

Motion and Sensor Models

CSCI 545 Introduction to Robotics
Instructor: Stefanos Nikolaidis

Motion Model

- What is the transition function?

$$P(x_t | x_{t-1}, u_t)$$

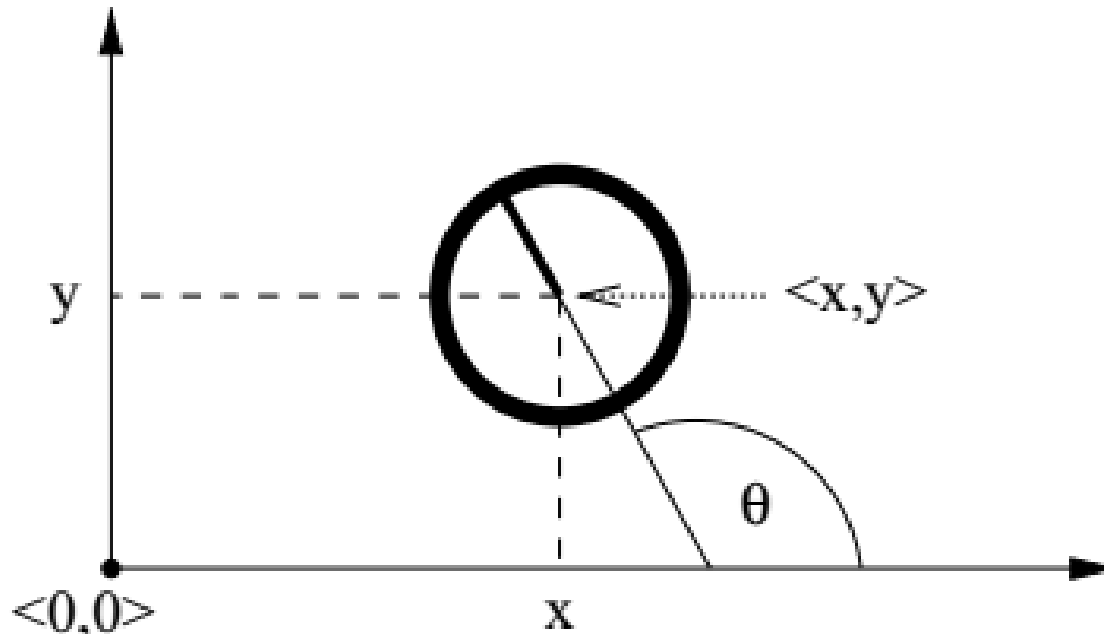
2D Mobile Robot



Motion Model

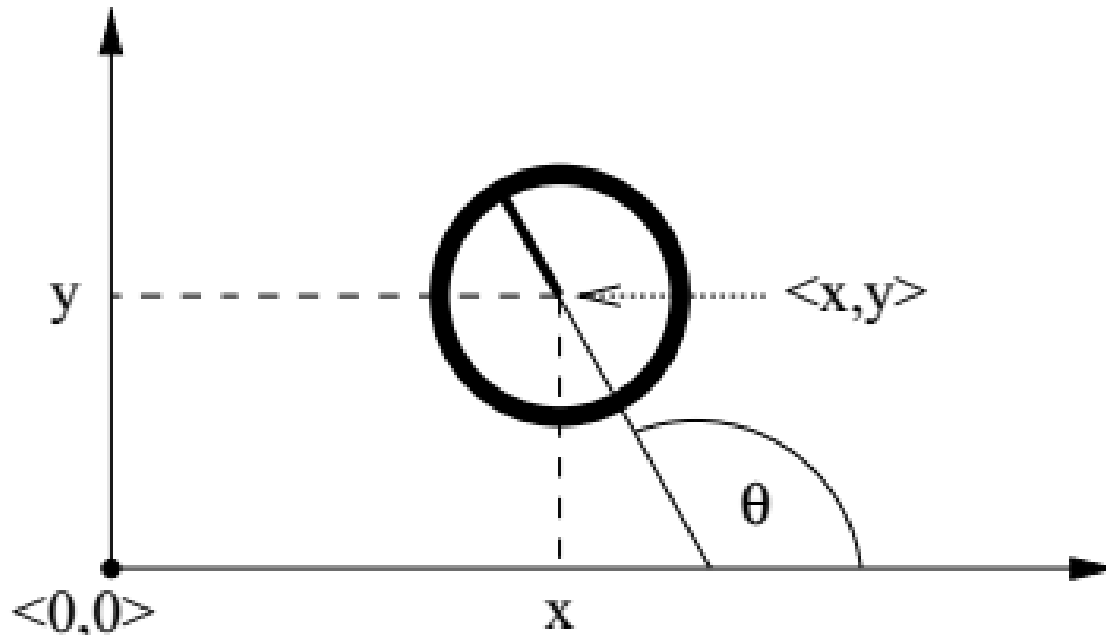
- What is the transition function?

$$P(x_t | x_{t-1}, u_t)$$



Robot Configuration

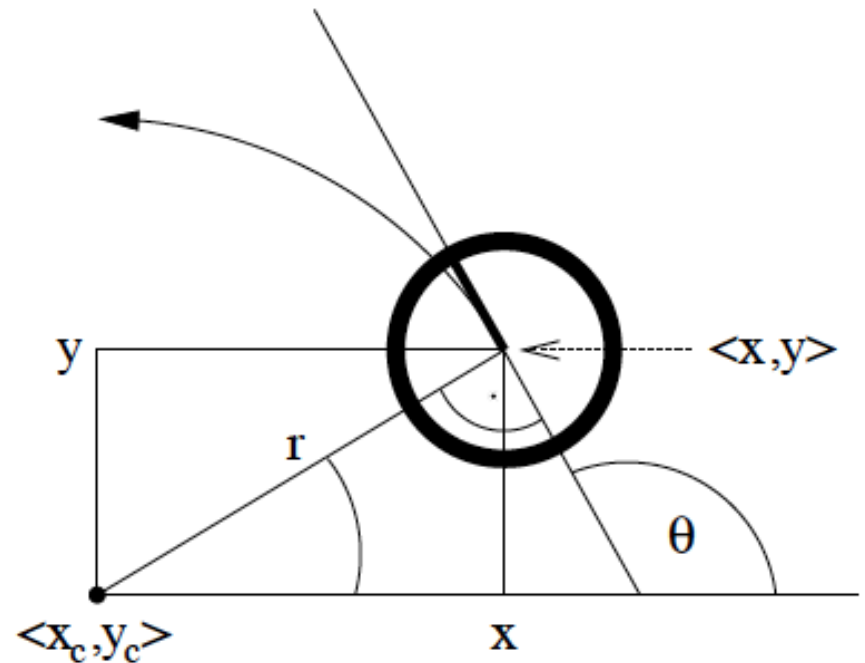
$$(x, y, \theta)$$



Velocity Motion Model

- We can assume that we provide two inputs, a translational and a rotational velocity

$$u_t = \begin{bmatrix} v_t \\ \omega_t \end{bmatrix}$$

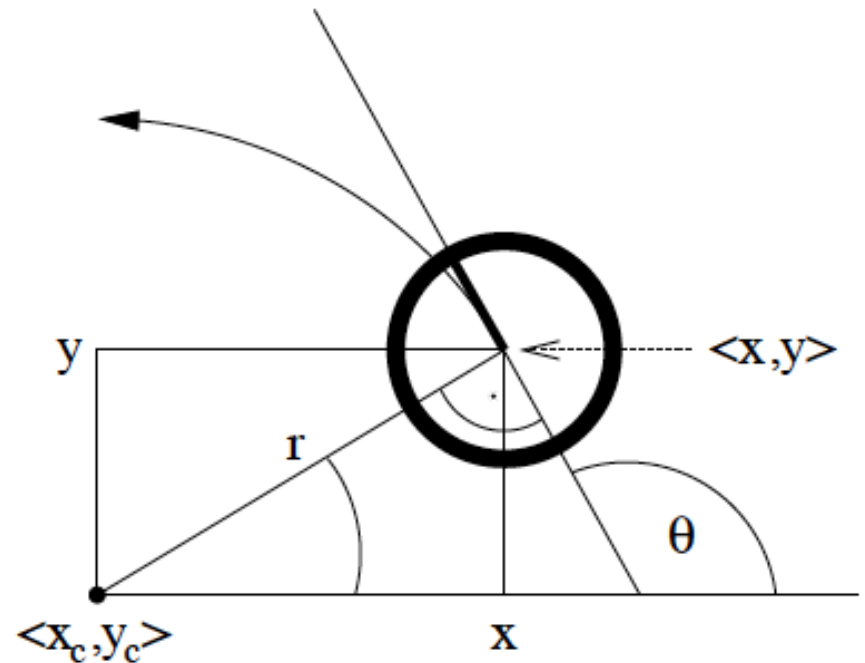


Velocity Motion Model

- We can assume that we provide two inputs, a translational and a rotational velocity

$$u_t = \begin{bmatrix} v_t \\ \omega_t \end{bmatrix}$$

$$r_t = \frac{v_t}{\omega_t}$$



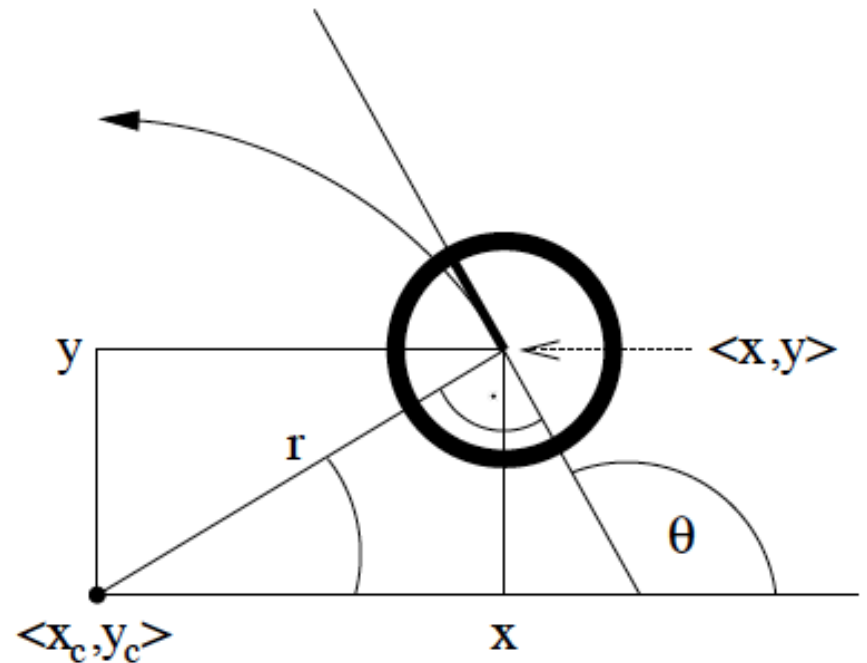
Velocity Motion Model

- We can assume that we provide two inputs, a translational and a rotational velocity

$$u_t = \begin{bmatrix} v_t \\ \omega_t \end{bmatrix}$$

$$r_t = \frac{v_t}{\omega_t}$$

$$(v_t = \omega_t \cdot r_t)$$



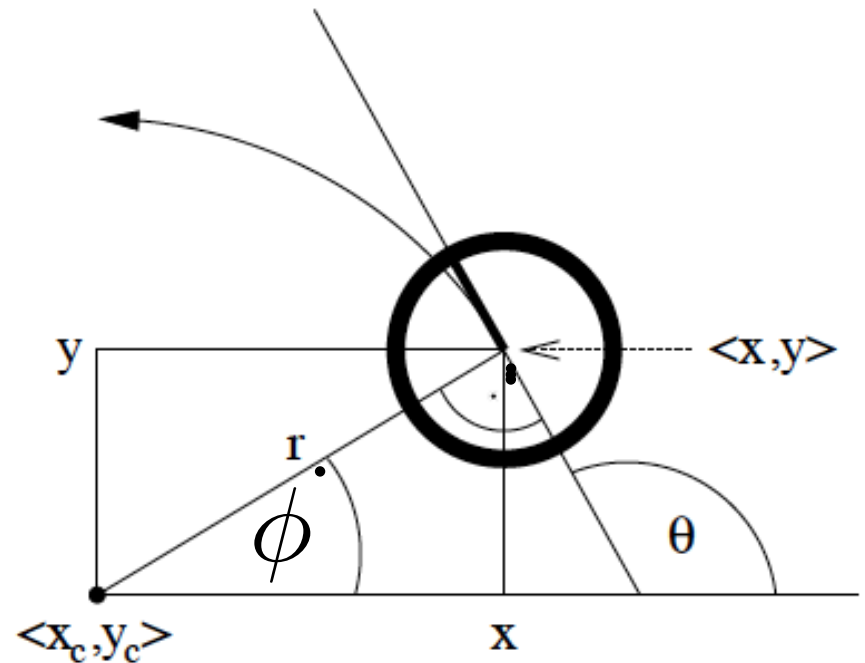
Velocity Motion Model

- We can assume that we provide two inputs, a translational and a rotational velocity

$$u_t = \begin{bmatrix} v_t \\ \omega_t \end{bmatrix}$$

$$\pi - \theta + \frac{\pi}{2} + \phi = \pi$$

$$\phi = \theta - \frac{\pi}{2}$$

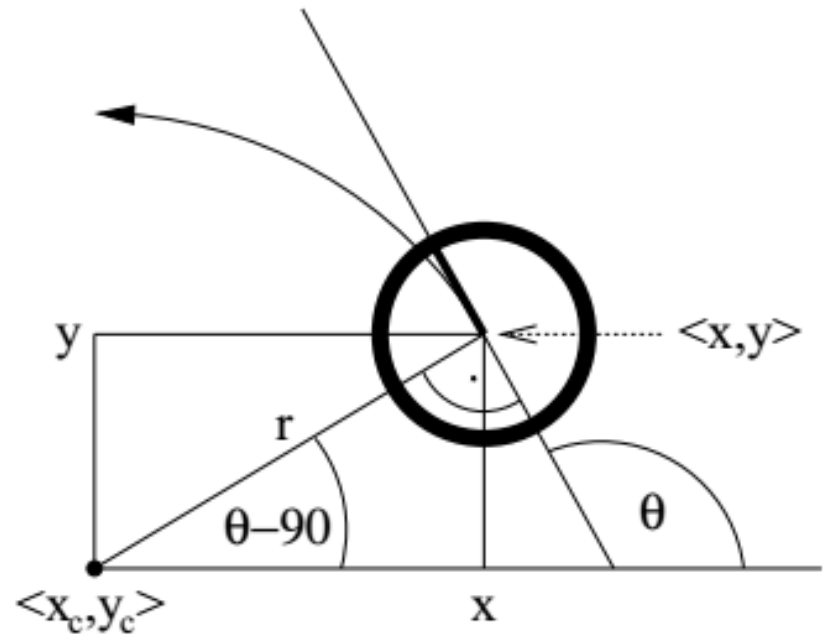


Velocity Motion Model

- We can assume that we provide two inputs, a translational and a rotational velocity

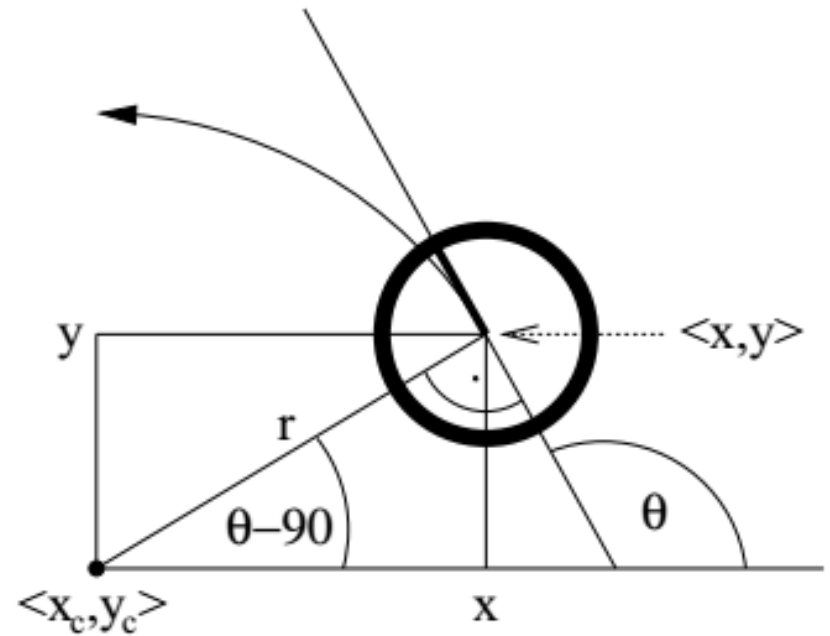
$$u_t = \begin{bmatrix} v_t \\ \omega_t \end{bmatrix}$$

$$r_t = \frac{v_t}{\omega_t}$$



Velocity Motion Model

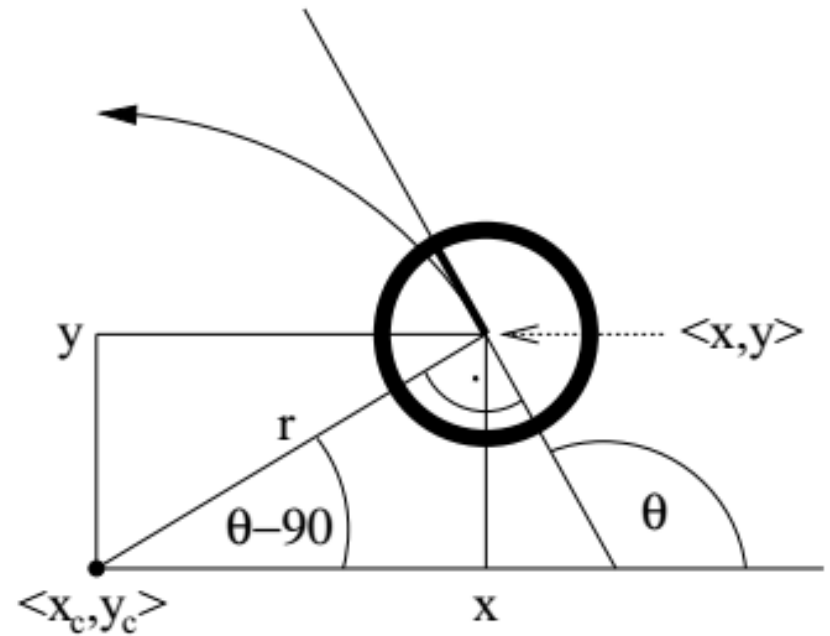
$$r_t = \frac{v_t}{\omega_t}$$



Velocity Motion Model

$$r_t = \frac{v_t}{\omega_t}$$

$$x_t = x_c + r_t \cos(\theta_t - \pi/2)$$

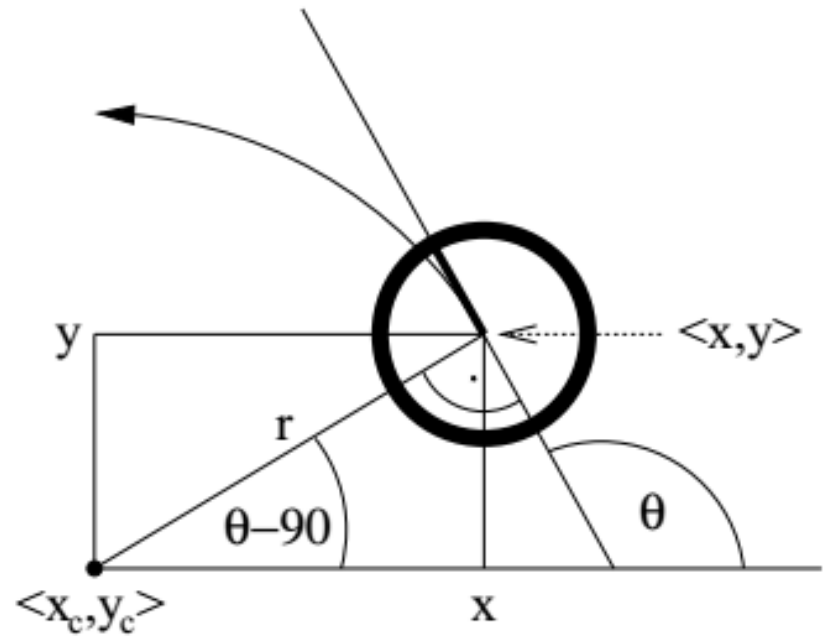


Velocity Motion Model

$$r_t = \frac{v_t}{\omega_t}$$

$$x_t = x_c + r_t \cos(\theta_t - \pi/2)$$

$$x_t = x_c + r_t \sin(\theta_t)$$



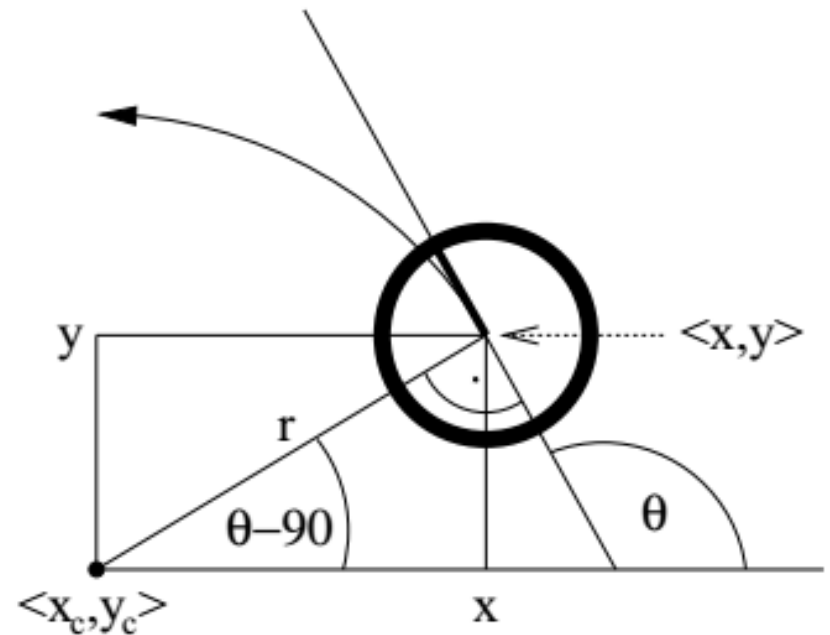
Velocity Motion Model

$$r_t = \frac{v_t}{\omega_t}$$

$$x_t = x_c + r_t \cos(\theta_t - \pi/2)$$

$$x_t = x_c + r_t \sin(\theta_t)$$

$$x_c = x_t - r_t \sin(\theta_t)$$



Velocity Motion Model

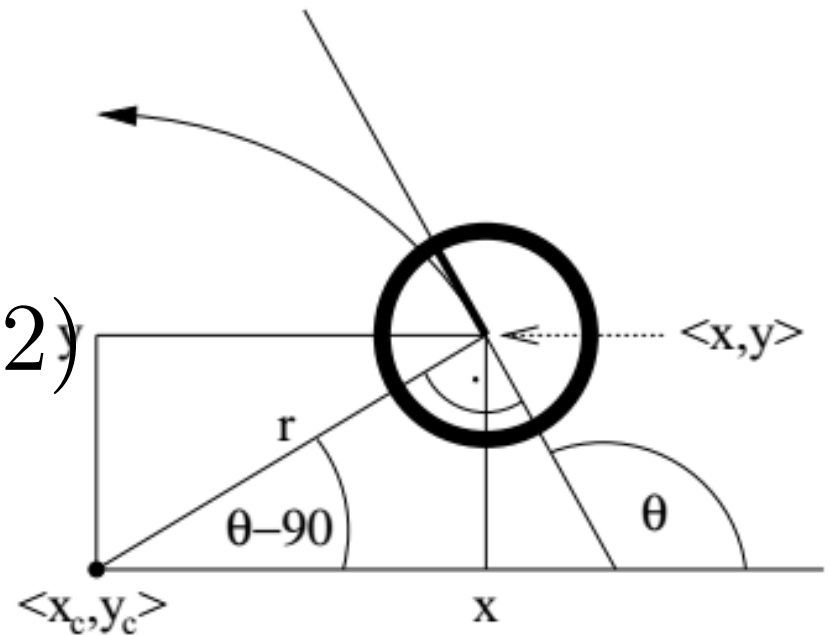
$$r_t = \frac{v_t}{\omega_t}$$

$$x_t = x_c + r_t \cos(\theta_t - \pi/2)$$

$$x_t = x_c + r_t \sin(\theta_t)$$

$$x_c = x_t - r_t \sin(\theta_t)$$

$$y_t = y_c + r_t \sin(\theta_t - \pi/2)$$



Velocity Motion Model

$$r_t = \frac{v_t}{\omega_t}$$

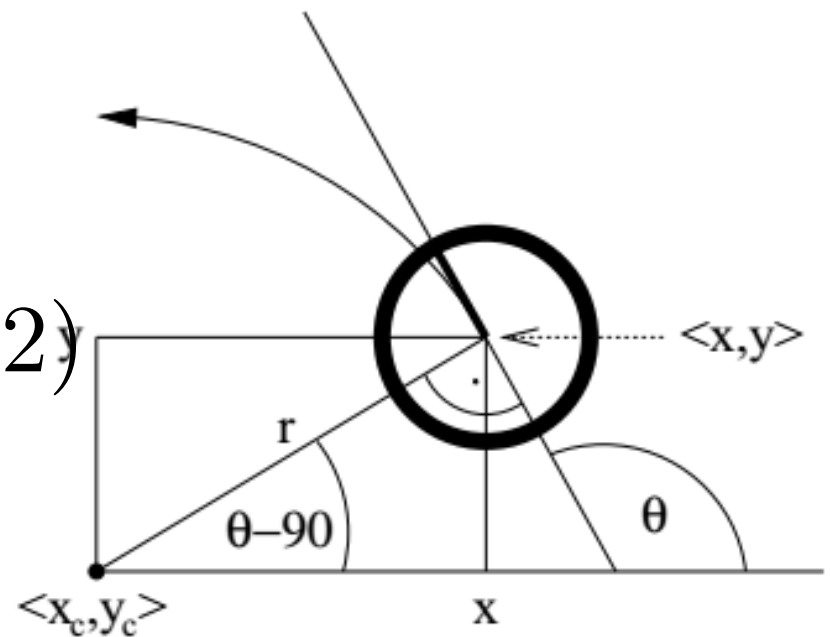
$$x_t = x_c + r_t \cos(\theta_t - \pi/2)$$

$$x_t = x_c + r_t \sin(\theta_t)$$

$$x_c = x_t - r_t \sin(\theta_t)$$

$$y_t = y_c + r_t \sin(\theta_t - \pi/2)$$

$$y_t = y_c - r_t \cos(\theta_t)$$



Velocity Motion Model

$$r_t = \frac{v_t}{\omega_t}$$

$$x_t = x_c + r_t \cos(\theta_t - \pi/2)$$

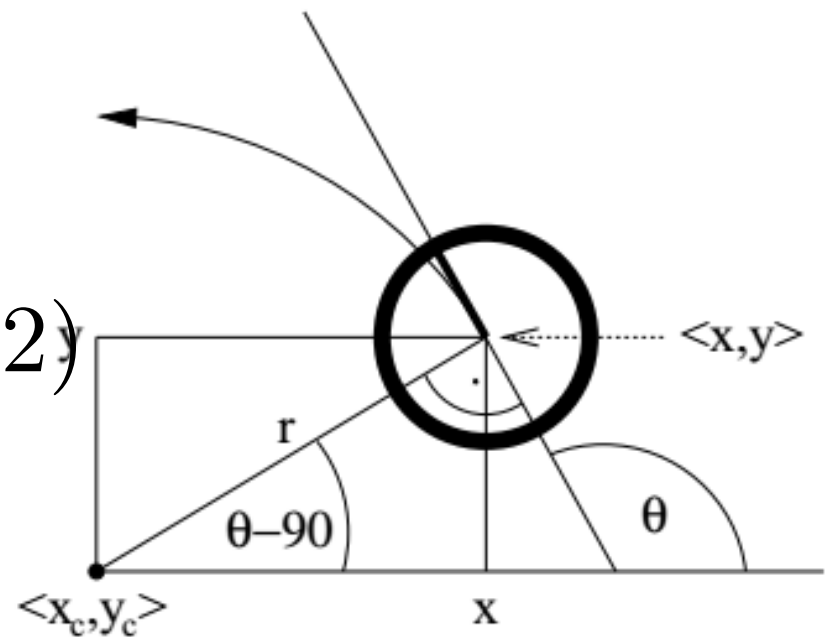
$$x_t = x_c + r_t \sin(\theta_t)$$

$$x_c = x_t - r_t \sin(\theta_t)$$

$$y_t = y_c + r_t \sin(\theta_t - \pi/2)$$

$$y_t = y_c - r_t \cos(\theta_t)$$

$$y_c = y_t + r_t \cos(\theta_t)$$

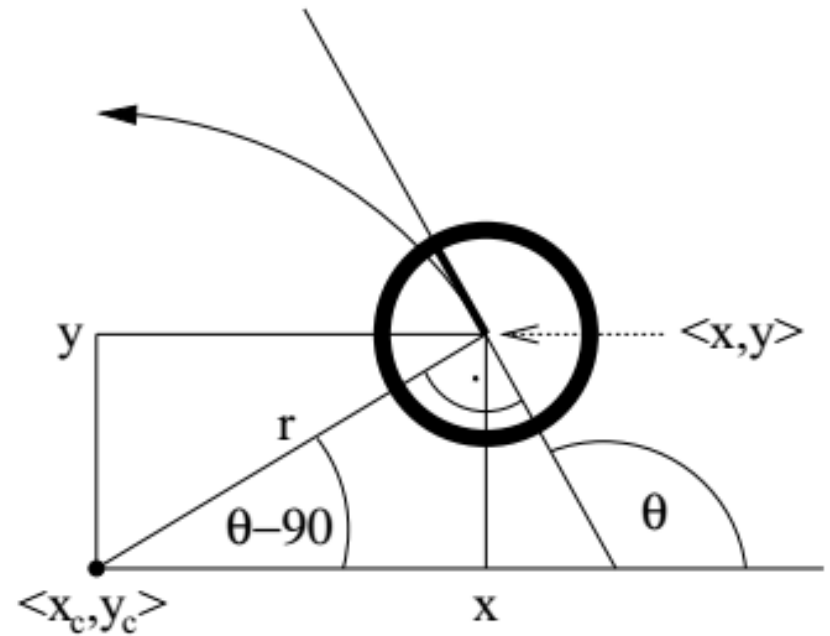


Velocity Motion Model

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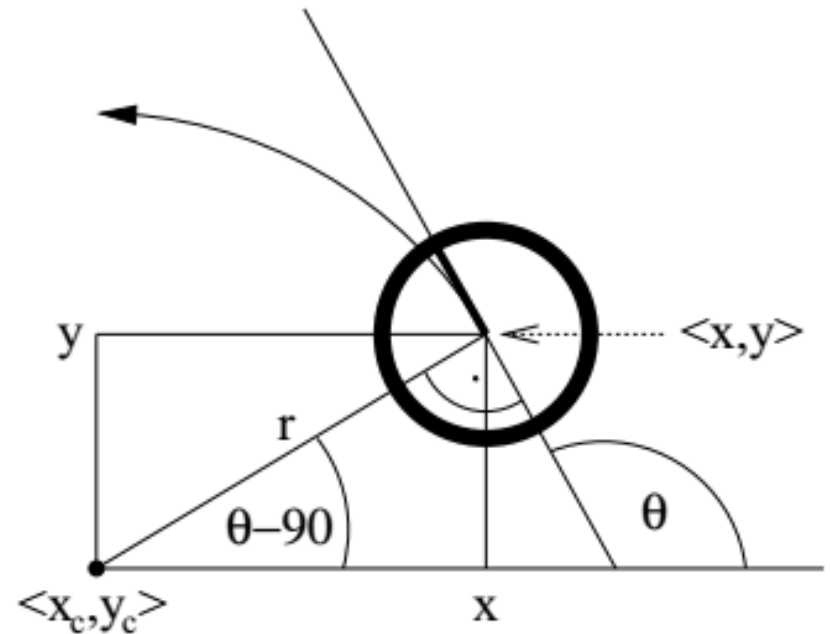
Velocity Motion Model

$$r_t = \frac{v_t}{\omega_t}$$

$$x_c = x_t - r_t \sin(\theta_t)$$

$$y_c = y_t + r_t \cos(\theta_t)$$

$$\theta_t = \theta_{t-1} + \omega_t * \Delta_t$$



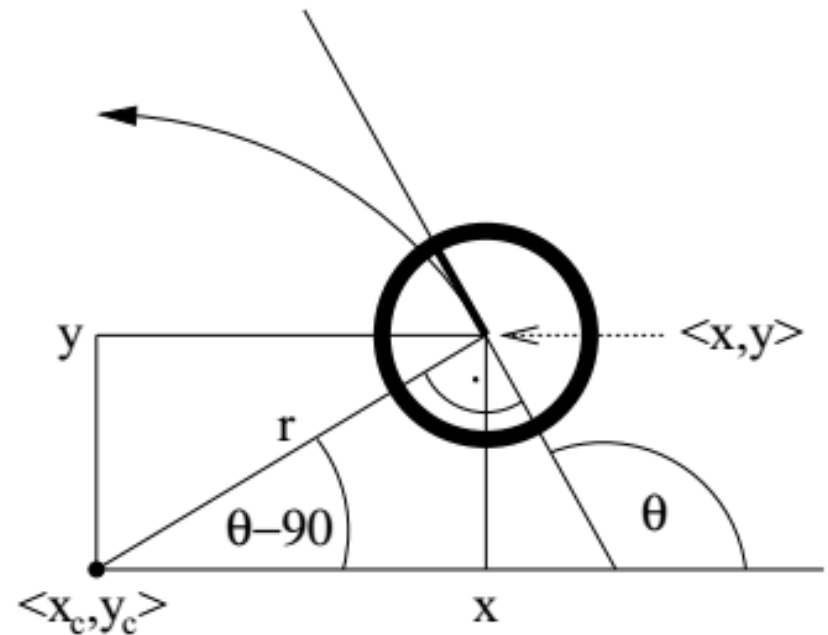
Velocity Motion Model

$$r_t = \frac{v_t}{\omega_t}$$

$$x_c = x_t - r_t \sin(\theta_t)$$

$$y_c = y_t + r_t \cos(\theta_t)$$

$$\theta_t = \theta_{t-1} + \omega_t * \Delta_t$$



Velocity Motion Model

$$r_t = \frac{v_t}{\omega_t}$$

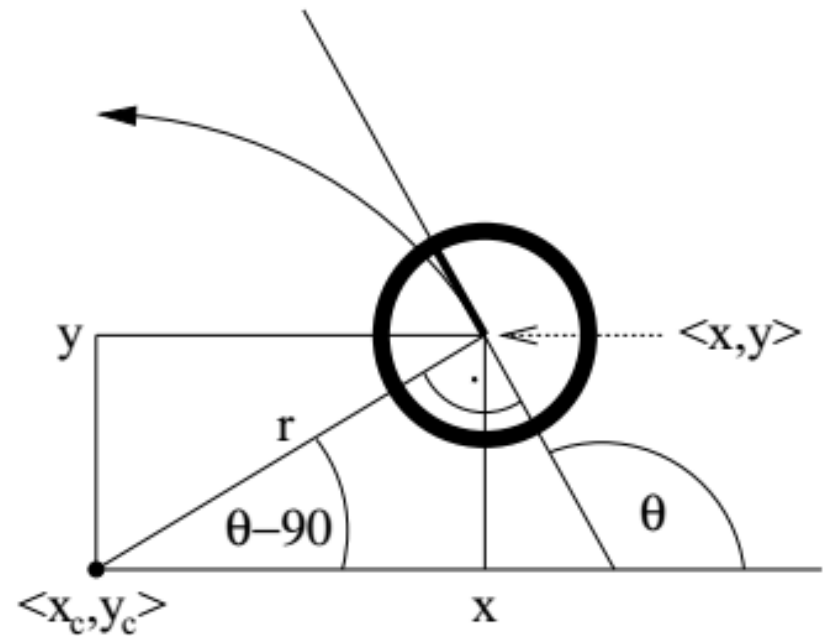
$$x_c = x_t - r_t \sin(\theta_t)$$

$$y_c = y_t + r_t \cos(\theta_t)$$

$$\theta_t = \theta_{t-1} + \omega_t * \Delta t$$

$$x_t = x_c + r_t \sin(\theta_t)$$

$$y_t = y_c - r_t \cos(\theta_t)$$



Velocity Motion Model

$$r_t = \frac{v_t}{\omega_t}$$

$$x_c = x_t - r_t \sin(\theta_t)$$

$$y_c = y_t + r_t \cos(\theta_t)$$

$$\theta_t = \theta_{t-1} + \omega_t * \Delta_t$$

$$x_t = x_c + r_t \sin(\theta_{t-1} + \omega_t * \Delta_t)$$

$$y_t = y_c - r_t \cos(\theta_{t-1} + \omega_t * \Delta_t)$$

Velocity Motion Model

$$r_t = \frac{v_t}{\omega_t}$$

$$x_c = x_t - r_t \sin(\theta_t)$$

$$y_c = y_t + r_t \cos(\theta_t)$$

$$\theta_t = \theta_{t-1} + \omega_t * \Delta_t$$

$$x_t = x_c + r_t \sin(\theta_{t-1} + \omega_t * \Delta_t)$$

$$y_t = y_c - r_t \cos(\theta_{t-1} + \omega_t * \Delta_t)$$

Velocity Motion Model

$$r_t = \frac{v_t}{\omega_t}$$

$$x_c = x_t - r_t \sin(\theta_t)$$

$$y_c = y_t + r_t \cos(\theta_t)$$

$$\theta_t = \theta_{t-1} + \omega_t * \Delta_t$$

$$x_t = x_{t-1} - r_t \sin(\theta_{t-1}) + r_t \sin(\theta_{t-1} + \omega_t * \Delta_t)$$

$$y_t = y_{t-1} + r_t \cos(\theta_{t-1}) - r_t \cos(\theta_{t-1} + \omega_t * \Delta_t)$$

Velocity Motion Model

$$\begin{pmatrix} \hat{v}_t \\ \hat{\omega}_t \end{pmatrix} = \begin{pmatrix} v_t \\ \omega_t \end{pmatrix} + \begin{pmatrix} \epsilon_{a_1 |v_t| + a_2 |\omega_t|} \\ \epsilon_{a_3 |v_t| + a_4 |\omega_t|} \end{pmatrix}$$

$$\theta_t = \theta_{t-1} + \hat{\omega}_t * \Delta_t$$

$$x_t = x_{t-1} - \frac{\hat{v}_t}{\hat{\omega}_t} \sin(\theta_{t-1}) + \frac{\hat{v}_t}{\hat{\omega}_t} \sin(\theta_{t-1} + \hat{\omega}_t * \Delta_t)$$

$$y_t = y_{t-1} + \frac{\hat{v}_t}{\hat{\omega}_t} \cos(\theta_{t-1}) - \frac{\hat{v}_t}{\hat{\omega}_t} \cos(\theta_{t-1} + \hat{\omega}_t * \Delta_t)$$

Velocity Motion Model

$$\begin{pmatrix} \hat{v}_t \\ \hat{\omega}_t \end{pmatrix} = \begin{pmatrix} v_t \\ \omega_t \end{pmatrix} + \begin{pmatrix} \epsilon a_1 |v_t| + a_2 |\omega_t| \\ \epsilon a_3 |v_t| + a_4 |\omega_t| \end{pmatrix}$$

$$\theta_t = \theta_{t-1} + \hat{\omega}_t * \Delta_t$$

$$x_t = x_{t-1} - \frac{\hat{v}_t}{\hat{\omega}_t} \sin(\theta_{t-1}) + \frac{\hat{v}_t}{\hat{\omega}_t} \sin(\theta_{t-1} + \hat{\omega}_t * \Delta_t)$$

$$y_t = y_{t-1} + \frac{\hat{v}_t}{\hat{\omega}_t} \cos(\theta_{t-1}) - \frac{\hat{v}_t}{\hat{\omega}_t} \cos(\theta_{t-1} + \hat{\omega}_t * \Delta_t)$$

Velocity Motion Model

1: **Algorithm** `sample_motion_model_velocity`(u_t, x_{t-1}):

2: $\hat{v} = v + \text{sample}(\alpha_1|v| + \alpha_2|\omega|)$

3: $\hat{\omega} = \omega + \text{sample}(\alpha_3|v| + \alpha_4|\omega|)$

4: $\hat{\gamma} = \text{sample}(\alpha_5|v| + \alpha_6|\omega|)$

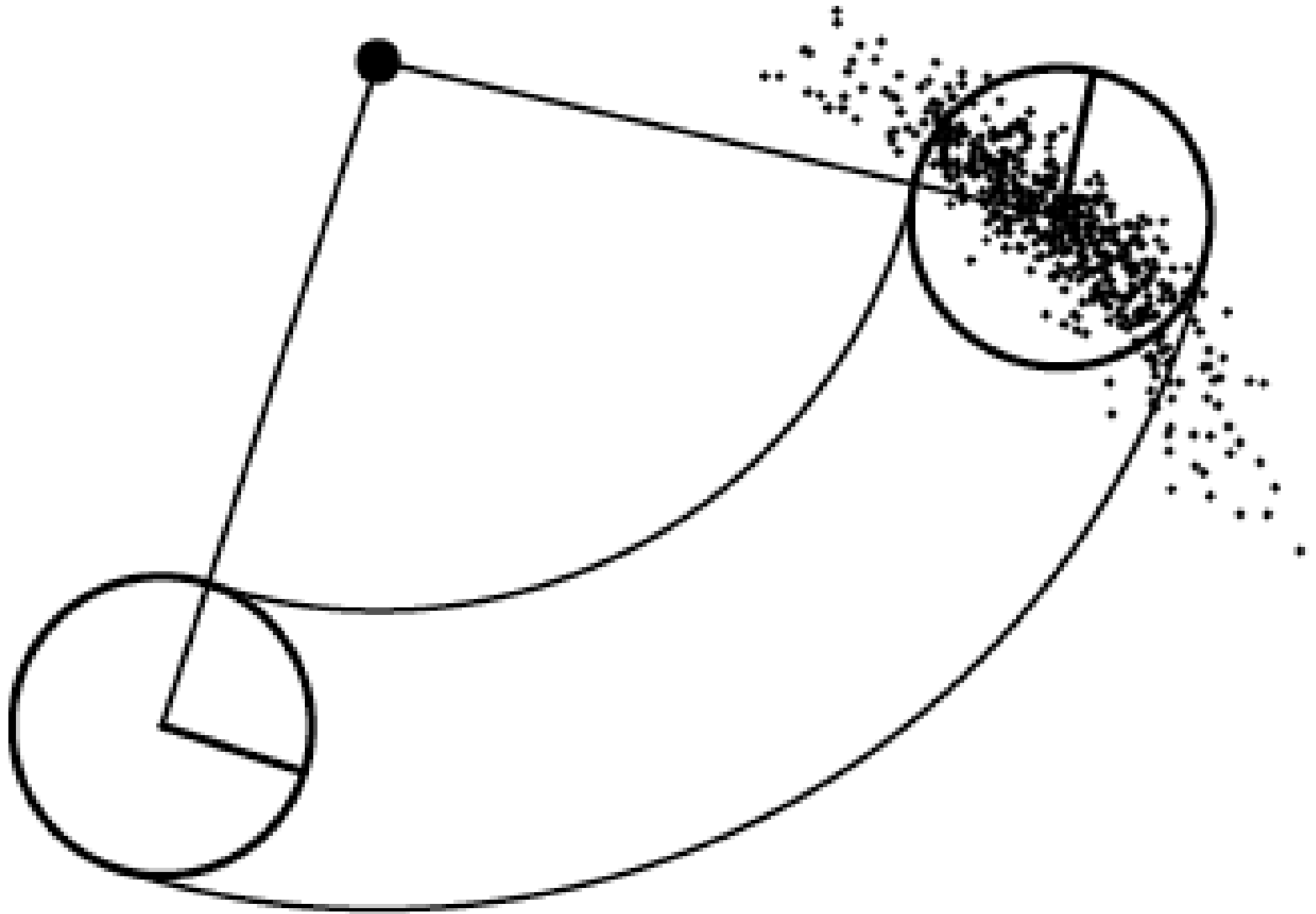
5: $x' = x - \frac{\hat{v}}{\hat{\omega}} \sin \theta + \frac{\hat{v}}{\hat{\omega}} \sin(\theta + \hat{\omega} \Delta t)$

6: $y' = y + \frac{\hat{v}}{\hat{\omega}} \cos \theta - \frac{\hat{v}}{\hat{\omega}} \cos(\theta + \hat{\omega} \Delta t)$

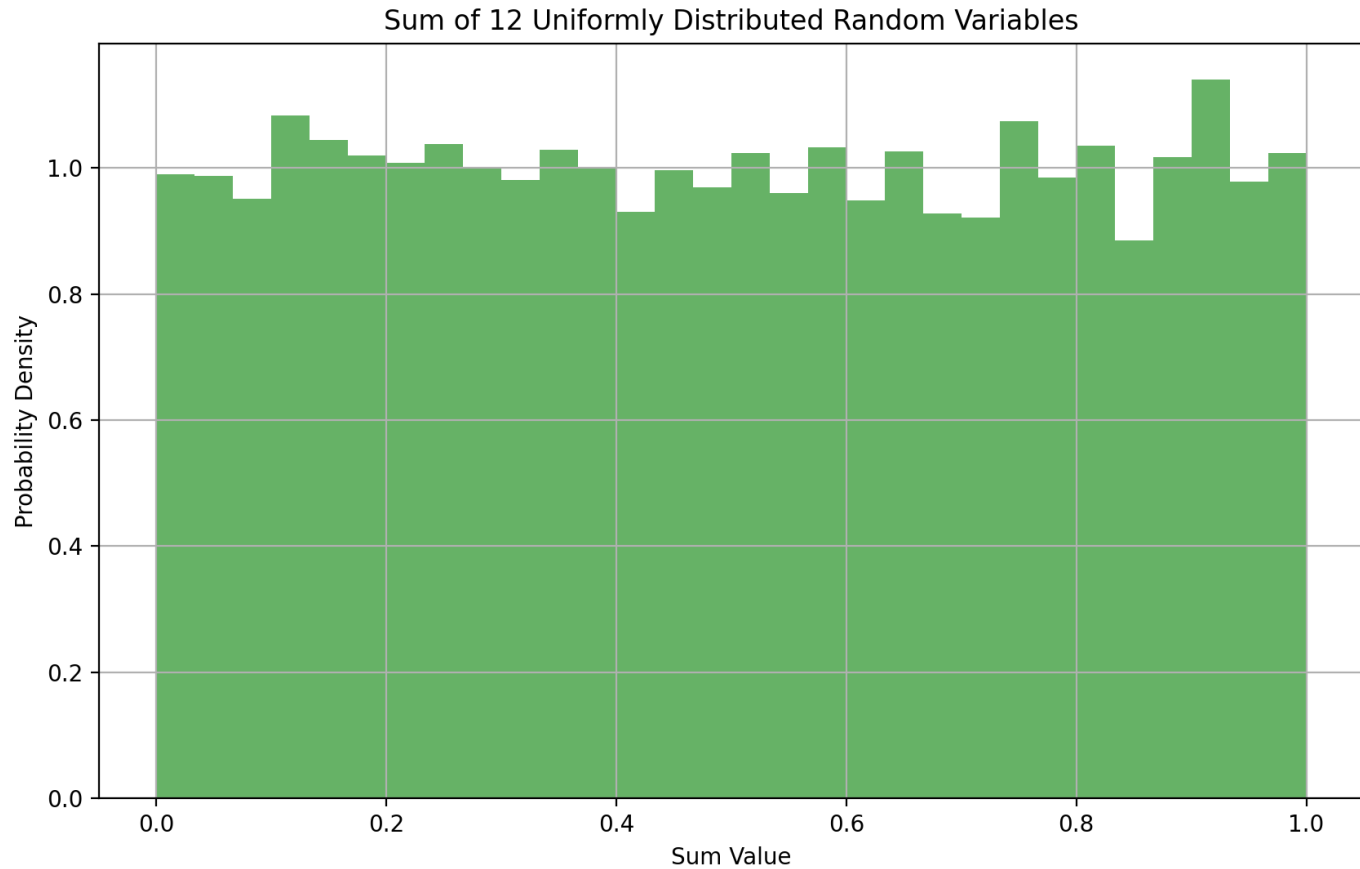
7: $\theta' = \theta + \hat{\omega} \Delta t + \hat{\gamma} \Delta t$

8: return $x_t = (x', y', \theta')^T$

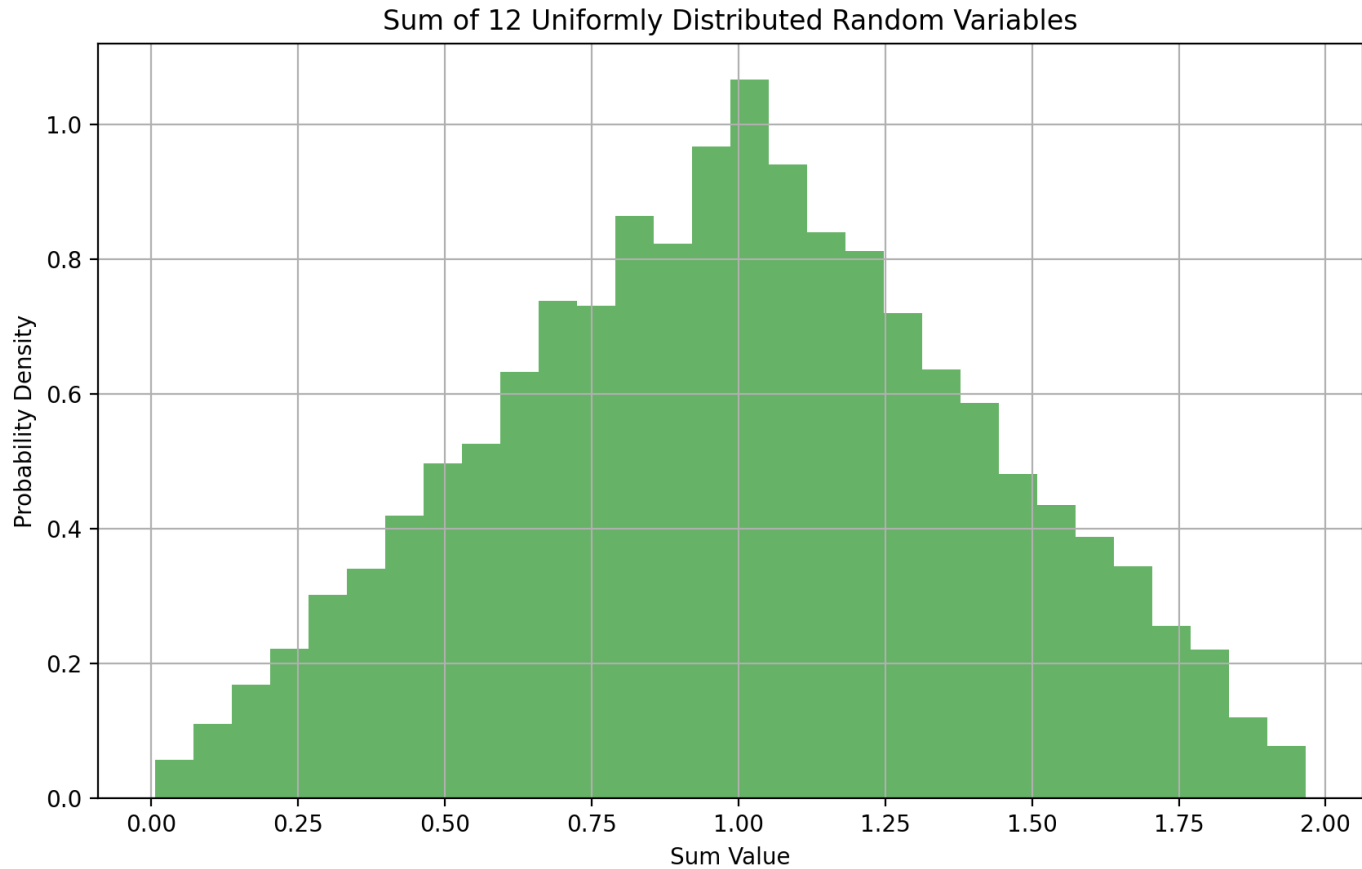
Velocity Motion Model



Sampling

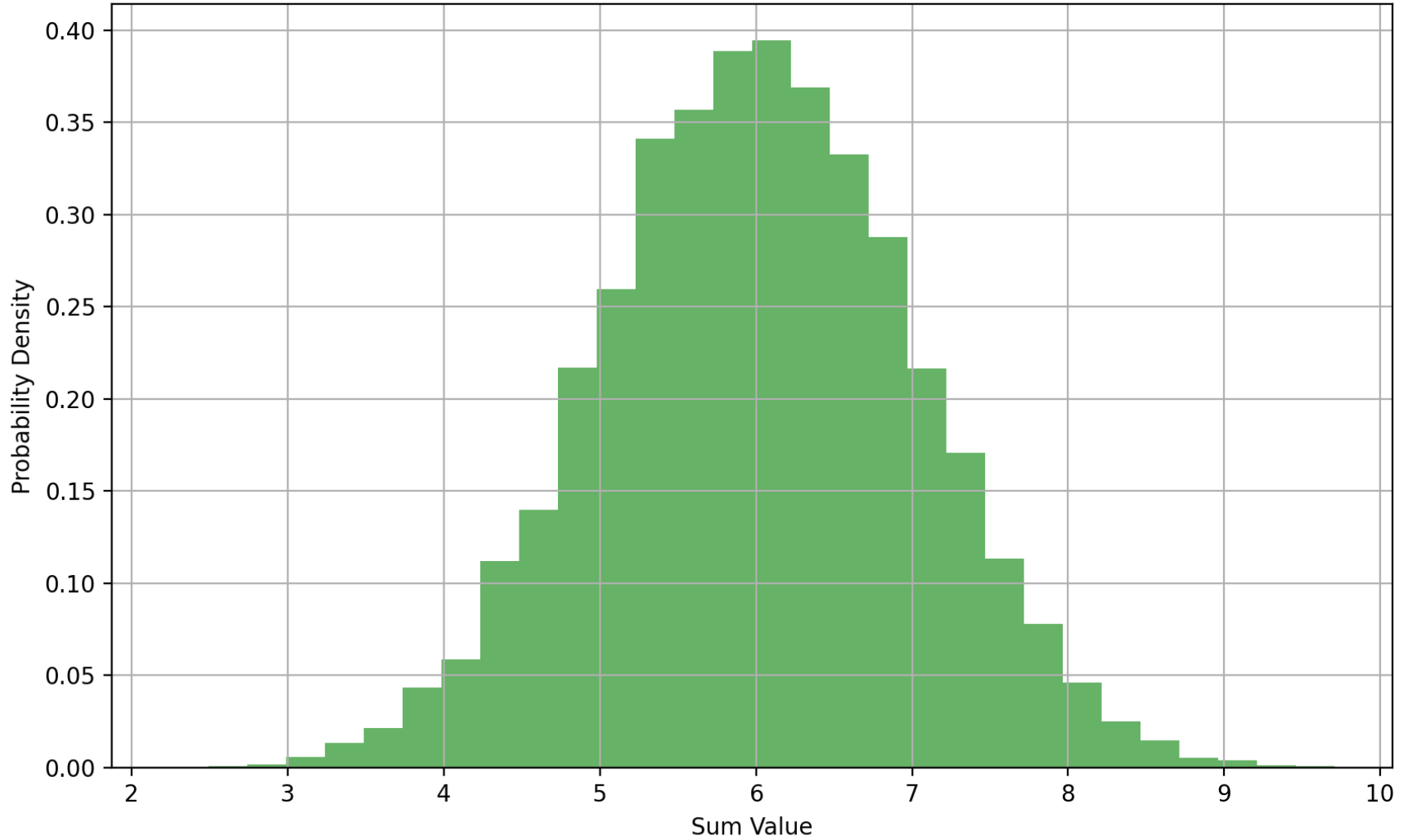


Sampling



Sampling

Sum of 12 Uniformly Distributed Random Variables

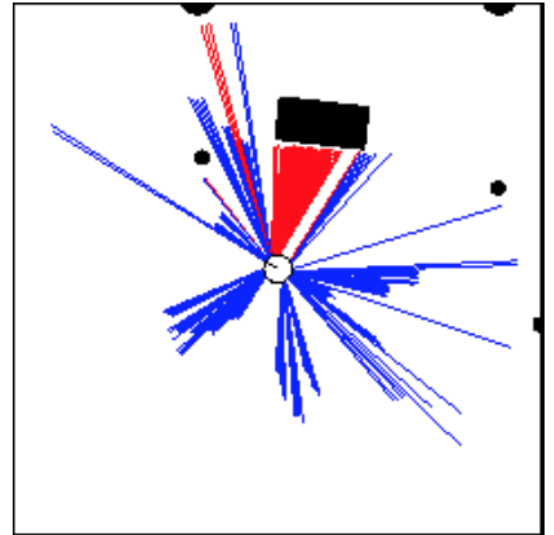
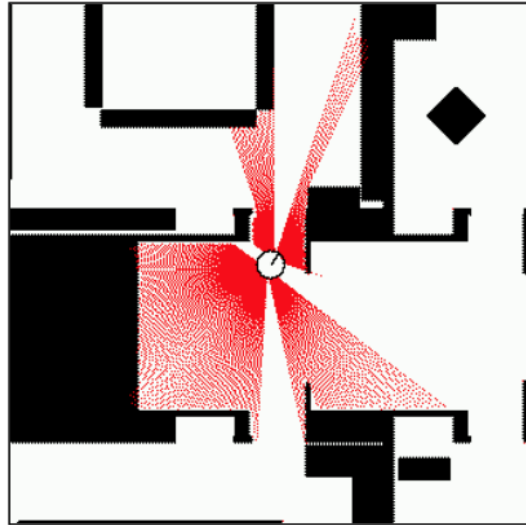
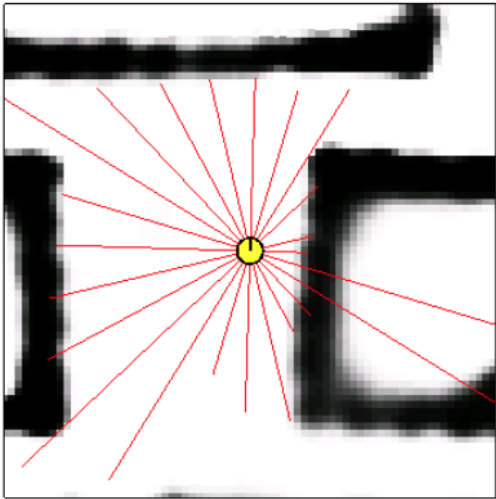


Sensor Model

- What is the observation function?

$$P(z_t | x_t)$$

LiDAR Measurements



Measurements

$$z_t = \{z_t^1, z_t^2, \dots, z_t^k\}$$

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$$z_t = \{z_t^1, z_t^2, \dots, z_t^k\}$$

$$p(z_t|x_t)$$

Measurements

$$z_t = \{z_t^1, z_t^2, \dots, z_t^k\}$$

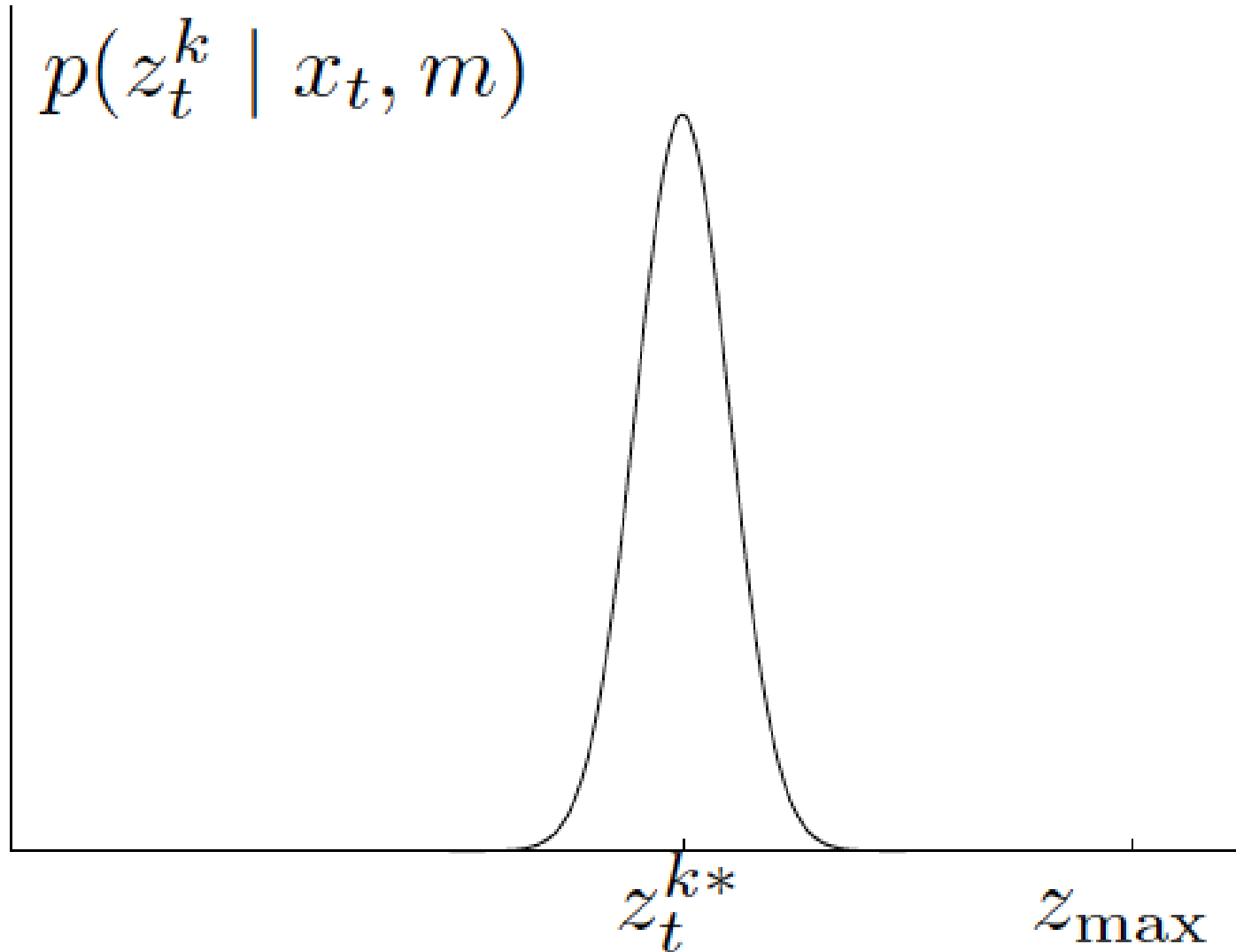
$$p(z_t | x_t, m)$$

Measurements

$$z_t = \{z_t^1, z_t^2, \dots, z_t^k\}$$

$$p(z_t|x_t, m) = p(z_t^1|x_t, m)p(z_t^2|x_t, m)\dots p(z_t^k|x_t, m)$$

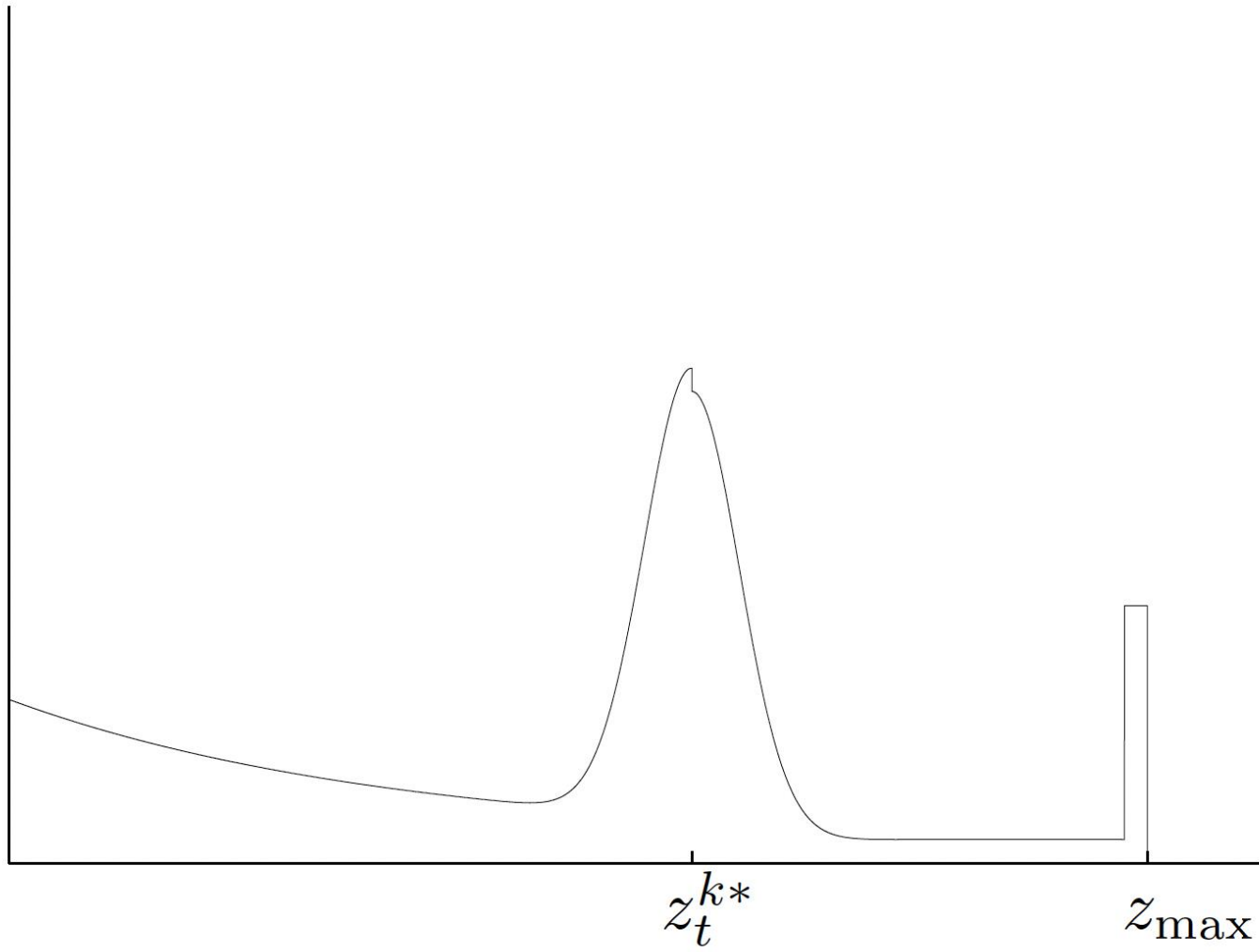
Sources of Error: Local Noise



Other Sources of Error:

- Unexpected Objects
- No Detection
- Random Errors

Sources of Error: Local Noise



Feature-Based Sensor Models



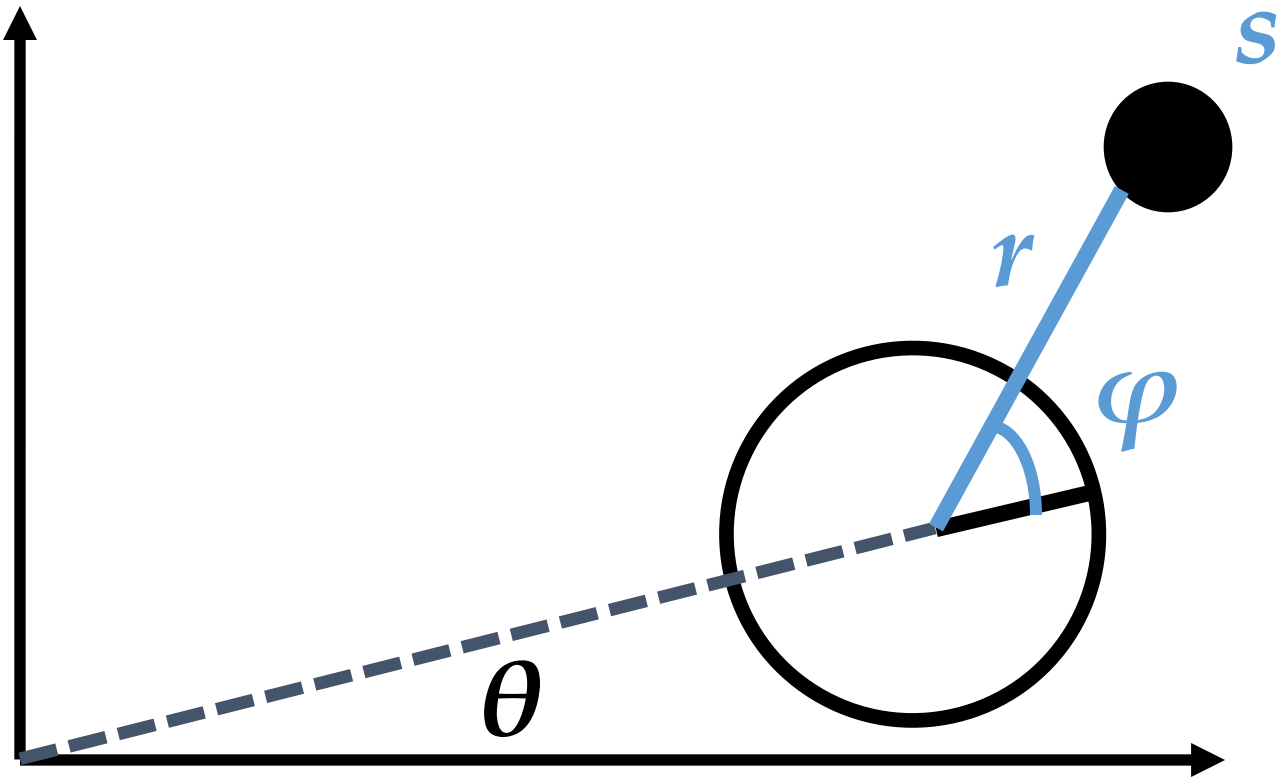
Fully Autonomous Robot Navigation based on a Prior Map without Artificial Landmarks

Feature-Based Sensor Models

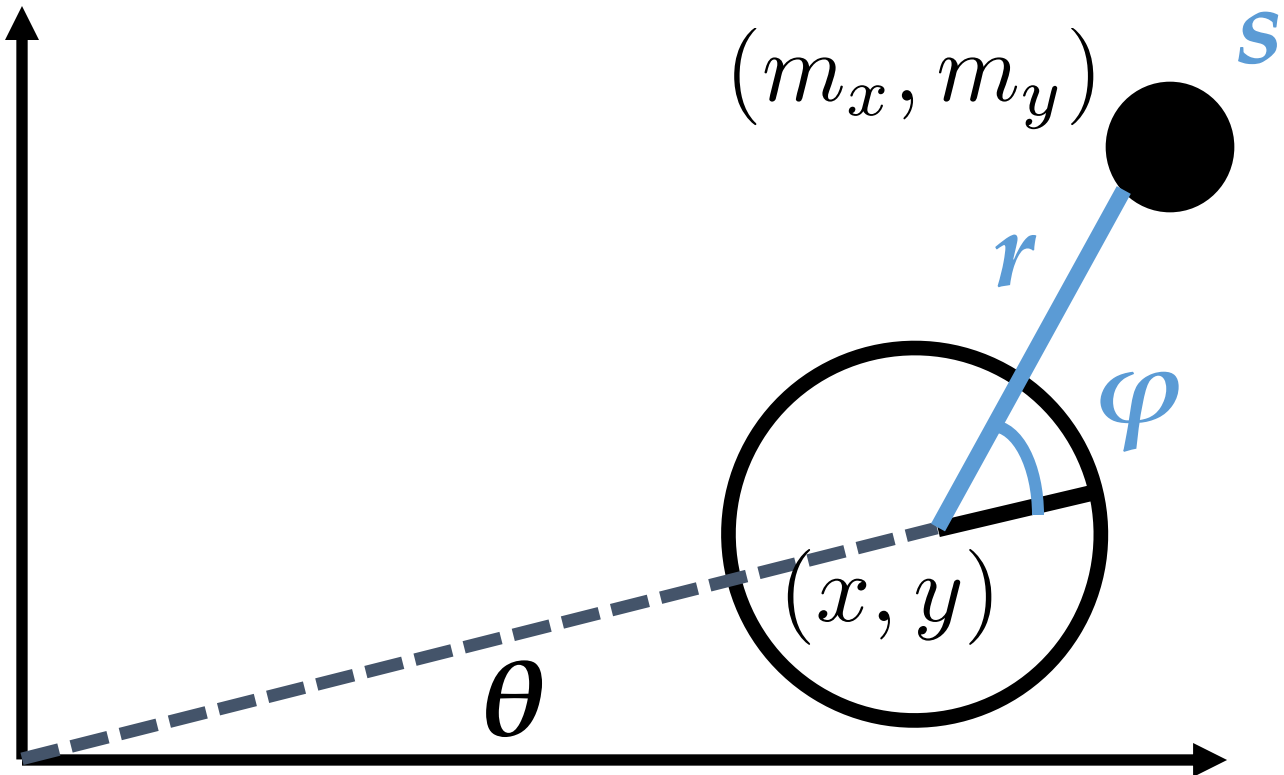
$$p(z_t | x_t, m) = p(z_t^1 | x_t, m) p(z_t^2 | x_t, m) \dots p(z_t^k | x_t, m)$$

$$p(z_t | x_t, m) = \prod_{i=1}^k p(r_t^i, \phi_t^i, s_t^i | x_t, m)$$

Feature-Based Sensor Models



Feature-Based Sensor Models



Feature-Based Sensor Models

$$\begin{pmatrix} r_t^i \\ \phi_t^i \\ s_t^i \end{pmatrix} = \begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \text{atan2}(m_{j,y} - y, m_{j,x} - x) - \theta \\ s_j \end{pmatrix} + \begin{pmatrix} \varepsilon_{\sigma_r^2} \\ \varepsilon_{\sigma_\phi^2} \\ \varepsilon_{\sigma_s^2} \end{pmatrix}$$

