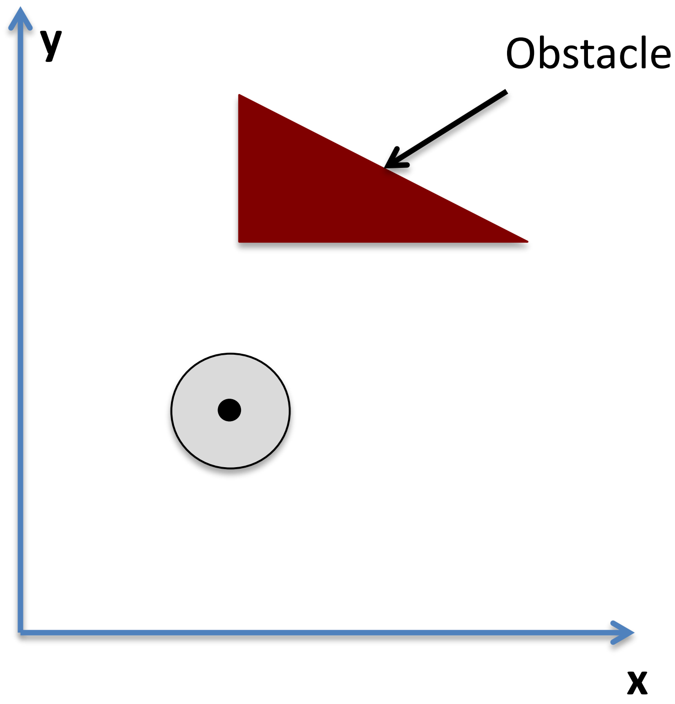
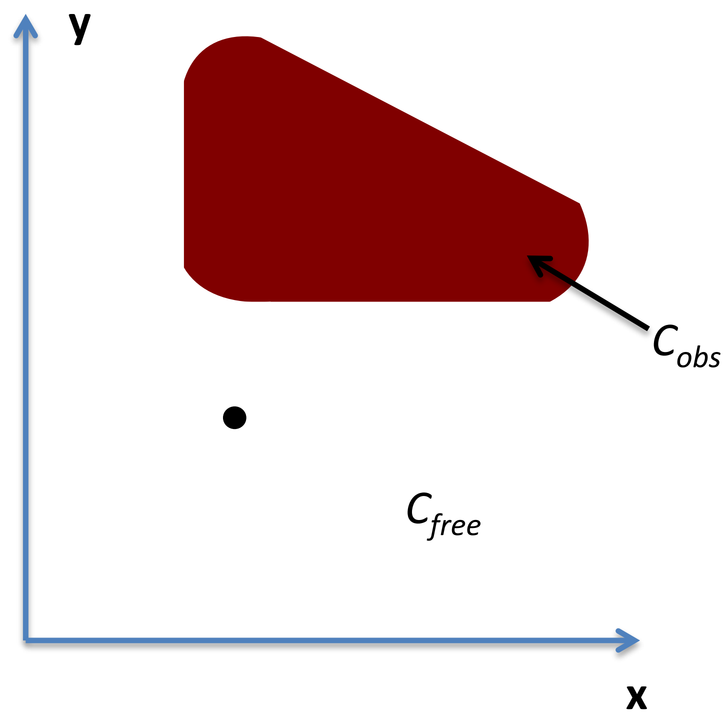


World



Configuration Space

$$\mathbf{q} = (x, y) \in \Re^2$$

Configuration Space Obstacles

Star algorithm

2D Workspace, translation

C_{obs}: convex polygon

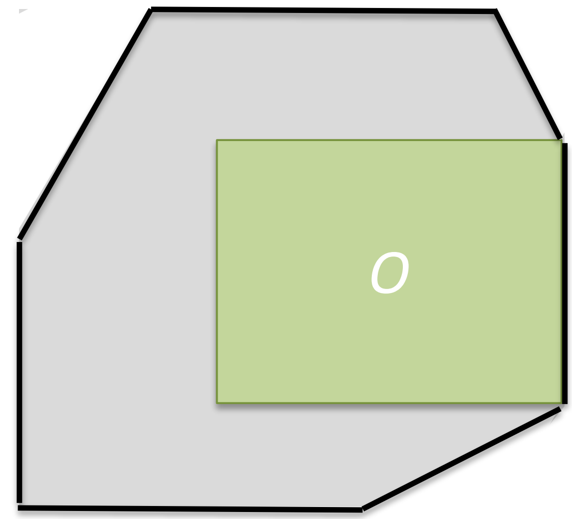
So far: identified edges

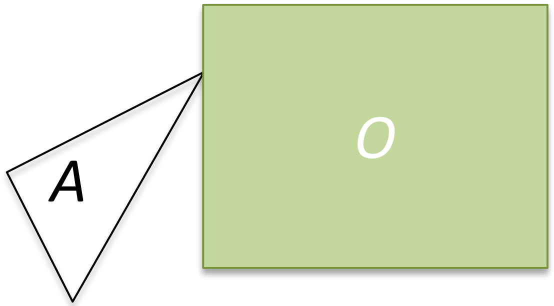
Left: solid representation of C_{obs}

Idea: Create convex obstacles by intersecting half-planes

Two types of contacts:

- Vertex of *A* and Edge of *O*





Configuration Space Obstacles

Star algorithm

2D Workspace, translation

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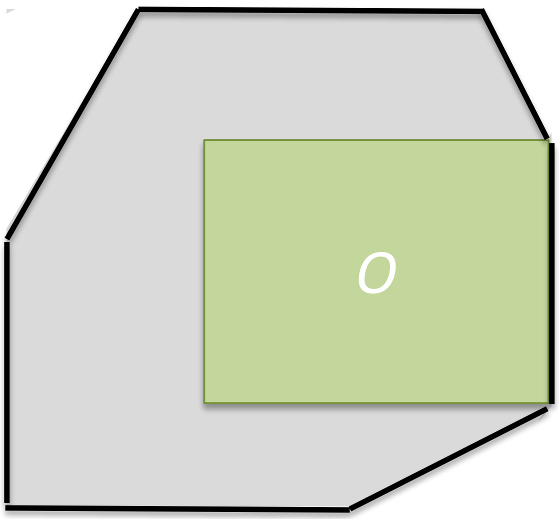
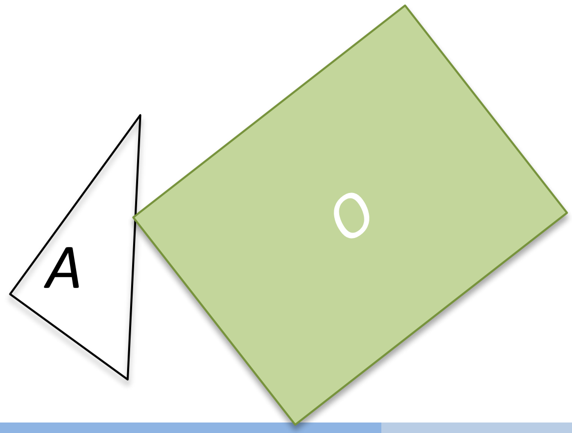
Two types of contacts:

- Vertex of *A* and Edge of *O*
- Edge of *A* and Vertex of *O*





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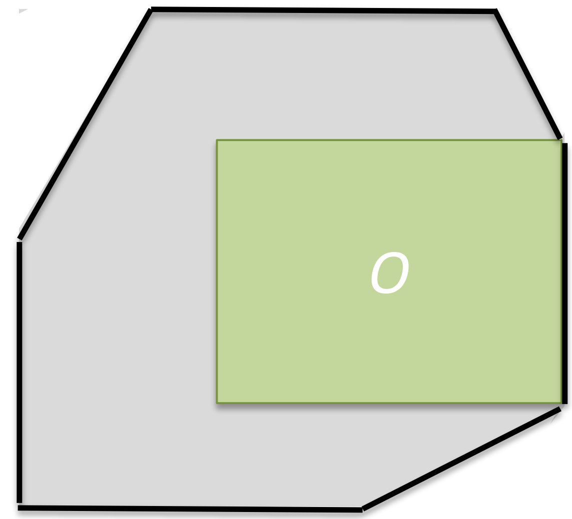
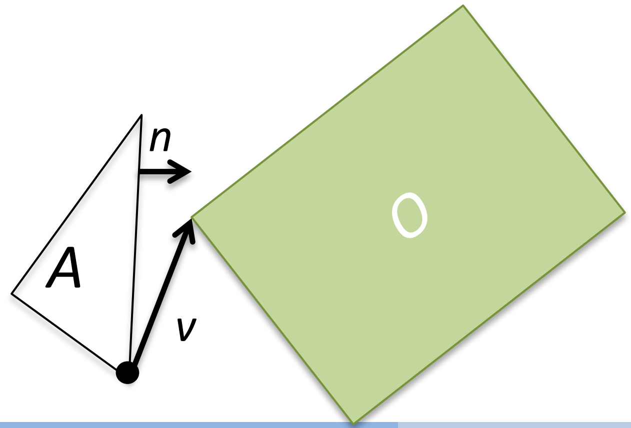


Contact occurs when *n* and *v* are perpendicular

$$n \cdot v = 0$$



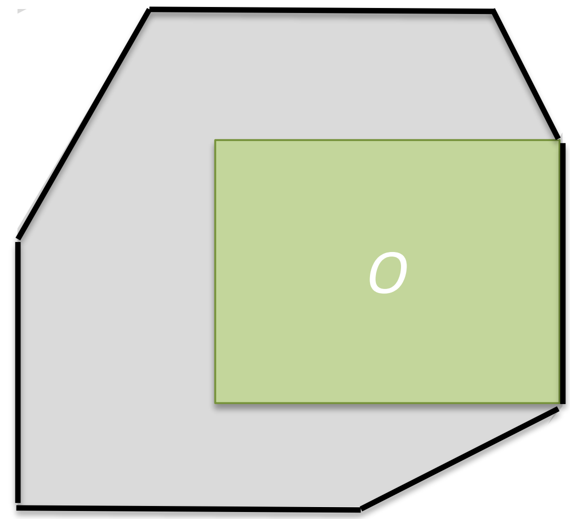
Configuration Space Obstacles

See example: Planning Algorithms, Chapter 4.3 (page 155) by Lavalle (link provided on web site)

<u>**Star algorithm**</u>

2D Workspace, translation

C_{obs}: convex polygon



So far: identified edges

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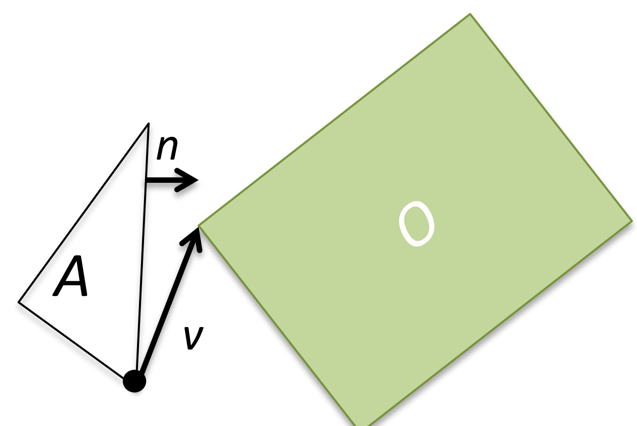
Idea: Create convex obstacles by intersecting half-planes

Contact occurs when *n* and *v* are perpendicular

$$n \cdot v(x_t, y_t) = 0$$

Function of kinematics
f(q) <=0

$$H = \{(x_t, y_t) \in C \mid n \cdot v(x_t, y_t) \leq 0\}$$

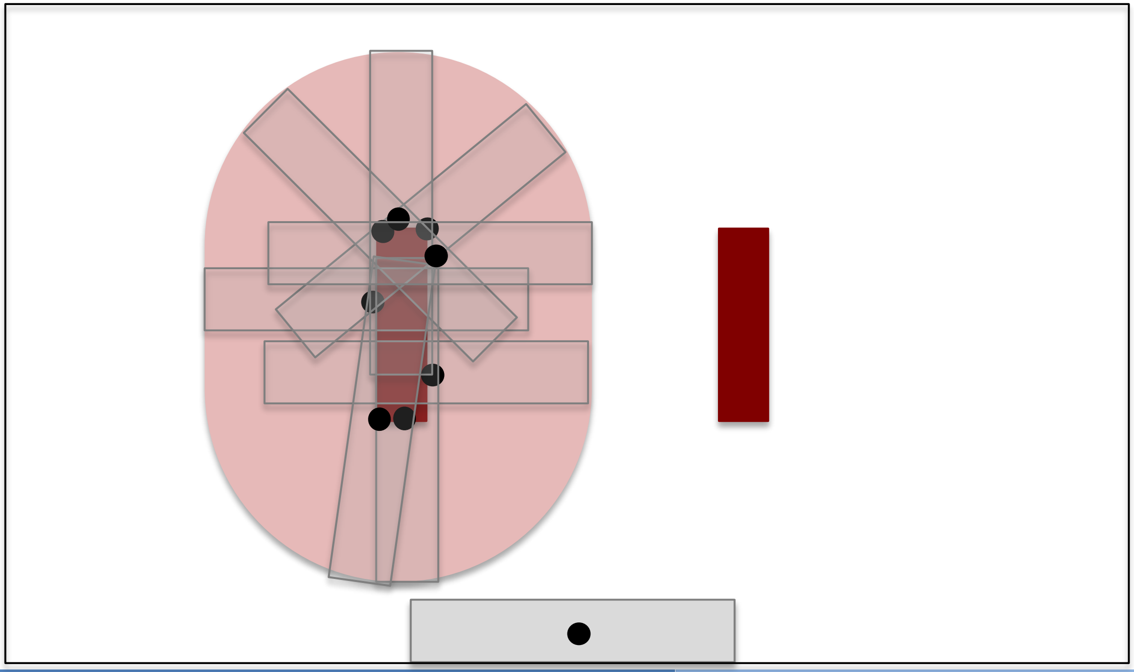


Configuration Space Obstacles

Star algorithm
3D Workspace, translation
C_{obs}: polyhedral

generalizes

What happens when the robot translates *and* rotates in the plane?



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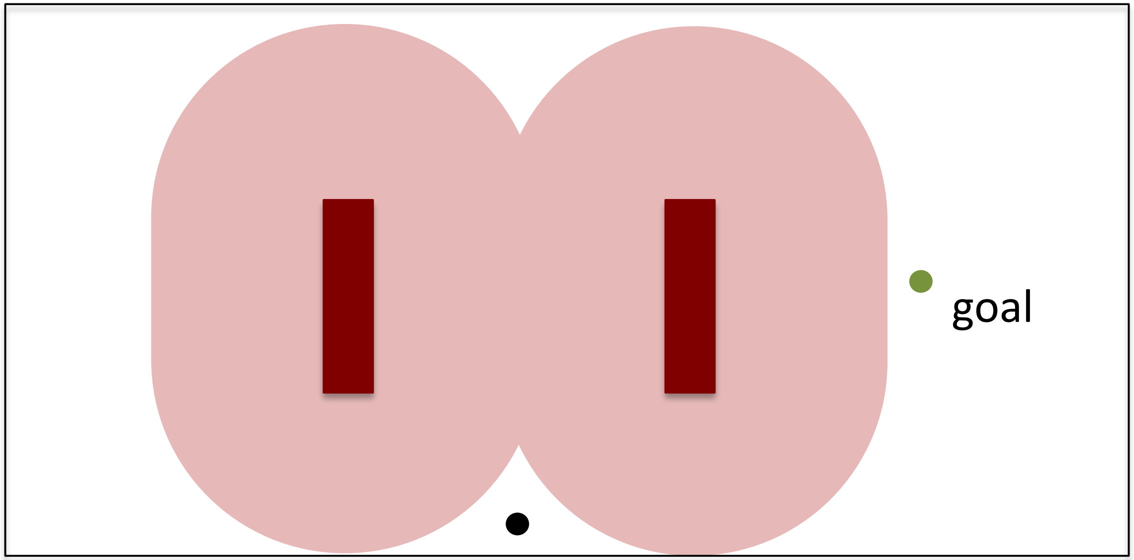

Need to compute C_{obs} based on the third dimension θ

Configuration Space Obstacles

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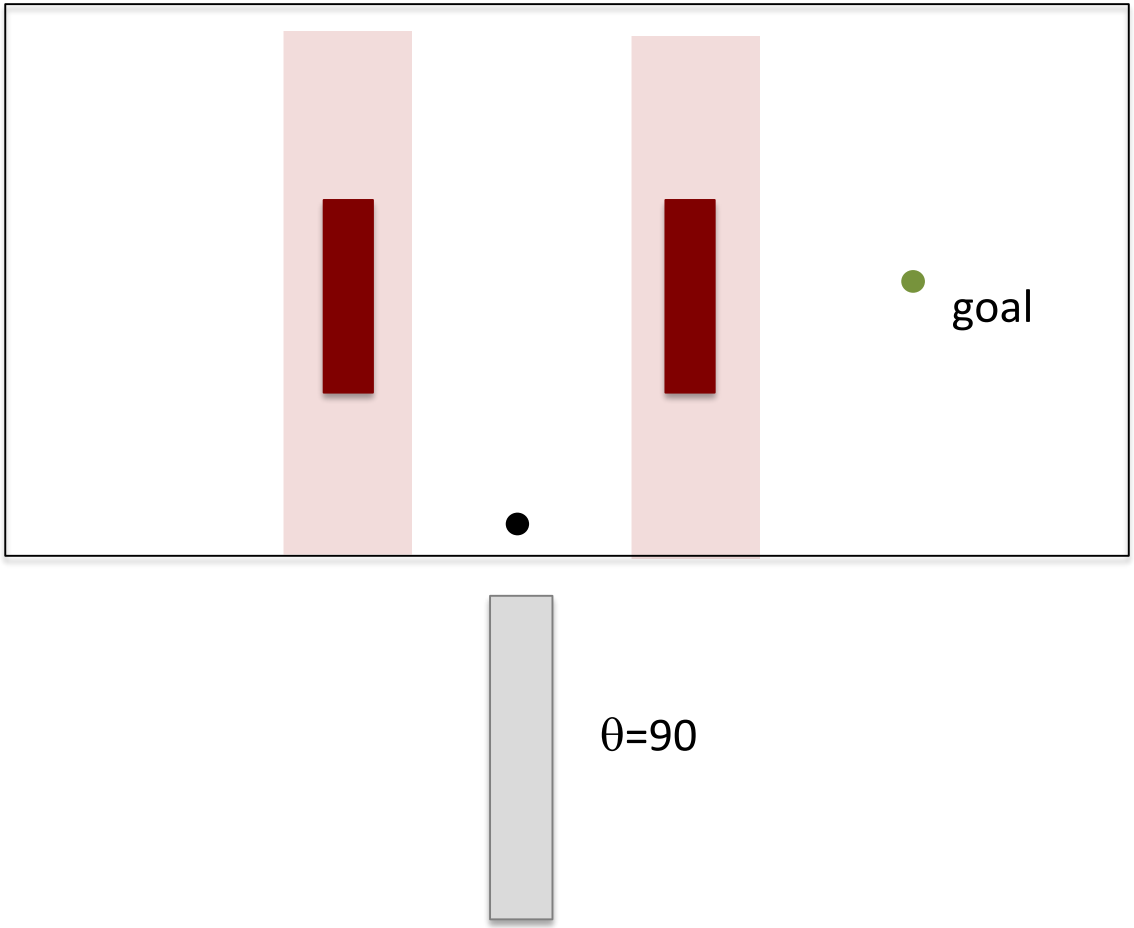

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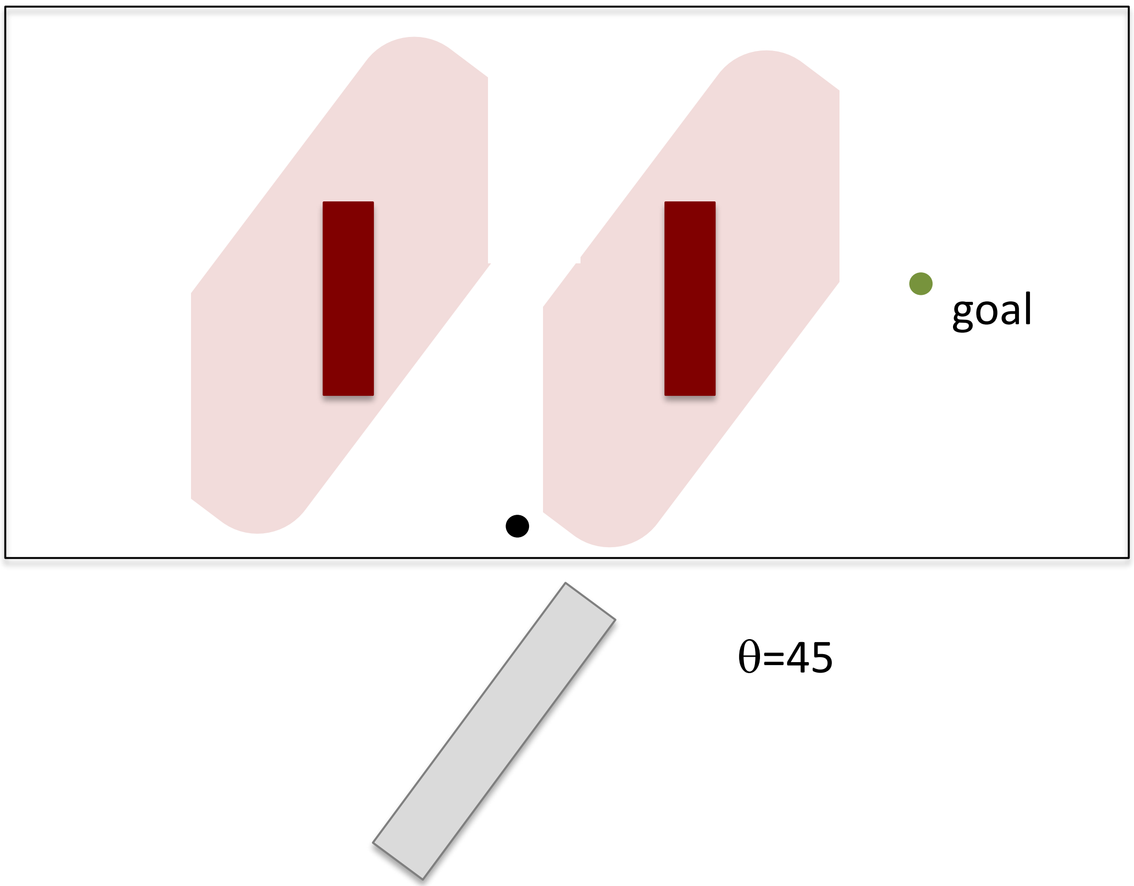

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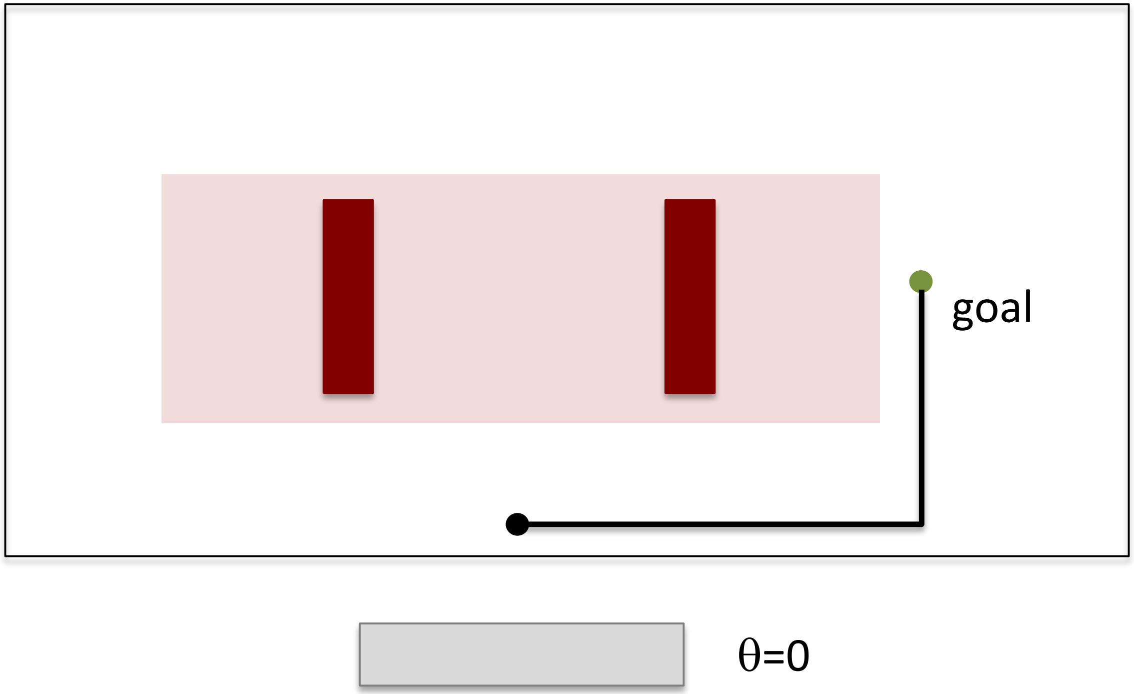

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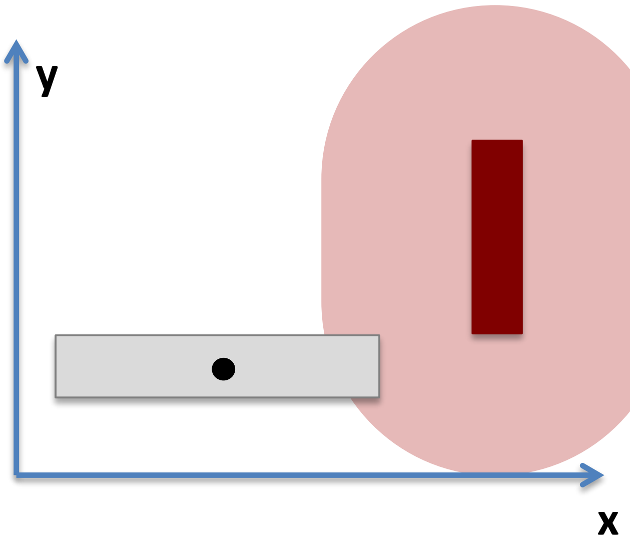
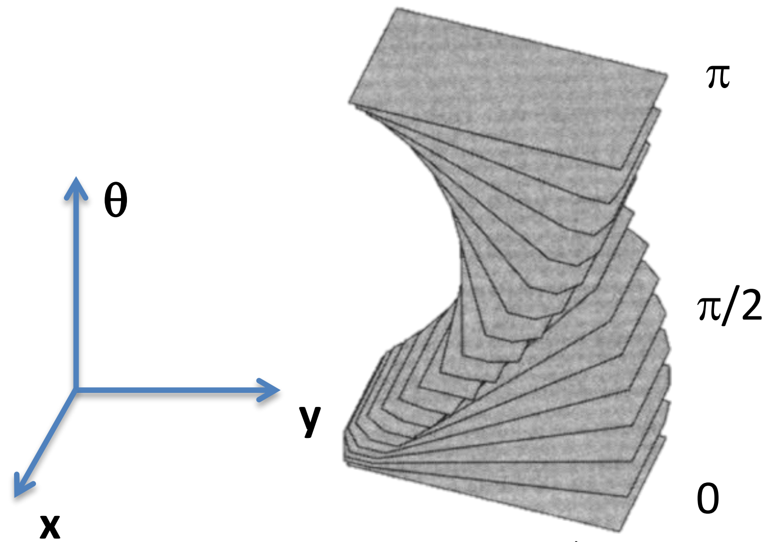




From Howie Choset

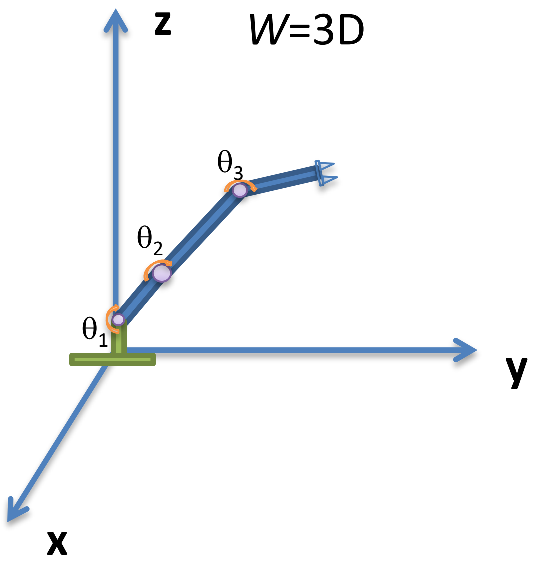
Need to consider orientation, i.e., need to consider all possible contact interactions between all pairs of object and robot features!

Problem: Exponential run times!

Articulated Robots

Types of collisions:

- **Objects**: collision with objects make certain configuration in configuration space unattainable due to possible collision
- **Robot**: links of the robot collide with each other



- World $W = \mathbb{R}^2$ or $W = \mathbb{R}^3$
- Obstacles $O \subset W$
- Robot $A = \{A_1, A_2, \dots, A_n\}$

Consider each link independently: $A_i(\mathbf{q})$

Collision pairs: $(i, j) \in P \quad \forall i \neq j$

Self collision:

$$\bigcup_{[i,j]\in P} \{q \in C \mid A_i(q) \cap A_j(q) \neq \emptyset\}$$

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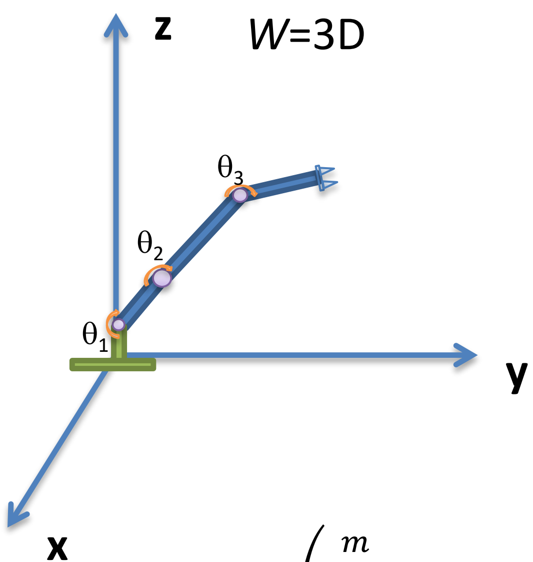

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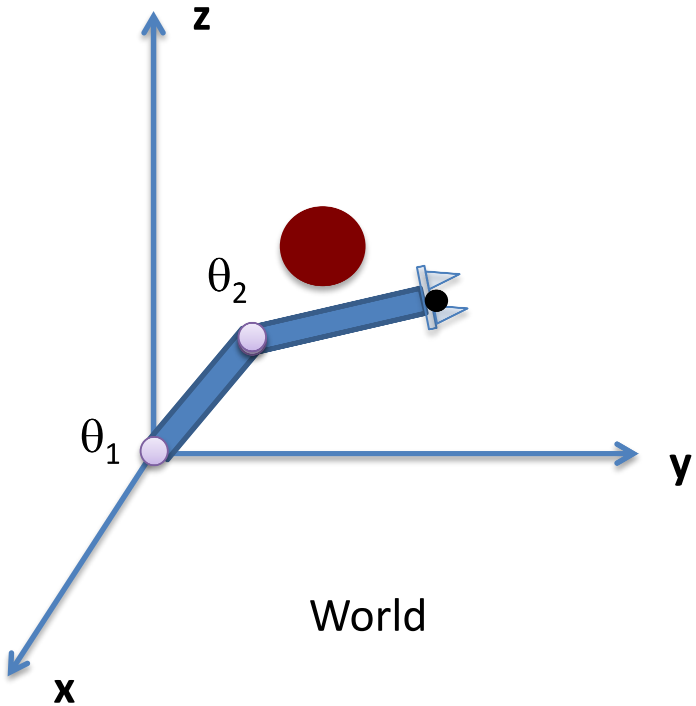
Consider each link independently: $A_i(\mathbf{q})$

Link – Object collision:

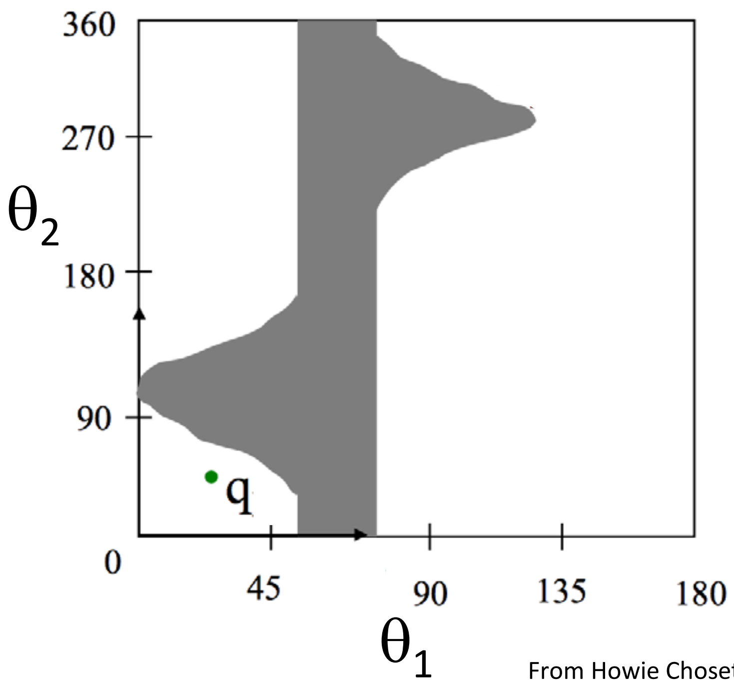
$$\cup_{i=1}^{m} \{q \in C \mid A_i(q) \cap O \neq \emptyset\}$$

$$C_{obs} = \left(\bigcup_{i=1}^{m} \{q \in C \mid A_i(q) \cap O \neq \emptyset\} \right) \cup \left(\bigcup_{[i,j] \in P} \{q \in C \mid A_i(q) \cap A_j(q) \neq \emptyset\} \right)$$

Articulated Robots



World



From Howie Choset

Configuration space