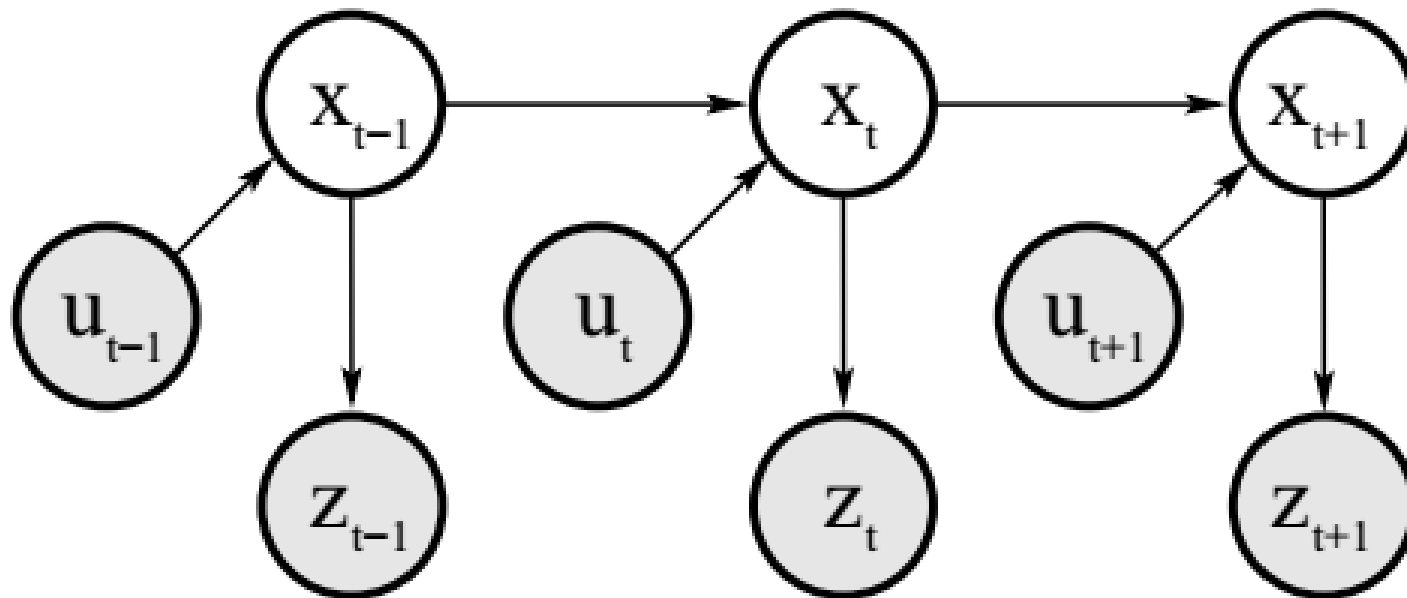


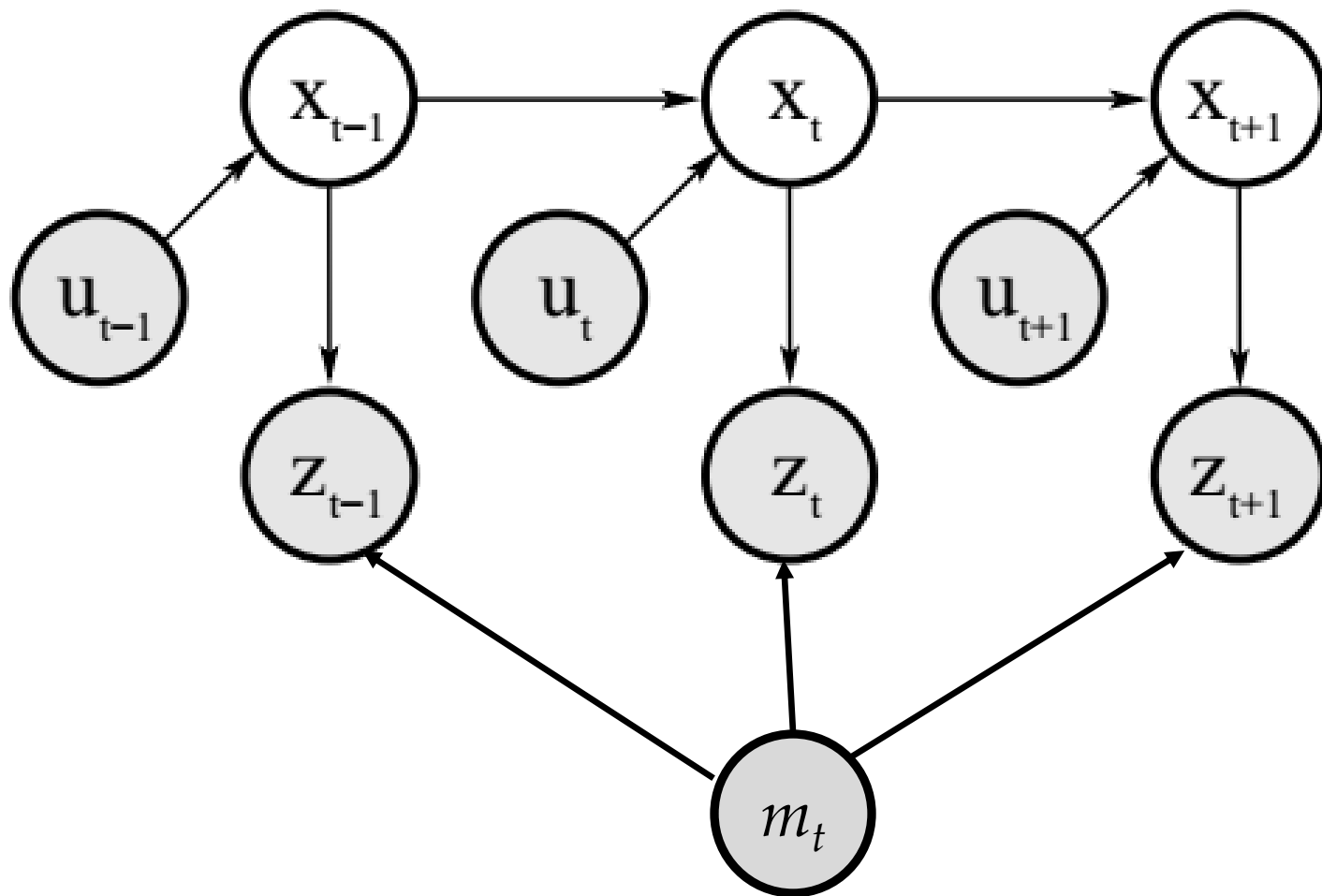
Localization and Mapping

CSCI 545 Introduction to Robotics
Instructor: Stefanos Nikolaidis

Localization



Localization



Dynamics

$$\begin{pmatrix} \hat{v}_t \\ \hat{\omega}_t \end{pmatrix} = \begin{pmatrix} v_t \\ \omega_t \end{pmatrix} + \begin{pmatrix} \epsilon_{a_1|v_t|+a_2|\omega_t|} \\ \epsilon_{a_3|v_t|+a_4|\omega_t|} \end{pmatrix}$$

$$x_t = x_{t-1} - \frac{\hat{v}_t}{\hat{\omega}_t} \sin(\theta_{t-1}) + \frac{\hat{v}_t}{\hat{\omega}_t} \sin(\theta_{t-1} + \hat{\omega}_t * \Delta_t)$$

$$y_t = y_{t-1} + \frac{\hat{v}_t}{\hat{\omega}_t} \cos(\theta_{t-1}) - \frac{\hat{v}_t}{\hat{\omega}_t} \cos(\theta_{t-1} + \hat{\omega}_t * \Delta_t)$$

$$\theta_t = \theta_{t-1} + \hat{\omega}_t * \Delta_t$$

Dynamics

$$x_t = x_{t-1} - \frac{v_t}{\omega_t} \sin(\theta_{t-1}) + \frac{v_t}{\omega_t} \sin(\theta_{t-1} + \omega_t * \Delta_t) + \epsilon_x$$


$$y_t = y_{t-1} + \frac{v_t}{\omega_t} \cos(\theta_{t-1}) - \frac{v_t}{\omega_t} \cos(\theta_{t-1} + \omega_t * \Delta_t) + \epsilon_y$$

$$\theta_t = \theta_{t-1} + \omega_t * \Delta_t + \epsilon_\theta$$

EKF

```
1:   Algorithm Extended_Kalman_filter( $\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$ ):  
2:      $\bar{\mu}_t = g(u_t, \mu_{t-1})$   
3:      $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$   
4:      $K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1}$   
5:      $\mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t))$   
6:      $\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$   
7:     return  $\mu_t, \Sigma_t$ 
```

EKF

1: **Algorithm Extended_Kalman_filter**($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$):
2:  $\bar{\mu}_t = g(u_t, \mu_{t-1})$
3: $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$
4: $K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1}$
5: $\mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t))$
6: $\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$
7: return μ_t, Σ_t

Dynamics

$$x_t = x_{t-1} - \frac{v_t}{\omega_t} \sin(\theta_{t-1}) + \frac{v_t}{\omega_t} \sin(\theta_{t-1} + \omega_t * \Delta_t) + \epsilon_x$$

$$y_t = y_{t-1} + \frac{v_t}{\omega_t} \cos(\theta_{t-1}) - \frac{v_t}{\omega_t} \cos(\theta_{t-1} + \omega_t * \Delta_t) + \epsilon_y$$

$$\theta_t = \theta_{t-1} + \omega_t * \Delta_t + \epsilon_\theta$$

EKF

$$\bar{\mu}_t = \mu_{t-1} + \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \mu_{t-1,\theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1,\theta} + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \mu_{t-1,\theta} - \frac{v_t}{\omega_t} \cos(\mu_{t-1,\theta} + \omega_t \Delta t) \\ \omega_t \Delta t \end{pmatrix}$$

EKF

```
1:   Algorithm Extended_Kalman_filter( $\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$ ):  
2:      $\bar{\mu}_t = g(u_t, \mu_{t-1})$   
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5:      $\mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t))$   
6:      $\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$   
7:     return  $\mu_t, \Sigma_t$ 
```

Dynamics

$$x_t = x_{t-1} - \frac{v_t}{\omega_t} \sin(\theta_{t-1}) + \frac{v_t}{\omega_t} \sin(\theta_{t-1} + \omega_t * \Delta_t) + \epsilon_x$$

$$y_t = y_{t-1} + \frac{v_t}{\omega_t} \cos(\theta_{t-1}) - \frac{v_t}{\omega_t} \cos(\theta_{t-1} + \omega_t * \Delta_t) + \epsilon_y$$

$$\theta_t = \theta_{t-1} + \omega_t * \Delta_t + \epsilon_\theta$$

EKF

$$G_t = \begin{pmatrix} 1 & 0 & -\frac{v_t}{\omega_t} \cos(\mu_{t-1,\theta}) + \frac{v_t}{\omega_t} \cos(\mu_{t-1,\theta} + \omega_t \Delta t) \\ 0 & 1 & -\frac{v_t}{\omega_t} \sin(\mu_{t-1,\theta}) + \frac{v_t}{\omega_t} \sin(\mu_{t-1,\theta} + \omega_t \Delta t) \\ 0 & 0 & 1 \end{pmatrix}$$

EKF

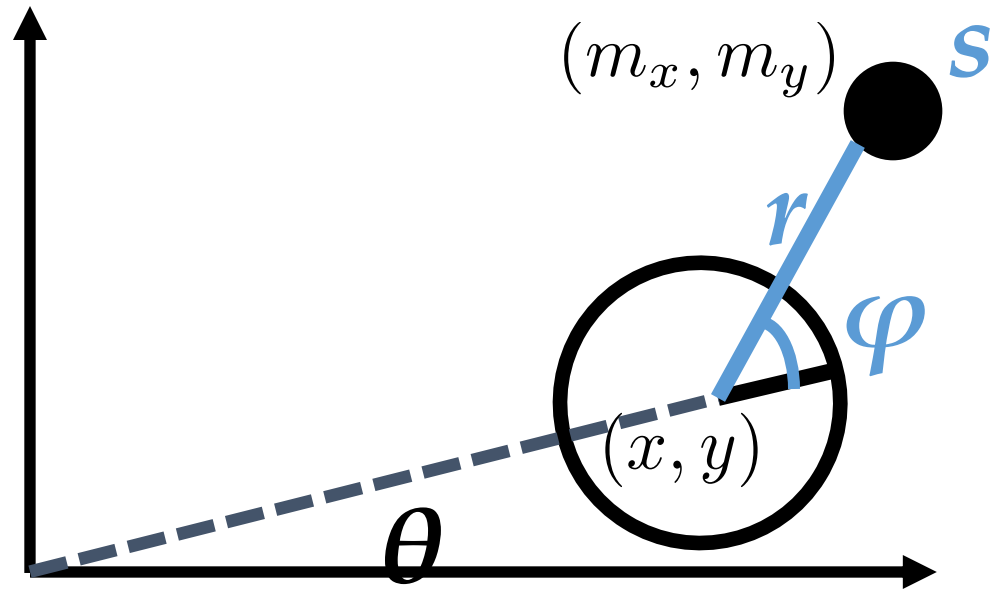
1: **Algorithm Extended_Kalman_filter**($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$):
2: $\bar{\mu}_t = g(u_t, \mu_{t-1})$
3: $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$
4: → $K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1}$
5: $\mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t))$
6: $\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$
7: return μ_t, Σ_t

Measurements

$$\underbrace{\begin{pmatrix} r_t^i \\ \phi_t^i \\ s_t^i \end{pmatrix}}_{z_t^i} = \underbrace{\begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \text{atan2}(m_{j,y} - y, m_{j,x} - x) - \theta \\ m_{j,s} \end{pmatrix}}_{h(x_t, j, m)} + \mathcal{N}(0, Q_t),$$

Feature-Based Sensor Models

$$\begin{pmatrix} r_t^i \\ \phi_t^i \\ s_t^i \end{pmatrix} = \begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \text{atan2}(m_{j,y} - y, m_{j,x} - x) - \theta \\ s_j \end{pmatrix} + \begin{pmatrix} \varepsilon_{\sigma_r^2} \\ \varepsilon_{\sigma_\phi^2} \\ \varepsilon_{\sigma_s^2} \end{pmatrix}$$



Measurements

$$\underbrace{\begin{pmatrix} r_t^i \\ \phi_t^i \\ s_t^i \end{pmatrix}}_{z_t^i} = \underbrace{\begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \text{atan2}(m_{j,y} - y, m_{j,x} - x) - \theta \\ m_{j,s} \end{pmatrix}}_{h(x_t, j, m)} + \mathcal{N}(0, Q_t),$$

EKF

$$Q_t = \begin{pmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & \sigma_s^2 \end{pmatrix}$$

$$H_t^i = \frac{\partial h(\bar{\mu}_t, j, m)}{\partial x_t} = \begin{pmatrix} \frac{\partial r_t^i}{\partial \bar{\mu}_{t,x}} & \frac{\partial r_t^i}{\partial \bar{\mu}_{t,y}} & \frac{\partial r_t^i}{\partial \bar{\mu}_{t,\theta}} \\ \frac{\partial \phi_t^i}{\partial \bar{\mu}_{t,x}} & \frac{\partial \phi_t^i}{\partial \bar{\mu}_{t,y}} & \frac{\partial \phi_t^i}{\partial \bar{\mu}_{t,\theta}} \\ \frac{\partial s_t^i}{\partial \bar{\mu}_{t,x}} & \frac{\partial s_t^i}{\partial \bar{\mu}_{t,y}} & \frac{\partial s_t^i}{\partial \bar{\mu}_{t,\theta}} \end{pmatrix} = \begin{pmatrix} -\frac{m_{j,x} - \bar{\mu}_{t,x}}{\sqrt{q}} & -\frac{m_{j,y} - \bar{\mu}_{t,y}}{\sqrt{q}} & 0 \\ \frac{m_{j,y} - \bar{\mu}_{t,y}}{q} & -\frac{m_{j,x} - \bar{\mu}_{t,x}}{q} & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$q_t = (m_{j,x} - \bar{\mu}_{t,x})^2 + (m_{j,y} - \bar{\mu}_{t,y})^2$$

EKF

```
1:   Algorithm Extended_Kalman_filter( $\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$ ):  
2:      $\bar{\mu}_t = g(u_t, \mu_{t-1})$   
3:      $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$   
4:      $K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1}$   
5:      $\mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t))$   
6:      $\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$   
7:     return  $\mu_t, \Sigma_t$ 
```

EKF

1: **Algorithm EKF_localization_known_correspondences**($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, c_t, m$):

2: $\theta = \mu_{t-1, \theta}$

$$3: G_t = \begin{pmatrix} 1 & 0 & -\frac{v_t}{\omega_t} \cos \theta + \frac{v_t}{\omega_t} \cos(\theta + \omega_t \Delta t) \\ 0 & 1 & -\frac{v_t}{\omega_t} \sin \theta + \frac{v_t}{\omega_t} \sin(\theta + \omega_t \Delta t) \\ 0 & 0 & 1 \end{pmatrix}$$

$$4: V_t = \begin{pmatrix} \frac{-\sin \theta + \sin(\theta + \omega_t \Delta t)}{\omega_t} & \frac{v_t(\sin \theta - \sin(\theta + \omega_t \Delta t))}{\omega_t^2} + \frac{v_t \cos(\theta + \omega_t \Delta t) \Delta t}{\omega_t} \\ \frac{\cos \theta - \cos(\theta + \omega_t \Delta t)}{\omega_t} & -\frac{v_t(\cos \theta - \cos(\theta + \omega_t \Delta t))}{\omega_t^2} + \frac{v_t \sin(\theta + \omega_t \Delta t) \Delta t}{\omega_t} \\ 0 & \Delta t \end{pmatrix}$$

$$5: M_t = \begin{pmatrix} \alpha_1 v_t^2 + \alpha_2 \omega_t^2 & 0 \\ 0 & \alpha_3 v_t^2 + \alpha_4 \omega_t^2 \end{pmatrix}$$

$$6: \bar{\mu}_t = \mu_{t-1} + \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \theta + \frac{v_t}{\omega_t} \sin(\theta + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \theta - \frac{v_t}{\omega_t} \cos(\theta + \omega_t \Delta t) \\ \omega_t \Delta t \end{pmatrix}$$

$$7: \bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + \underbrace{V_t M_t V_t^T}_R$$

$$8: Q_t = \begin{pmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & \sigma_s^2 \end{pmatrix} R$$

9: *for all observed features* $z_t^i = (r_t^i \phi_t^i s_t^i)^T$ *do*

10: $j = c_t^i$

$$11: q = (m_{j,x} - \bar{\mu}_{t,x})^2 + (m_{j,y} - \bar{\mu}_{t,y})^2$$

$$12: \hat{z}_t^i = \begin{pmatrix} \sqrt{q} \\ \text{atan2}(m_{j,y} - \bar{\mu}_{t,y}, m_{j,x} - \bar{\mu}_{t,x}) - \bar{\mu}_{t,\theta} \\ m_{j,s} \end{pmatrix}$$

$$13: H_t^i = \begin{pmatrix} -\frac{m_{j,x} - \bar{\mu}_{t,x}}{\sqrt{q}} & -\frac{m_{j,y} - \bar{\mu}_{t,y}}{\sqrt{q}} & 0 \\ \frac{m_{j,y} - \bar{\mu}_{t,y}}{q} & -\frac{m_{j,x} - \bar{\mu}_{t,x}}{q} & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$14: S_t^i = H_t^i \bar{\Sigma}_t [H_t^i]^T + Q_t$$

$$15: K_t^i = \bar{\Sigma}_t [H_t^i]^T [S_t^i]^{-1}$$

$$16: \bar{\mu}_t = \bar{\mu}_t + K_t^i (z_t^i - \hat{z}_t^i)$$

$$17: \bar{\Sigma}_t = (I - K_t^i H_t^i) \bar{\Sigma}_t$$

18: *endfor*

$$19: \mu_t = \bar{\mu}_t$$

$$20: \Sigma_t = \bar{\Sigma}_t$$

$$21: p_{z_t} = \prod_i \det(2\pi S_t^i)^{-\frac{1}{2}} \exp\left\{-\frac{1}{2} (z_t^i - \hat{z}_t^i)^T [S_t^i]^{-1} (z_t^i - \hat{z}_t^i)\right\}$$

22: *return* μ_t, Σ_t, p_{z_t}

Example

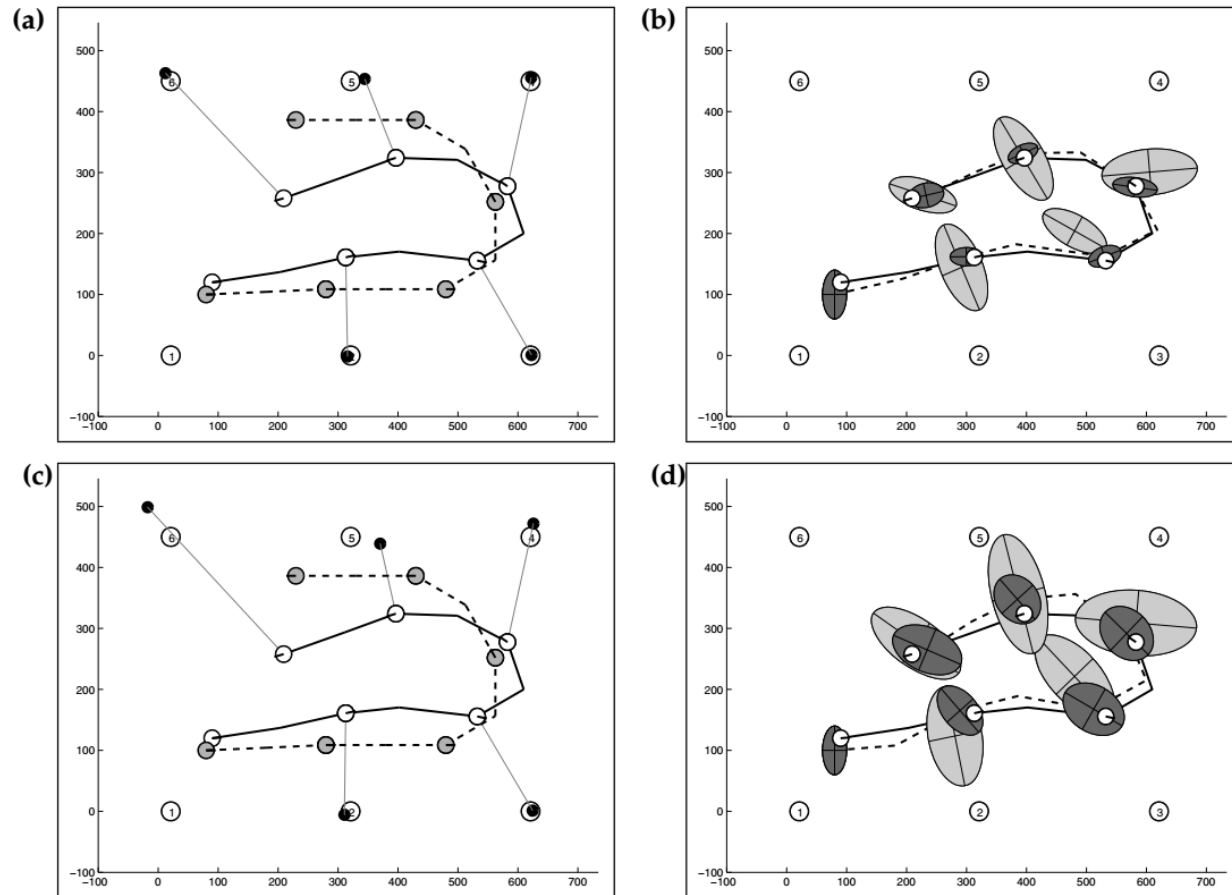


Figure 7.11 EKF-based localization with an accurate (upper row) and a less accurate (lower row) landmark detection sensor. The dashed lines in the left panel indicate the robot trajectories as estimated from the motion controls. The solid lines represent the true robot motion resulting from these controls. Landmark detections at five locations are indicated by the thin lines. The dashed lines in the right panels show the corrected robot trajectories, along with uncertainty before (light gray, $\bar{\Sigma}_t$) and after (dark gray, Σ_t) incorporating a landmark detection.

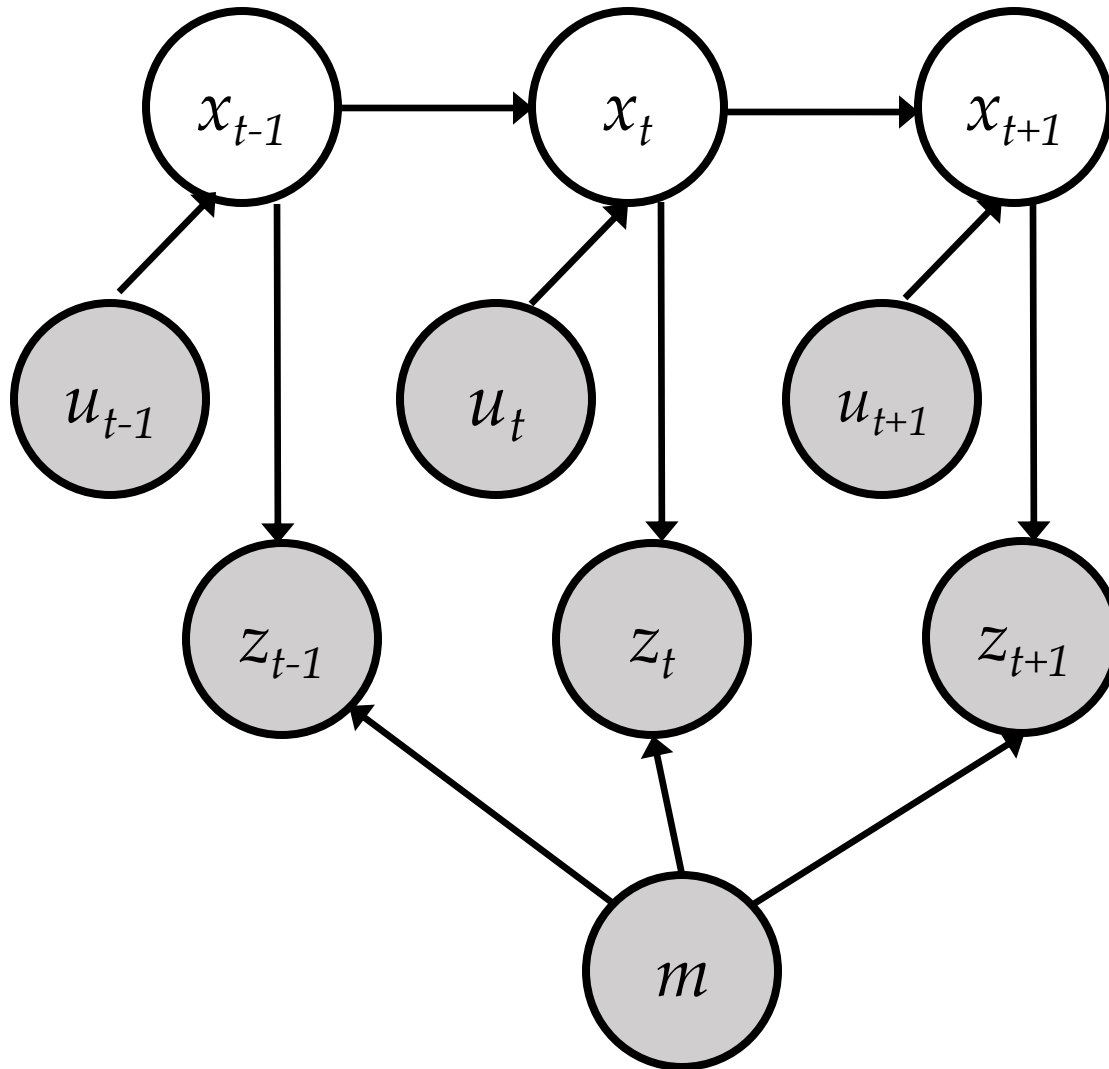
Localization with Unknown Correspondences

- How to do EKF-localization when the correspondences are not known?

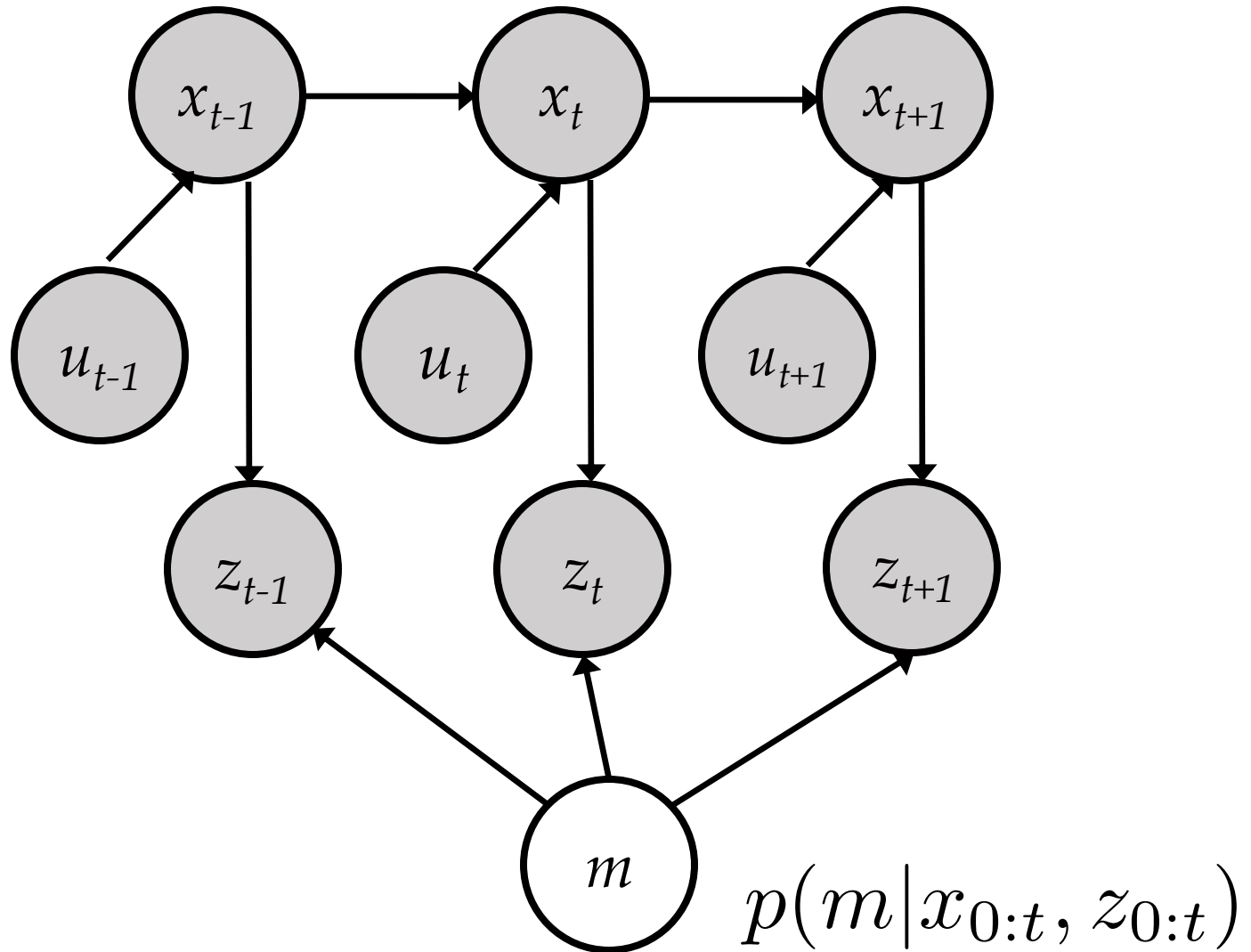
Localization with Unknown Correspondences

- How to do EKF-localization when the correspondences are not known?
- Maximum Likelihood Estimation

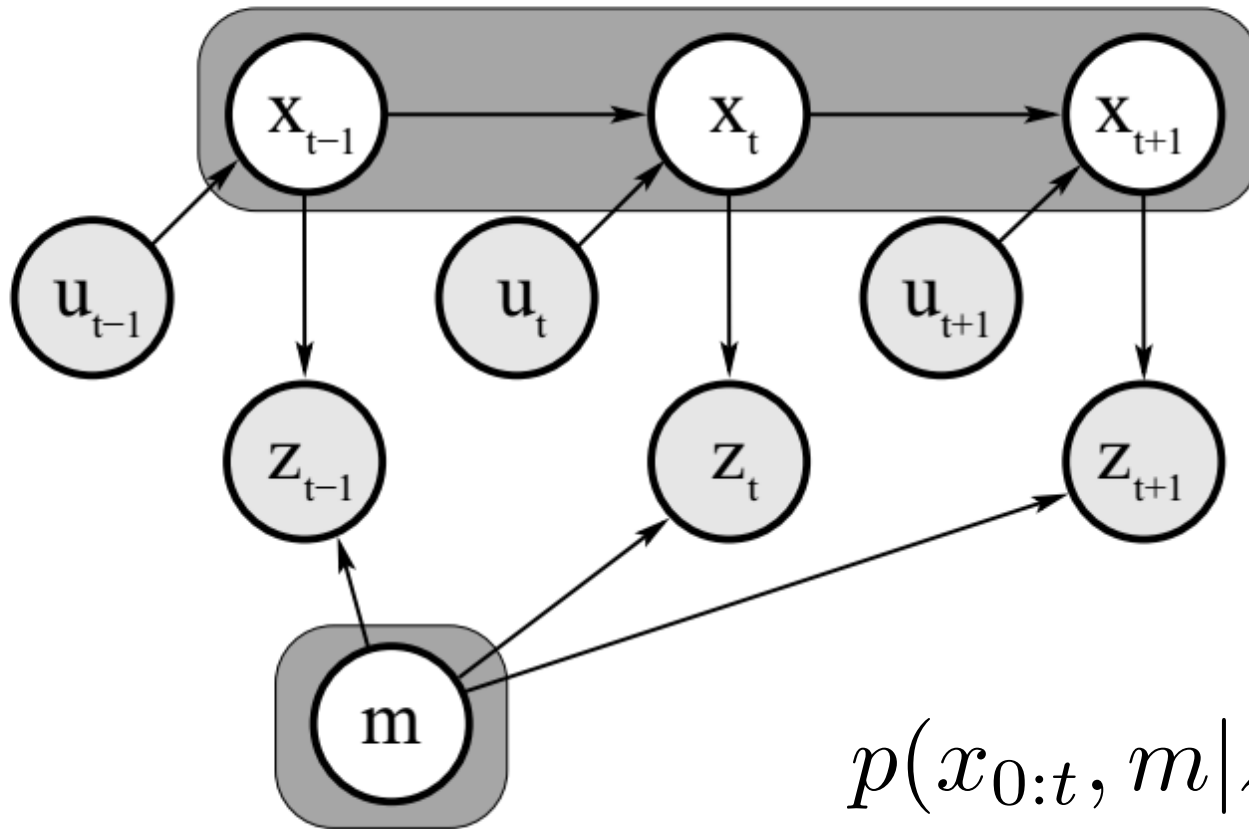
Localization



Mapping



Simultaneous Localization and Mapping



Simultaneous Localization and Mapping

Simultaneous Localization and Mapping

Autonomous Aerial Navigation in Confined Indoor Environments

Shaojie Shen, Nathan Michael, Vijay Kumar



EKF - SLAM

$$\begin{aligned} y_t &= \begin{pmatrix} x_t \\ m \end{pmatrix} \\ &= (x \ y \ \theta \ m_{1,x} \ m_{1,y} \ s_1 \ m_{2,x} \ m_{2,y} \ s_2 \ \dots \ m_{N,x} \ m_{N,y} \ s_N)^T \end{aligned}$$

Assumptions:

- Static map
- Known set of landmarks
- Known correspondences

EKF

```
1:   Algorithm Extended_Kalman_filter( $\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$ ):  
2:      $\bar{\mu}_t = g(u_t, \mu_{t-1})$   
3:      $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$   
4:      $K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1}$   
5:      $\mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t))$   
6:      $\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$   
7:     return  $\mu_t, \Sigma_t$ 
```

Initial Mean and Covariance

$$\mu_0 = (0 \ 0 \ 0 \ \dots \ 0)^T$$

$$\Sigma_0 = \begin{pmatrix} 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \infty & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & \infty \end{pmatrix}$$

Dynamics Model

$$y_t = y_{t-1} + \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \theta + \frac{v_t}{\omega_t} \sin(\theta + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \theta - \frac{v_t}{\omega_t} \cos(\theta + \omega_t \Delta t) \\ \omega_t \Delta t + \gamma_t \Delta t \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

Dynamics Model

$$y_t = y_{t-1} + \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \theta + \frac{v_t}{\omega_t} \sin(\theta + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \theta - \frac{v_t}{\omega_t} \cos(\theta + \omega_t \Delta t) \\ \omega_t \Delta t + \gamma_t \Delta t \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$y_t = y_{t-1} + F_x^T \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \theta + \frac{v_t}{\omega_t} \sin(\theta + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \theta - \frac{v_t}{\omega_t} \cos(\theta + \omega_t \Delta t) \\ \omega_t \Delta t + \gamma_t \Delta t \end{pmatrix}$$

$$F_x = \begin{pmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & \underbrace{0 & \dots & 0}_{3N \text{ columns}} \end{pmatrix}$$

Dynamics Model

$$y_t = y_{t-1} + \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \theta + \frac{v_t}{\omega_t} \sin(\theta + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \theta - \frac{v_t}{\omega_t} \cos(\theta + \omega_t \Delta t) \\ \omega_t \Delta t + \gamma_t \Delta t \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$y_t = y_{t-1} + \underbrace{F_x^T \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \theta + \frac{v_t}{\omega_t} \sin(\theta + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \theta - \frac{v_t}{\omega_t} \cos(\theta + \omega_t \Delta t) \\ \omega \Delta t \end{pmatrix}}_{g(u_t, y_{t-1})} + \mathcal{N}(0, F_x^T R_t F_x)$$

$$F_x = \begin{pmatrix} 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ & & & \underbrace{0 & \dots & 0}_{3N \text{ columns}} \end{pmatrix}$$

Dynamics Model

$$y_t = y_{t-1} + \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \theta + \frac{v_t}{\omega_t} \sin(\theta + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \theta - \frac{v_t}{\omega_t} \cos(\theta + \omega_t \Delta t) \\ \omega_t \Delta t + \gamma_t \Delta t \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$g(u_t, y_{t-1}) \approx g(u_t, \mu_{t-1}) + G_t (y_{t-1} - \mu_{t-1})$$

EKF-SLAM

```

1: Algorithm EKF_SLAM_known_correspondences( $\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, c_t$ ):
2:    $F_x = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & \underbrace{0 \cdots 0}_{3N} \end{pmatrix}$ 
3:    $\bar{\mu}_t = \mu_{t-1} + F_x^T \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \mu_{t-1, \theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1, \theta} + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \mu_{t-1, \theta} - \frac{v_t}{\omega_t} \cos(\mu_{t-1, \theta} + \omega_t \Delta t) \\ \omega_t \Delta t \end{pmatrix}$ 
4:    $G_t = I + F_x^T \begin{pmatrix} 0 & 0 & -\frac{v_t}{\omega_t} \cos \mu_{t-1, \theta} + \frac{v_t}{\omega_t} \cos(\mu_{t-1, \theta} + \omega_t \Delta t) \\ 0 & 0 & -\frac{v_t}{\omega_t} \sin \mu_{t-1, \theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1, \theta} + \omega_t \Delta t) \\ 0 & 0 & 0 \end{pmatrix} F_x$ 
5:    $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + F_x^T R_t F_x$ 
6:    $Q_t = \begin{pmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & \sigma_s^2 \end{pmatrix}$ 
7:   for all observed features  $z_t^i = (r_t^i \ \phi_t^i \ s_t^i)^T$  do
8:      $j = c_t^i$ 
9:     if landmark  $j$  never seen before
10:       $\begin{pmatrix} \bar{\mu}_{j,x} \\ \bar{\mu}_{j,y} \\ \bar{\mu}_{j,s} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \\ s_t^i \end{pmatrix} + \begin{pmatrix} r_t^i \cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i \sin(\phi_t^i + \bar{\mu}_{t,\theta}) \\ 0 \end{pmatrix}$ 
11:    endif
12:     $\delta = \begin{pmatrix} \delta_x \\ \delta_y \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{j,x} - \bar{\mu}_{t,x} \\ \bar{\mu}_{j,y} - \bar{\mu}_{t,y} \end{pmatrix}$ 
13:     $q = \delta^T \delta$ 
14:     $z_t^i = \begin{pmatrix} \text{atan2}(\delta_y, \delta_x) - \bar{\mu}_{t,\theta} \\ \bar{\mu}_{j,s} \end{pmatrix}$ 
15:     $F_{x,j} = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 1 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & \underbrace{0 \cdots 0}_{3j-3} & 0 & 0 & 1 & \underbrace{0 \cdots 0}_{3N-3j} \end{pmatrix}$ 
16:     $H_t^i = \frac{1}{q} \begin{pmatrix} -\sqrt{q} \delta_x & -\sqrt{q} \delta_y & 0 & +\sqrt{q} \delta_x & \sqrt{q} \delta_y & 0 \\ \delta_y & -\delta_x & -q & -\delta_y & +\delta_x & 0 \\ 0 & 0 & 0 & 0 & 0 & q \end{pmatrix} F_{x,j}$ 
17:     $K_t^i = \bar{\Sigma}_t H_t^{iT} (H_t^i \bar{\Sigma}_t H_t^{iT} + Q_t)^{-1}$ 
18:     $\bar{\mu}_t = \bar{\mu}_t + K_t^i (z_t^i - \hat{z}_t^i)$ 
19:     $\bar{\Sigma}_t = (I - K_t^i H_t^i) \bar{\Sigma}_t$ 
20:  endfor
21:   $\mu_t = \bar{\mu}_t$ 
22:   $\Sigma_t = \bar{\Sigma}_t$ 
23:  return  $\mu_t, \Sigma_t$ 

```

Table 10.1 The EKF algorithm for the SLAM problem (with known correspondences).

Measurement Model

$$z_t^i = \underbrace{\begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \text{atan2}(m_{j,y} - y, m_{j,x} - x) - \theta \\ m_{j,s} \end{pmatrix}}_{h(y_t, j)} + \mathcal{N}\left(0, \underbrace{\begin{pmatrix} \sigma_r & 0 & 0 \\ 0 & \sigma_\phi & 0 \\ 0 & 0 & \sigma_s \end{pmatrix}}_{Q_t}\right)$$

$$h(y_t, j) \approx h(\bar{\mu}_t, j) + H_t^i (y_t - \bar{\mu}_t)$$

Measurement Model

$$z_t^i = \underbrace{\begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \text{atan2}(m_{j,y} - y, m_{j,x} - x) - \theta \\ m_{j,s} \end{pmatrix}}_{h(y_t, j)} + \mathcal{N}\left(0, \underbrace{\begin{pmatrix} \sigma_r & 0 & 0 \\ 0 & \sigma_\phi & 0 \\ 0 & 0 & \sigma_s \end{pmatrix}}_{Q_t}\right)$$

$$h(y_t, j) \approx h(\bar{\mu}_t, j) + H_t^i (y_t - \bar{\mu}_t)$$

What if landmark is seen for the first time?

Measurement Model

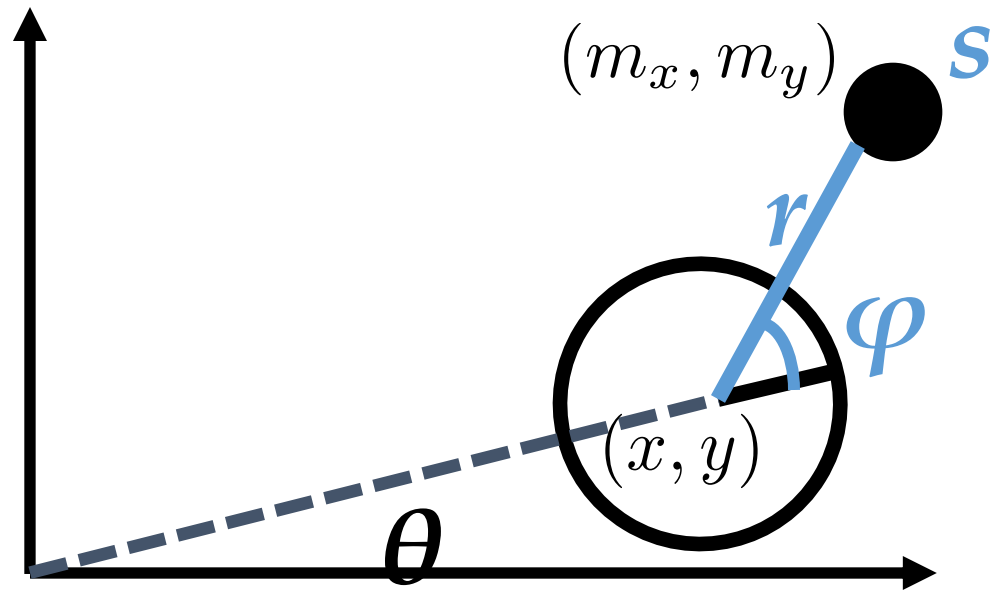
$$z_t^i = \underbrace{\begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \text{atan2}(m_{j,y} - y, m_{j,x} - x) - \theta \\ m_{j,s} \end{pmatrix}}_{h(y_t, j)} + \mathcal{N}\left(0, \underbrace{\begin{pmatrix} \sigma_r & 0 & 0 \\ 0 & \sigma_\phi & 0 \\ 0 & 0 & \sigma_s \end{pmatrix}}_{Q_t}\right)$$

$$h(y_t, j) \approx h(\bar{\mu}_t, j) + H_t^i (y_t - \bar{\mu}_t)$$

What if landmark is seen for the first time?

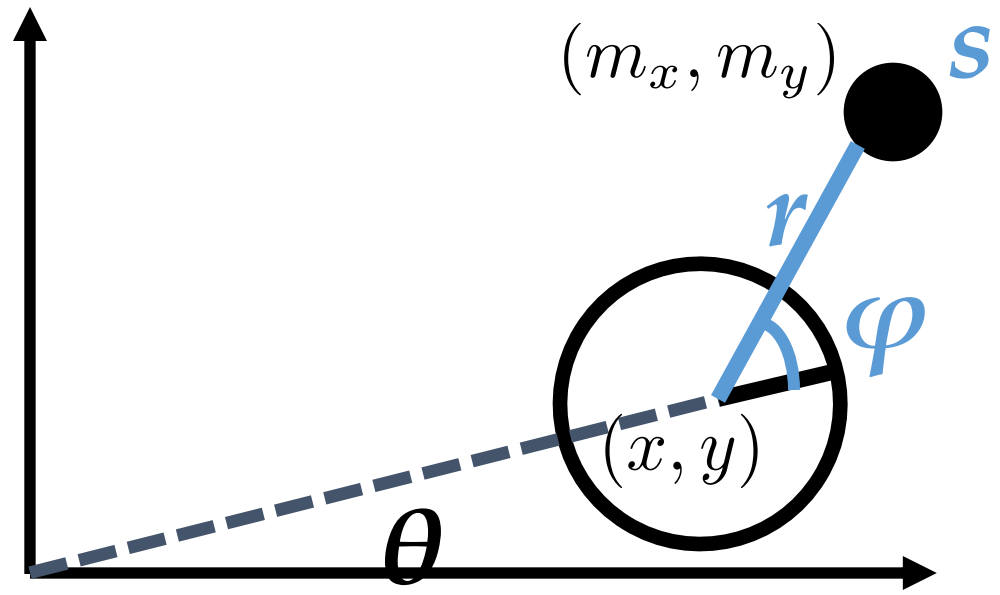
Estimate it based on the robot's position!

Measurement Model



Measurement Model

$$\begin{pmatrix} \bar{\mu}_{j,x} \\ \bar{\mu}_{j,y} \\ \bar{\mu}_{j,s} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \\ s_t^i \end{pmatrix} + \begin{pmatrix} r_t^i \cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i \sin(\phi_t^i + \bar{\mu}_{t,\theta}) \\ 0 \end{pmatrix}$$



Measurement Model

$$z_t^i = \underbrace{\begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \text{atan2}(m_{j,y} - y, m_{j,x} - x) - \theta \\ m_{j,s} \end{pmatrix}}_{h(y_t, j)} + \mathcal{N}\left(0, \underbrace{\begin{pmatrix} \sigma_r & 0 & 0 \\ 0 & \sigma_\phi & 0 \\ 0 & 0 & \sigma_s \end{pmatrix}}_{Q_t}\right)$$

$$h(y_t, j) \approx h(\bar{\mu}_t, j) + H_t^i (y_t - \bar{\mu}_t)$$

What if landmark is seen for the first time?

Estimate it based on the robot's position!

Otherwise use previous estimate

EKF-SLAM

```

1: Algorithm EKF_SLAM_known_correspondences( $\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, c_t$ ):
2:    $F_x = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & \underbrace{0 \cdots 0}_{3N} \end{pmatrix}$ 
3:    $\bar{\mu}_t = \mu_{t-1} + F_x^T \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \mu_{t-1, \theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1, \theta} + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \mu_{t-1, \theta} - \frac{v_t}{\omega_t} \cos(\mu_{t-1, \theta} + \omega_t \Delta t) \\ \omega_t \Delta t \end{pmatrix}$ 
4:    $G_t = I + F_x^T \begin{pmatrix} 0 & 0 & -\frac{v_t}{\omega_t} \cos \mu_{t-1, \theta} + \frac{v_t}{\omega_t} \cos(\mu_{t-1, \theta} + \omega_t \Delta t) \\ 0 & 0 & -\frac{v_t}{\omega_t} \sin \mu_{t-1, \theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1, \theta} + \omega_t \Delta t) \\ 0 & 0 & 0 \end{pmatrix} F_x$ 
5:    $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + F_x^T R_t F_x$ 
6:    $Q_t = \begin{pmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & \sigma_s^2 \end{pmatrix}$ 
7:   for all observed features  $z_t^i = (r_t^i \ \phi_t^i \ s_t^i)^T$  do
8:      $j = c_t^i$ 
9:     if landmark  $j$  never seen before
10:       $\begin{pmatrix} \bar{\mu}_{j,x} \\ \bar{\mu}_{j,y} \\ \bar{\mu}_{j,s} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \\ s_t^i \end{pmatrix} + \begin{pmatrix} r_t^i \cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i \sin(\phi_t^i + \bar{\mu}_{t,\theta}) \\ 0 \end{pmatrix}$ 
11:    endif
12:     $\delta = \begin{pmatrix} \delta_x \\ \delta_y \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{j,x} - \bar{\mu}_{t,x} \\ \bar{\mu}_{j,y} - \bar{\mu}_{t,y} \end{pmatrix}$ 
13:     $q = \delta^T \delta$ 
14:     $z_t^i = \begin{pmatrix} \text{atan2}(\delta_y, \delta_x) - \bar{\mu}_{t,\theta} \\ \bar{\mu}_{j,s} \end{pmatrix}$ 
15:     $F_{x,j} = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 1 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & \underbrace{0 \cdots 0}_{3j-3} & 0 & 0 & 1 & \underbrace{0 \cdots 0}_{3N-3j} \end{pmatrix}$ 
16:     $H_t^i = \frac{1}{q} \begin{pmatrix} -\sqrt{q} \delta_x & -\sqrt{q} \delta_y & 0 & +\sqrt{q} \delta_x & \sqrt{q} \delta_y & 0 \\ \delta_y & -\delta_x & -q & -\delta_y & +\delta_x & 0 \\ 0 & 0 & 0 & 0 & 0 & q \end{pmatrix} F_{x,j}$ 
17:     $K_t^i = \bar{\Sigma}_t H_t^{iT} (H_t^i \bar{\Sigma}_t H_t^{iT} + Q_t)^{-1}$ 
18:     $\bar{\mu}_t = \bar{\mu}_t + K_t^i (z_t^i - \bar{z}_t^i)$ 
19:     $\bar{\Sigma}_t = (I - K_t^i H_t^i) \bar{\Sigma}_t$ 
20:  endfor
21:   $\mu_t = \bar{\mu}_t$ 
22:   $\Sigma_t = \bar{\Sigma}_t$ 
23:  return  $\mu_t, \Sigma_t$ 

```

Table 10.1 The EKF algorithm for the SLAM problem (with known correspondences).

EKF-SLAM

```

1: Algorithm EKF_SLAM_known_correspondences( $\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, c_t$ ):
2:    $F_x = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & \underbrace{0 \cdots 0}_{3N} \end{pmatrix}$ 
3:    $\bar{\mu}_t = \mu_{t-1} + F_x^T \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \mu_{t-1, \theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1, \theta} + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \mu_{t-1, \theta} - \frac{v_t}{\omega_t} \cos(\mu_{t-1, \theta} + \omega_t \Delta t) \\ \omega_t \Delta t \end{pmatrix}$ 
4:    $G_t = I + F_x^T \begin{pmatrix} 0 & 0 & -\frac{v_t}{\omega_t} \cos \mu_{t-1, \theta} + \frac{v_t}{\omega_t} \cos(\mu_{t-1, \theta} + \omega_t \Delta t) \\ 0 & 0 & -\frac{v_t}{\omega_t} \sin \mu_{t-1, \theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1, \theta} + \omega_t \Delta t) \\ 0 & 0 & 0 \end{pmatrix} F_x$ 
5:    $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + F_x^T R_t F_x$ 
6:    $Q_t = \begin{pmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & \sigma_s^2 \end{pmatrix}$ 
7:   for all observed features  $z_t^i = (r_t^i \ \phi_t^i \ s_t^i)^T$  do
8:      $j = c_t^i$ 
9:     if landmark  $j$  never seen before
10:       $\begin{pmatrix} \bar{\mu}_{j,x} \\ \bar{\mu}_{j,y} \\ \bar{\mu}_{j,s} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \\ s_t^i \end{pmatrix} + \begin{pmatrix} r_t^i \cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i \sin(\phi_t^i + \bar{\mu}_{t,\theta}) \\ 0 \end{pmatrix}$ 
11:    endif
12:     $\delta = \begin{pmatrix} \delta_x \\ \delta_y \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{j,x} - \bar{\mu}_{t,x} \\ \bar{\mu}_{j,y} - \bar{\mu}_{t,y} \end{pmatrix}$ 
13:     $q = \delta^T \delta$ 
14:     $z_t^i = \begin{pmatrix} \sqrt{q} \\ \text{atan2}(\delta_y, \delta_x) - \bar{\mu}_{t,\theta} \\ \bar{\mu}_{j,s} \end{pmatrix}$ 
15:     $F_{x,j} = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 1 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & \underbrace{0 \cdots 0}_{3j-3} & 0 & 0 & 1 & \underbrace{0 \cdots 0}_{3N-3j} \end{pmatrix}$ 
16:     $H_t^i = \frac{1}{q} \begin{pmatrix} -\sqrt{q} \delta_x & -\sqrt{q} \delta_y & 0 & +\sqrt{q} \delta_x & \sqrt{q} \delta_y & 0 \\ \delta_y & -\delta_x & -q & -\delta_y & +\delta_x & 0 \\ 0 & 0 & 0 & 0 & 0 & q \end{pmatrix} F_{x,j}$ 
17:     $K_t^i = \bar{\Sigma}_t H_t^{iT} (H_t^i \bar{\Sigma}_t H_t^{iT} + Q_t)^{-1}$ 
18:     $\bar{\mu}_t = \bar{\mu}_t + K_t^i (z_t^i - \bar{z}_t^i)$ 
19:     $\bar{\Sigma}_t = (I - K_t^i H_t^i) \bar{\Sigma}_t$ 
20:  endfor
21:   $\mu_t = \bar{\mu}_t$ 
22:   $\Sigma_t = \bar{\Sigma}_t$ 
23:  return  $\mu_t, \Sigma_t$ 

```

Table 10.1 The EKF algorithm for the SLAM problem (with known correspondences).

Measurement Model

$$z_t^i = \underbrace{\begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \text{atan2}(m_{j,y} - y, m_{j,x} - x) - \theta \\ m_{j,s} \end{pmatrix}}_{h(y_t, j)} + \mathcal{N}\left(0, \underbrace{\begin{pmatrix} \sigma_r & 0 & 0 \\ 0 & \sigma_\phi & 0 \\ 0 & 0 & \sigma_s \end{pmatrix}}_{Q_t}\right)$$

$$h(y_t, j) \approx h(\bar{\mu}_t, j) + H_t^i (y_t - \bar{\mu}_t)$$

$$H_t^i = h_t^i F_{x,j}$$

EKF-SLAM

```

1: Algorithm EKF_SLAM_known_correspondences( $\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, c_t$ ):
2:    $F_x = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & \underbrace{0 \cdots 0}_{3N} \end{pmatrix}$ 
3:    $\bar{\mu}_t = \mu_{t-1} + F_x^T \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \mu_{t-1, \theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1, \theta} + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \mu_{t-1, \theta} - \frac{v_t}{\omega_t} \cos(\mu_{t-1, \theta} + \omega_t \Delta t) \\ \omega_t \Delta t \end{pmatrix}$ 
4:    $G_t = I + F_x^T \begin{pmatrix} 0 & 0 & -\frac{v_t}{\omega_t} \cos \mu_{t-1, \theta} + \frac{v_t}{\omega_t} \cos(\mu_{t-1, \theta} + \omega_t \Delta t) \\ 0 & 0 & -\frac{v_t}{\omega_t} \sin \mu_{t-1, \theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1, \theta} + \omega_t \Delta t) \\ 0 & 0 & 0 \end{pmatrix} F_x$ 
5:    $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + F_x^T R_t F_x$ 
6:    $Q_t = \begin{pmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & \sigma_s^2 \end{pmatrix}$ 
7:   for all observed features  $z_t^i = (r_t^i \ \phi_t^i \ s_t^i)^T$  do
8:      $j = c_t^i$ 
9:     if landmark  $j$  never seen before
10:       $\begin{pmatrix} \bar{\mu}_{j,x} \\ \bar{\mu}_{j,y} \\ \bar{\mu}_{j,s} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \\ s_t^i \end{pmatrix} + \begin{pmatrix} r_t^i \cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i \sin(\phi_t^i + \bar{\mu}_{t,\theta}) \\ 0 \end{pmatrix}$ 
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12:     $\delta = \begin{pmatrix} \delta_x \\ \delta_y \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{j,x} - \bar{\mu}_{t,x} \\ \bar{\mu}_{j,y} - \bar{\mu}_{t,y} \end{pmatrix}$ 
13:     $q = \delta^T \delta$ 
14:     $\hat{z}_t^i = \begin{pmatrix} \sqrt{q} \\ \text{atan2}(\delta_y, \delta_x) - \bar{\mu}_{t,\theta} \\ \bar{\mu}_{j,s} \end{pmatrix}$ 
15:     $F_{x,j} = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 1 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & \underbrace{0 \cdots 0}_{3j-3} & 0 & 0 & 1 & \underbrace{0 \cdots 0}_{3N-3j} \end{pmatrix}$ 
16:     $H_t^i = \frac{1}{q} \begin{pmatrix} -\sqrt{q} \delta_x & -\sqrt{q} \delta_y & 0 & +\sqrt{q} \delta_x & \sqrt{q} \delta_y & 0 \\ \delta_y & -\delta_x & -q & -\delta_y & +\delta_x & 0 \\ 0 & 0 & 0 & 0 & 0 & q \end{pmatrix} F_{x,j}$ 
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19:     $\bar{\Sigma}_t = (I - K_t^i H_t^i) \bar{\Sigma}_t$ 
20:  endfor
21:   $\mu_t = \bar{\mu}_t$ 
22:   $\Sigma_t = \bar{\Sigma}_t$ 
23:  return  $\mu_t, \Sigma_t$ 

```

Table 10.1 The EKF algorithm for the SLAM problem (with known correspondences).

EKF-SLAM

```

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3:    $\bar{\mu}_t = \mu_{t-1} + F_x^T \begin{pmatrix} -\frac{v_t}{\omega_t} \sin \mu_{t-1, \theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1, \theta} + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos \mu_{t-1, \theta} - \frac{v_t}{\omega_t} \cos(\mu_{t-1, \theta} + \omega_t \Delta t) \\ \omega_t \Delta t \end{pmatrix}$ 
4:    $G_t = I + F_x^T \begin{pmatrix} 0 & 0 & -\frac{v_t}{\omega_t} \cos \mu_{t-1, \theta} + \frac{v_t}{\omega_t} \cos(\mu_{t-1, \theta} + \omega_t \Delta t) \\ 0 & 0 & -\frac{v_t}{\omega_t} \sin \mu_{t-1, \theta} + \frac{v_t}{\omega_t} \sin(\mu_{t-1, \theta} + \omega_t \Delta t) \\ 0 & 0 & 0 \end{pmatrix} F_x$ 
5:    $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + F_x^T R_t F_x$ 
6:    $Q_t = \begin{pmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & \sigma_s^2 \end{pmatrix}$ 
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11:     endif
12:      $\delta = \begin{pmatrix} \delta_x \\ \delta_y \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{j,x} - \bar{\mu}_{t,x} \\ \bar{\mu}_{j,y} - \bar{\mu}_{t,y} \end{pmatrix}$ 
13:      $q = \delta^T \delta$ 
14:      $\hat{z}_t^i = \begin{pmatrix} \sqrt{q} \\ \text{atan2}(\delta_y, \delta_x) - \bar{\mu}_{t,\theta} \\ \bar{\mu}_{j,s} \end{pmatrix}$ 
15:      $F_{x,j} = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 1 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & \underbrace{0 \cdots 0}_{3j-3} & 0 & 0 & 1 & \underbrace{0 \cdots 0}_{3N-3j} \end{pmatrix}$ 
16:      $H_t^i = \frac{1}{q} \begin{pmatrix} -\sqrt{q} \delta_x & -\sqrt{q} \delta_y & 0 & +\sqrt{q} \delta_x & \sqrt{q} \delta_y & 0 \\ \delta_y & -\delta_x & -q & -\delta_y & +\delta_x & 0 \\ 0 & 0 & 0 & 0 & 0 & q \end{pmatrix} F_{x,j}$ 
17:      $K_t^i = \bar{\Sigma}_t H_t^{iT} (H_t^i \bar{\Sigma}_t H_t^{iT} + Q_t)^{-1}$ 
18:      $\bar{\mu}_t = \bar{\mu}_t + K_t^i (z_t^i - \hat{z}_t^i)$ 
19:      $\bar{\Sigma}_t = (I - K_t^i H_t^i) \bar{\Sigma}_t$ 
20:   endfor
21:    $\mu_t = \bar{\mu}_t$ 
22:    $\Sigma_t = \bar{\Sigma}_t$ 
23:   return  $\mu_t, \Sigma_t$ 

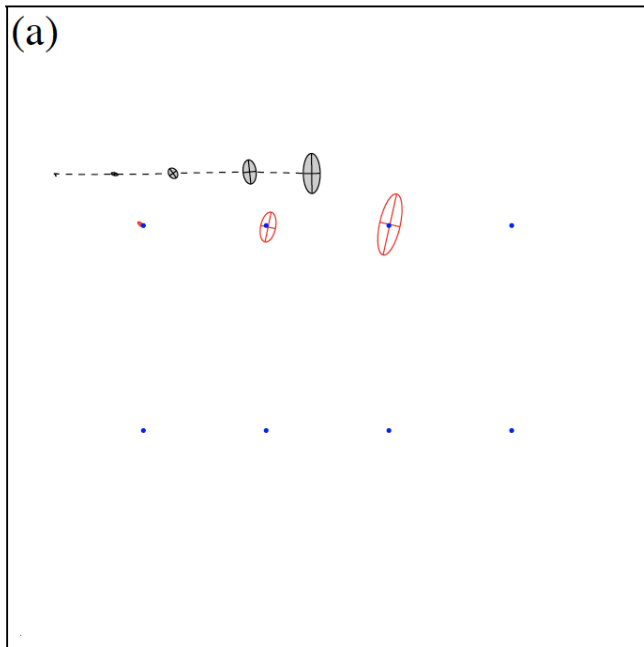
```

Table 10.1 The EKF algorithm for the SLAM problem (with known correspondences).

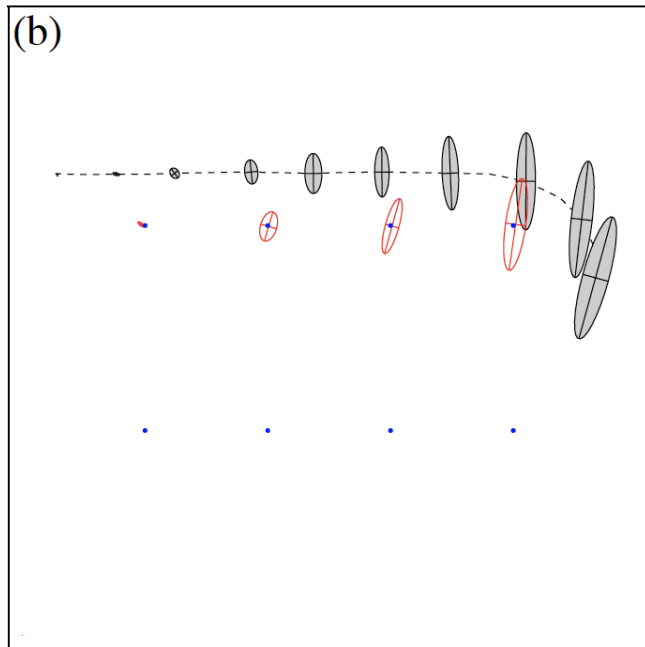
EKF-SLAM Intuition

- Kalman gain is fully populated for all state variables.
- Observing the landmark improves the robot's pose estimate, which in turn improves the estimates of other landmarks.

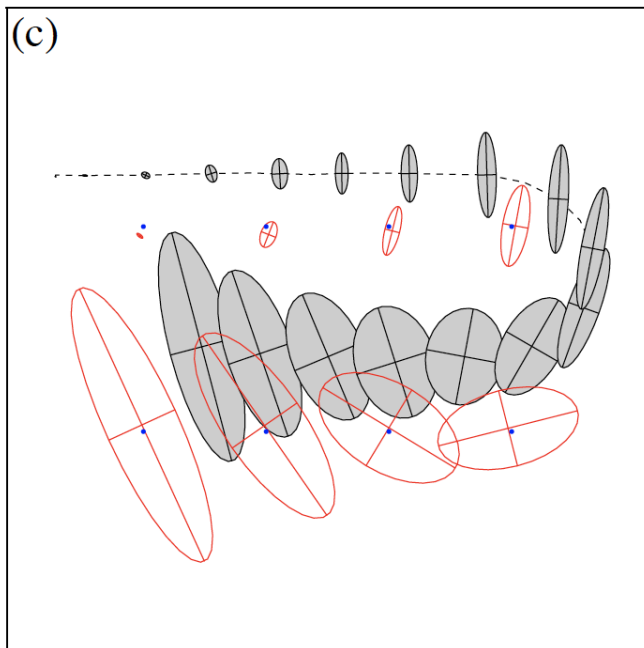
(a)



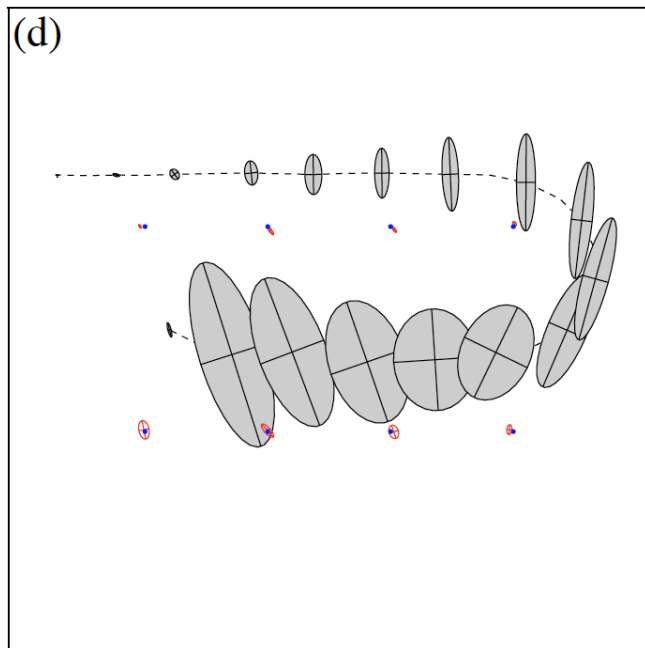
(b)



(c)



(d)



Explanation

- The position of the first landmark is well known
- Observing the first landmark again reduces uncertainty on the robot's pose
- This results in reduction of uncertainty in poses of other landmarks – from correlations represented by the non-diagonal elements of Σ_t

Explanation

- The position of the first landmark is well known
- Observing the first landmark again reduces uncertainty on the robot's pose
- This results in reduction of uncertainty in poses of other landmarks – from correlations represented by the non-diagonal elements of Σ_t
- Why does the uncertainty on previous robot poses not decrease?