HW1: Math Fundamentals

Due Date: 09/11, 11:59pm (PST)

1. Given the matrix
$$A = \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}$$
find the eigenvalues and eigenvectors. Do it by hand and confirm it with Python.

2. A matrix A is called symmetric iff $A^T = A$ and skew-symmetric iff $A^T = -A$. Prove that:
   (a) The sum of two symmetric matrices is a symmetric matrix
   (b) The sum of two skew-symmetric matrices is a skew-symmetric matrix

3. Express each of the following events in terms of the events A, B and C:
   (a) at least one of the events A, B, C occurs;
   (b) at most one of the events A, B, C occurs;
   (c) none of the events A, B, C occurs;
   (d) all three events A, B, C occur;
   (e) exactly one of the events A, B, C occurs;
   (f) events A and B occur, but not C;

   In each case draw the corresponding Venn diagrams.

4. Give one example in *robotics* where the Markov assumption is used (correctly or not). Is the assumption valid or not? Please explain in no more than three sentences.

5. Give one example in *the real world* where the Markov assumption is used (correctly or not). Is the assumption valid or not? Please explain in no more than three sentences.

6. Determine the value of α such that

$$f_X(x) = \frac{\alpha}{e^x + e^{-x}}$$

is a *valid* probability density function.
Hint:

$$\int \frac{1}{e^x + e^{-x}} = \text{arctan}(e^x) + C$$

7. Two random vectors $\mathbf{x_1}$ are said to be uncorrelated if:

$$P = E\{(\mathbf{x_1} - \bar{\mathbf{x}}_\mathbf{1})(\mathbf{x_2} - \bar{\mathbf{x}}_\mathbf{2})^T\} = 0$$

Show that

(a) Independent random vectors are uncorrelated.
Hint: Using the definition of independence, we have:

$$E[\mathbf{x_1}\mathbf{x_2}^T] = E[\mathbf{x_1}]E[\mathbf{x_2}]^T$$

(b) Uncorrelated jointly Gaussian random vectors are independent.
Hint: The off-diagonals of the covariance matrix are 0 if $\mathbf{x_1}$ and $\mathbf{x_2}$ are uncorrelated.